

Gas-hydrodynamics of gas-liquid flow in the reservoir-well system

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Abstract – A hydrodynamic model of gas-liquid flow in the reservoir-well system is developed and the boundary problem is solved.

As the volume of liquid in the mixture is very small (5-10%), its effect on the gas-liquid mixture flow was considered only in terms of density change effect. It is assumed that liquid and gas particles move in the pipe at the same speed and there is no mass transfer.

The gas is assumed to be ideal and an isothermal process is considered.

The solution of some equations in initial and boundary conditions is found and the continuity condition at the well bottom is written, $P_c(t)$ - bottom hole pressure is found. The borehole productivity is then determined by setting $P_{q,a}(t)$ (taken from experiment).

The density ρ of the gas-liquid mixture is determined by the following formula:

$$\rho_{mixture} = \frac{\rho_{atm}(1 + \eta)}{P_{atm}} P \quad (1)$$

The following initial and boundary conditions are known during percolation of the gas-liquid mixture in the reservoir:

$$P|_{t=0} = P_k, r = R_k \quad (2)$$

$$P|_{r=r_c} = P_c(t) \quad (3)$$

$$\frac{\partial P}{\partial r}|_{r=R_k} = 0 \quad (4)$$

The problem was resolved on the basis of material balance.

The mass of the gas-liquid mixture in the layer at any instant can be found as:

$$G_0 = 2\pi hm \int_{r_c}^{R_k} \rho_{mixture} r dr \quad (5)$$

If we consider formula (1) in formula (2), we get:

$$G_0 = 2\pi hm \frac{(1 + \eta)\rho_{atm}}{P_{atm}} \int_{r_c}^{R_k} P r dr \quad (6)$$

Let us choose the pressure according to the boundary conditions (3) and (4) as follows:

$$P = P_c(t) + A(t)f(r) \quad (7)$$

We can choose the function $f(r)$ satisfying the boundary conditions (3) and (4) as follows:

$$f(r) = \ln \frac{r}{r_c} - \frac{r}{R_k} + \frac{r_c}{R_k} \quad (8)$$

Let's write expression (8) in formula (7), and it in expression (6), we get:

$$G_0 = 2\pi h m \frac{(1 + \eta)\rho_{atm}}{P_{atm}} [P_c(t) \frac{R_k^2 - r_c^2}{2} + \frac{R_k^2}{2} DA(t)] \quad (9)$$

The amount of gas-liquid mixture coming from the formation to the well in a unit time can be found by the following formula:

$$G = \frac{dG_0}{dt} \quad (10)$$

Then, if we substitute expression (9) in formula (10), we get:

$$G_0 = 2\pi h m \frac{(1 + \eta)\rho_{atm}}{P_{atm}} [\dot{P}_c \frac{R_k^2 - r_c^2}{2} + \frac{R_k^2}{2} D\dot{A}(t)] \quad (11)$$

On the other hand:

$$G = -k \frac{P_k(0) + P_c(0)}{\rho_{mixture} \beta} \pi r_c h \frac{\partial P}{\partial r} \Big|_{r=r_c} \quad (12)$$

Equating expressions (11) and (12), we get:

$$\dot{A}(t) + \alpha A(t) = -\frac{1}{D} \dot{P}_c(t) \quad (13)$$

Here:

$$\alpha = k \frac{P_k(0) + P_c(0)}{\rho_{mixture} m R_k^2 D (1 + \eta)}$$

The convective limit was taken into account in the equation of mixture flow in the tube.

Then, based on the above assumptions, the equations of flow and continuity of the gas-liquid mixture are as follows:

$$\begin{aligned} -\frac{\partial P}{\partial x} &= \frac{\partial Q}{\partial t} + V_0 \frac{\partial Q}{\partial x} + 2\alpha Q \\ -\frac{\partial P}{\partial t} &= C^2 \frac{\partial Q}{\partial x} \end{aligned} \quad (14)$$

$$Q = \rho_{mixture} u$$

Here, u is the expression of gas-liquid flow averaged over the cross-section of the pipe.

The initial and boundary conditions are as follows:

$$P|_{t=0} = f(x) \quad (15)$$

$$\frac{\partial P}{\partial t}|_{t=0} = 0 \quad (16)$$

$$P|_{x=0} = P_c(t) \quad (17)$$

$$P|_{x=l} = P_{q,a}(t) \quad (18)$$

It was being find solving of (14) equations in (15), (16) initial and (17), (18) boundary conditions and writing the continuity condition at the bottom of the well, $P_c(t)$ - bottom hole pressure is found. Subsequently, the well rate is determined by setting $P_{q,a}(t)$ (taking it from experiment).

Keywords – Gas-Liquid Mixture, Formation-Well System, Bottom Pressure of The Well, Percolation, Mass And Output of The Well
