

Non-Integer Order Proportional Integral Control for Time Delay Plant with Single Fractional Order

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Abstract – This paper studies the tuning of non-integer order proportional integral controller for time delay plants having one fractional order in its denominator polynomial. The well-known first order plus time delay plant is reorganized to have the order of the operator s to be an arbitrary real number. The main aim is to tune the parameters of the fractional order proportional integral controller to make the mentioned plant behavior to be stable and robust against unexpected changes in the gain of the system. The approach here is a graphical point of view on the Bode diagram. The two crossover points in the Bode diagram, the gain crossover frequency and the phase crossover frequency are tuned towards the desire of the researcher. This brought us to extend or to restrict the distance between these two crossover frequencies and thus, to provide the stability and robustness of the system. The design process is an analytical way from beginning to the end. Hence, the method is more reliable when compared with the methods based on optimization. This paper presents the findings of the first step in the research. The formulas to design the controller are found and the results are shown on an illustrative example.

Keywords – Non-Integer Order; Fractional Order; Time Delay Plant; Proportional Integral; Controller

I. INTRODUCTION

It has been a difficult field of study for decades to tune various controller types for various plant species. Over the years, several control structure types have been proposed for diverse uses. For instance, the well-known Proportional Integral Derivative (PID) controller [1], the fuzzy controller [2], the adaptive controller [3], the lag-lead controller [4], the two-feedback controller [5], the sliding mode control for nonlinear models [6], etc. Most of these controller types were also modified to have non-integer orders in their structure. One of the most common controllers in this type can be named as the Fractional Order PID (FOPID) Controller [7]. As well as the FOPID controller, deficiency of the derivative or the integral operator brought us the Fractional Order Proportional Integral (FOPI) and the Fractional Order Proportional Derivative

(FOPD) Controllers [8], [9]. In the literature, these structures are employed to successfully manage a variety of plants.

The First Order plus Time Delay (FOPTD) model, one of the listed plants, is commonly used to define a variety of processes [10]-[12]. This paper pretends the FOPTD plant to have its single s operator in non-integer order. The FOPI controller will be tuned to control the system. In other words, the order of the operator s in the plant is not 1 and it can take any real number. Also, the order of the integral operator of the FOPI controller is a real number. In this case, the time and frequency domain behavior of the system differs from its original state. Besides it becomes more challenging to tune a controller for such plant to provide it stability and robustness. It is more challenging because the non-integer order requires fine tuning and also the FOPI controller has

one more parameter to tune when compared to the classical Proportional Integral (PI) controller.

The way of providing stability and robustness to the system is based on tuning the gain and phase crossover frequencies towards one's desire. The main aim is to straighten the phase curve between these frequency points. This will prevent the phase curve to drop below the -180° line and prohibit the system to pass to the instability area. Also, the system will gain robustness against unexpected changes in the gain.

This paper is organized in the following way. Section II reminds the transfer functions of the FOPI controller and the time delay plant with single fractional order. Third section gives the FOPI controller design procedure for fractional order plant with time delay. Illustrative examples are given in the fourth section to clarify the given method and some concluding remarks are discussed in Section V.

II. PLANT AND THE CONTROLLER

General representation of the time delay plant with single fractional order is,

$$P(s) = \frac{K}{1 + s^\alpha T} e^{-Ls}. \quad (1)$$

Here, $\alpha \in (0, 2)$ is the fractional order, K is the gain, T is the time constant and L is the delay. Similarly, the FOPI controller is,

$$C(s) = k_p + \frac{k_i}{s^\lambda}, \quad \lambda \in (0, 2). \quad (2)$$

Thus, the system can be written as multiplication of the controller and the plant.

The method in this study is a frequency domain approach, so the frequency domain response of the system can be obtain by using the equation $G(j\omega) = C(j\omega)P(j\omega)$. Here, the frequency response of the controller is calculated as below.

$$C(j\omega) = k_p + \frac{k_i}{(j\omega)^\lambda} \quad (3)$$

Similarly, the frequency response of the plant is,

$$P(j\omega) = \frac{K}{T(j\omega)^\alpha + 1} e^{-L(j\omega)} = |P(j\omega)| e^{j\angle P(j\omega)} \quad (4)$$

Now, we can define the frequency properties of a system. In the Bode diagram of any system, the gain crossover frequency is the frequency value where the gain curve cuts the 0dB line. In this point, the absolute value of the system is 1. Similarly, the phase of the system in this point is $PM - \pi$. Here, PM is the phase margin and is the difference of the value of the phase curve with the -180° line. As known, π radians equals to 180° .

The phase crossover frequency is the frequency value where the phase curve in the Bode plot cuts the -180° line. The absolute value of the system in the phase crossover frequency is $10^{GM/20}$. The GM stands for the gain margin and is the difference between the value of the gain curve and the 0dB in the phase crossover frequency. Phase of the system at the phase crossover frequency is $-\pi$. With this information, the design of the FOPI controller can be done.

III. DESIGNING THE CONTROLLER

Now, we have to match the frequency response of the system with the frequency properties given in the previous section. The absolute values of the plant and the controller were calculated before. Then, the absolute value of the system can be found by $|G(j\omega)| = |C(j\omega)P(j\omega)| = |C(j\omega)||P(j\omega)|$. Similarly, the phase of the system can be found by $\angle G(j\omega) = \angle C(j\omega)P(j\omega) = \angle C(j\omega) + \angle P(j\omega)$.

The first step is to tune the parameters of the controller to provide the frequency specifications on the gain crossover frequency. We have to replace ω with ω_{gc} in the equation denoting the absolute value of the system and equalize it to 1. Here, ω_{gc} stands for the gain crossover frequency. Similarly we have to make the same replacement in the equation showing the phase of the system and equalize it to $PM - \pi$. By solving these equations together, we can obtain the controller parameters to satisfy the frequency properties in the gain crossover frequency. Here, the parameters of the controller can be obtained as given below.

$$k_{p1} = \pm \frac{\sqrt{1+T^2(\omega_{cg}^2)^\alpha + 2T(\omega_{cg}^2)^{\alpha/2} \cos(\pi\alpha/2 + \alpha \arg(\omega_{cg}))} \sec(\varphi_1) \sin(\varphi_1 + \pi\lambda/2)}{K \sqrt{\sec(\varphi_1)^2 \sin(\pi\lambda/2)^2}}, \quad (5)$$

$$k_{i1} = \mp \frac{\omega_{cg}^\lambda \sqrt{1+T^2(\omega_{cg}^2)^\alpha + 2T(\omega_{cg}^2)^{\alpha/2} \cos(\pi\alpha/2 + \alpha \arg(\omega_{cg}))} \tan(\varphi_1)}{K \sqrt{\sec(\varphi_1)^2 \sin(\pi\lambda/2)^2}}, \quad (6)$$

$$\varphi_1 = PM - \pi + \arctan \left(\frac{T(\omega_{cg}^2)^{\alpha/2} \sin(\pi\alpha/2 + \alpha \arg(\omega_{cg}))}{1+T(\omega_{cg}^2)^{\alpha/2} \cos(\pi\alpha/2 + \alpha \arg(\omega_{cg}))} \right) + L\omega_{cg}. \quad (7)$$

The next step is to find the controller parameters to satisfy the frequency properties on the phase crossover frequency. For this, we have to equalize the absolute value of the system to $10^{GM/20}$. Here, ω is ω_{pc} and shows the phase crossover frequency. Similarly, we have to equalize the

phase of the system to $-\pi$. Solving of these equations together gives us the controller parameters to achieve the frequency properties on the phase crossover frequency. Below are the controller parameters in this case.

$$k_{p2} = \pm \frac{10^{GM/20} \sqrt{1+T^2(\omega_{cp}^2)^\alpha + 2T(\omega_{cp}^2)^{\alpha/2} \cos(\pi\alpha/2 + \alpha \arg(\omega_{cp}))} \sec(\varphi_2) \sin(\varphi_2 + \pi\lambda/2)}{K \sqrt{\sec(\varphi_2)^2 \sin(\pi\lambda/2)^2}}, \quad (8)$$

$$k_{i2} = \mp \frac{10^{GM/20} \omega_{cp}^\lambda \sqrt{1+T^2(\omega_{cp}^2)^\alpha + 2T(\omega_{cp}^2)^{\alpha/2} \cos(\pi\alpha/2 + \alpha \arg(\omega_{cp}))} \tan(\varphi_2)}{K \sqrt{\sec(\varphi_2)^2 \sin(\pi\lambda/2)^2}}, \quad (9)$$

$$\varphi_2 = -\pi + \arctan \left(\frac{T(\omega_{cp}^2)^{\alpha/2} \sin(\pi\alpha/2 + \alpha \arg(\omega_{cp}))}{1+T(\omega_{cp}^2)^{\alpha/2} \cos(\pi\alpha/2 + \alpha \arg(\omega_{cp}))} \right) + L\omega_{cp}. \quad (10)$$

Now we have to combine the controller parameters found for the gain crossover frequency and the phase crossover frequency. Numerical solutions of the two findings in the range $\lambda \in (0,2)$ will help us here. We can calculate two curves in this range. The intersection point of the two curves gives us the common fractional order of the FOPI controller. Thus, this is the controller satisfying all frequency properties at the same time. Let us study on a practical example.

IV. AN EXAMPLE

Consider the following plant from [13].

$$P(s) = \frac{66.16}{12.72s^{0.5} + 1} e^{-1.93s} \quad (27)$$

Frequency specifications for this plant are selected as $\omega_{cg} = 0.1 \text{ rad/sec}$ and $\omega_{cp} = 1.2 \text{ rad/sec}$. The phase margin is desired to be $PM = 60^\circ$. These variables are placed in their related equations and two different FOPI

controllers are calculated. Then the plot in Fig. 1 is used to find the common fractional order λ to combine the two controllers.

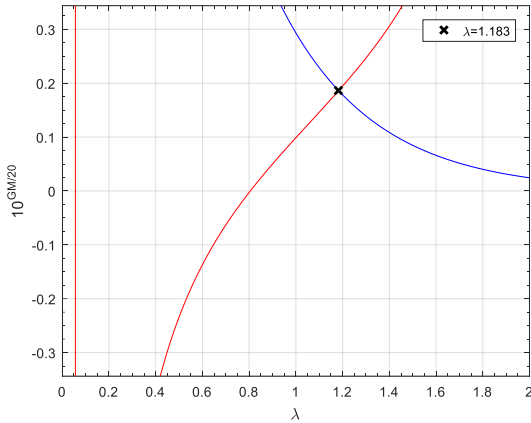


Fig. 1. Finding the common fractional order λ .

In Fig. 1, the red curve represents the plot of $10^{GM/20}$ w.r.t. $\lambda \in (0, 2)$. Similarly, blue curve is obtained by plotting $10^{GM/20}$ w.r.t. $\lambda \in (0, 2)$. These two curves intersect at $\lambda = 1.183$ thus, this is the fractional order of the controller for $PM = 60^\circ$. Also, the gain margin is calculated as $GM = -14.5996 \text{ dB}$. In this case, following FOPI controller is obtained.

$$C_{60^\circ}(s) = 0.042174 + \frac{0.00471537}{s^{1.18299}} \quad (28)$$

Let us check the Bode plot and the step response to prove the equations. Fig. 2 illustrates the Bode plot of the plant controlled with $C_{60^\circ}(s)$. The figure clearly shows that gain and phase crossover frequency requirements are successfully achieved. Also the phase margin is tuned as $PM = 60^\circ$. Gain margin information in the title of the figure is rounded up to $GM = 15 \text{ dB}$.

Proportional gain of $C_{60^\circ}(s)$ is iterated in the rate of $\pm 25\%$ to test the robustness of the system against gain changes. Step responses of the original system and the systems with $\pm 25\%$ of the proportional gain constant is comparatively plotted in Fig. 3. It can be seen in Fig. 3 that there occurs a little amount of change in the peak response of the plot against $\pm 25\%$ iteration of the controller gain.

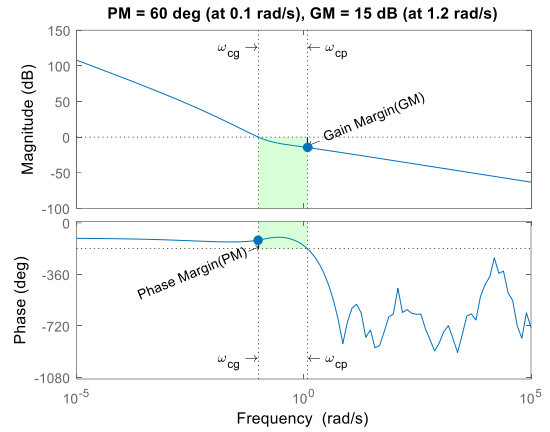


Fig. 2. Bode plot of the system $C_{60^\circ}(s)P(s)$ for $PM = 60^\circ$.

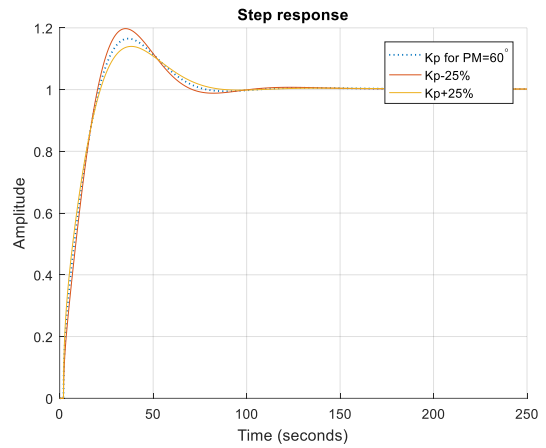


Fig. 3. Step response of the system $C_{60^\circ}(s)P(s)$ with $\pm 25\%$ iteration of the proportional gain constant.

It can be concluded that the derived formulas successfully achieve the desired gain crossover frequency, the phase crossover frequency and the phase margin. Also, by the successful selection of these parameters, the system shows a stable response and an improved robustness against the change in the proportional gain.

V. CONCLUSIONS

This study investigates the tuning of a non-integer order proportional integral controller for time delay plants with a polynomial denominator with a single fractional order. The well-known first order plus time delay plant has been reconfigured with the operators' order being any random real number. The primary goal is to fine-tune the settings of the fractional order proportional integral controller to make the behavior of the

stated plant robust and steady in the face of unforeseen changes in the system gain. Here, the Bode diagram is approached from a graphical point of view. Gain crossover frequency and phase crossover frequency, the two crossover points in the Bode diagram, are set to the researcher's preferences. As a result, we were able to increase or decrease the separation between these two crossover frequencies, so enhancing the system's stability and resilience. The entire design process is analytical from start to finish. As a result, as compared to approaches focused on optimization, the method is more trustworthy. The formulae for designing the controller are discovered, and the outcomes are displayed on an example for illustration. The research's initial findings are presented in this publication. The method is thought to be applied on more examples from the literature for future work. Also, the method will be further developed.

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