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PHYSICAL EXAMPLES OF NONLINEAR DYNAMICAL SYSTEMS

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Abstract **–** In this study, some important physical systems, which are examples of dynamic systems that exhibit chaotic behavior, are given. These are field models with spinor fields, which are especially important in particle physics. In this study, Duffing, Thirring and Dirac-Gursey systems will be discussed. The Duffing equation is a famous nonlinear differential equation that exemplifies a dynamical system that exhibits chaotic behavior. Various forms of the Duffing equation are used to describe many nonlinear systems. The Duffing equation serves as a test model for us to understand the dynamics of the nonlinear spinor field models we are working on. Thirring system is a relativistic field theory for the interaction of fermions. It is two-dimensional massless system. Since the Thirring system is the simplest nonlinear spinor system, it has an important place in particle physics as a toy model. The Dirac-Gursey system was proposed in 1956 to realize the dream of Heisenberg and his friends. It is the first nonlinear spinor wave equation with four dimensions and conformal invariance. Due to these properties, it has a wider symmetry than the Dirac equation and the equations proposed by Heisenberg et al. Gursey became the first physicist to test conformal invariance in spinor field theories with this system he proposed in 1956.

Keywords – Duffing, Gursey, Thirring, System, Nonlinear, Dynamical, Chaos

I. INTRODUCTION

As it is known, systems whose properties change over time in accordance with certain rules are called dynamic systems [1]. The real structure of nature is suitable for nonlinear properties, and linear models give limited information about the structure of nature. In this respect, it is very important that the rules that a dynamic system is subject to during its evolution should be nonlinear.

The analytical solution of nonlinear differential equations is often not possible and it is tried to be solved using certain approximation theories. However, the numerical results obtained with the used programming languages give information about the nonlinear dynamic structure. For this reason, numerical methods and the phase spaces of the solutions obtained from these methods are applied to have an opinion about the dynamics and evolution of nonlinear equation solutions.

In this work, three systems with nonlinear dynamics and playing an important role in the literature were studied. The first of these is the famous Duffing system [2], which also plays a role in the examination of many important physical problems. The other two systems are Thirring [3] and Dirac-Gursey systems [4], which play a very important role in particle physics and quantum field theory and exhibit soliton-type solutions.

II. Materials and Method

Some important physical systems, which are examples of dynamic systems that exhibit chaotic behavior, are given below:

Duffing System:

The Duffing equation is a famous nonlinear differential equation that exemplifies a dynamic system that exhibits chaotic behaviours [2]. Various forms of the Duffing equation are used to describe many non-linear systems. The most general form of the equation:

$$\ddot{x}+\delta\dot{x}\pm w_0{}^2x+\beta x^3=\gamma\cos(wt+\phi) \qquad (2.1)$$

is in the form. When we do not take the damping and force terms

$$\ddot{x}\pm w_0{}^2x+\beta x^3=0 \qquad (2.2)$$

This equation exhibits chaotic behavior depending on parameter selections.

Thirring System:

The Lagrangian function of this Thirring system [3]:

$$L=i\bar{\psi}\sigma_\mu\partial_\mu\psi+\frac{g}{2}(\bar{\psi}\psi)^2 \qquad (2.3)$$

g is the positive coupling constant and $\psi(x)$ is the fermion field.

The equation of motion of system

$$i\sigma_\mu\partial_\mu\psi+g\left(\bar{\psi}\psi\right)\psi=0. \qquad (2.4)$$

Heisenberg ansatz [5] is given as

$$\psi=\left[ix_\mu\gamma_\mu\chi(s)+\varphi(s)\right]C \qquad (2.5)$$

$\chi(s)$ and $\varphi(s)$ are real functions of $s=x^2+t^2$ and C is an arbitrary constant. Inserting Eq. (2.5) into Eq. (2.4):

$$\chi(s)+s\frac{d\chi(s)}{ds}+\alpha\left[s\chi(s)^2+\varphi(s)^2\right]\varphi(s)=0 \qquad (2.6)$$

$$\frac{d\varphi(s)}{ds}-\alpha\left[s\chi(s)^2+\varphi(s)^2\right]\chi(s)=0 \qquad (2.7)$$

By writing $\chi=As^{-\sigma}F(u)$ and $\varphi=Bs^{-\tau}G(u)$, with $u\equiv\ln s$ and $\sigma=\tau+\frac{1}{2}$, $t=\frac{1}{4}$ and $A^2=B^2$ [6], we obtain below nonlinear system under driven force:

$$\dot{F}=-\frac{1}{4}F+\frac{1}{2}G[F^2+G^2] \qquad (2.8)$$

$$\dot{G}=\frac{1}{4}G-\frac{1}{2}F[F^2+G^2]+\frac{A}{2}\cos\omega t \qquad (2.9)$$

here w is the frequency and A is the amplitude of driven force. This equations system exhibits chaotic behaviours depending on system parameters.

Dirac-Gursey System:

This spinor field equation proposed in 1956 is the first nonlinear spinor wave equation with conformal invariance [4]. Soliton-type solutions are found by adding the mass term to the equation for certain values of the coupling constant. Soliton solutions of the expanded form of Dirac-Gursey spinor equation and Wu-Yang type monopole solutions were found [5,6].

Lagrangian function of Dirac-Gursey spinor wave equation system with mass:

$$L=\overline{\psi(i}\gamma_\mu\partial_\mu\psi)+g(\bar{\psi}\psi)^{\frac{4}{3}}-m(\bar{\psi}\psi) \qquad (2.10)$$

g is the positive coupling constant and $\psi(x)$ is the fermion field.

The equation of motion of system

$$(i\gamma_\mu \partial_\mu \psi) = -\frac{4}{3} g(\overline{\psi}\psi)^{1/3} + m\psi \quad (2.11)$$

To find the soliton solutions of the equation, substituting the following soler pre-solution

$$\psi = \begin{bmatrix} g(r)\begin{pmatrix}1\\0\end{pmatrix} \\ if(r)\begin{pmatrix}\cos\theta \\ e^{i\phi}\sin\theta\end{pmatrix}\end{bmatrix} e^{-iwt} \quad (2.12)$$

and when necessary simplifications are made

$$f'(r) + \frac{2}{r} f(r) + (m-w)g(r) - \frac{4}{3}\alpha(g^2(r) - f^2(r))^{\frac{1}{3}} g(r) = 0$$
$$g'(r) + (m+w)f(r) - \frac{4}{3}\alpha(g^2(r) - f^2(r))^{\frac{1}{3}} f(r) = 0$$

(2.13)

system of equations is obtained [5,6,7].

To make the system of equations dimensionless;

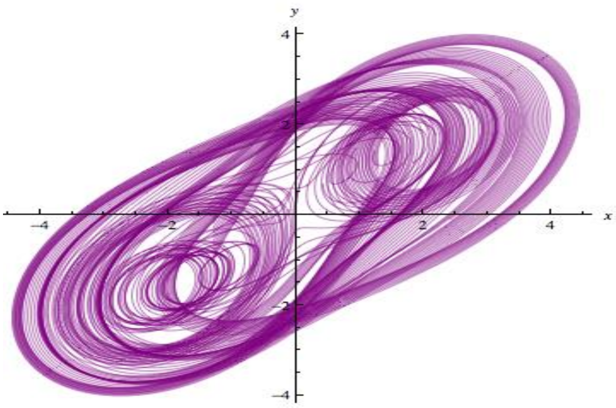
$$f(r) = \left[\frac{m+w}{\frac{4}{3}\alpha}\right]^{\frac{3}{2}} F(\rho),\; g(r) = \left[\frac{m+w}{\frac{4}{3}\alpha}\right]^{\frac{3}{2}} G(\rho),\; f'(r) = \frac{df}{dr} \text{ and } g'(r) = \frac{d}{d}$$

And dimensionless nonlinear system of equations is obtained as below:

$$f'(\rho) + \frac{2}{\rho} f(\rho) + \upsilon g(\rho) - (g^2(\rho) - f^2(\rho))^{\frac{1}{3}} g(\rho) = 0 \quad (2.14)$$
$$g'(\rho) + f(\rho) - (g^2(\rho) - f^2(\rho))^{\frac{1}{3}} f(\rho) = 0$$

III. RESULTS

In this section, drawn numerical results will be given for our three systems, which can exhibit regular or chaotic behaviors according to the system parameter values. It is better understood in the light of numerical results that each of the Duffing, Thirring and Dirac-Gursey systems are examples of dynamic systems that exhibit chaotic behavior, especially under the influence of external forces [7].



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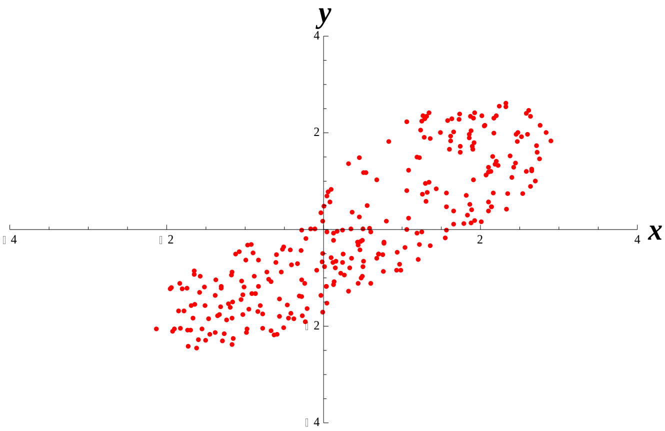

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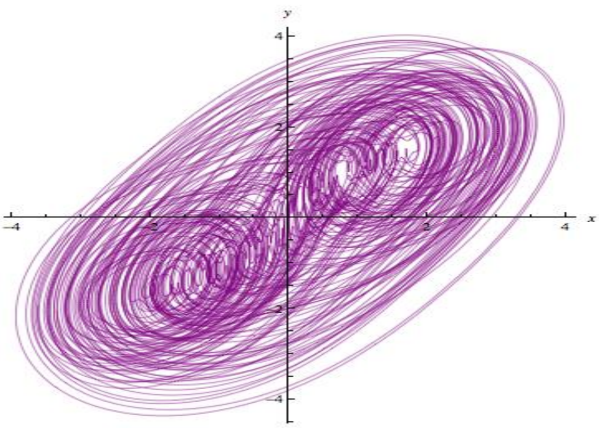
Figure 1: Duffing System (a) Phase Space Example (b) Poincare Section Example



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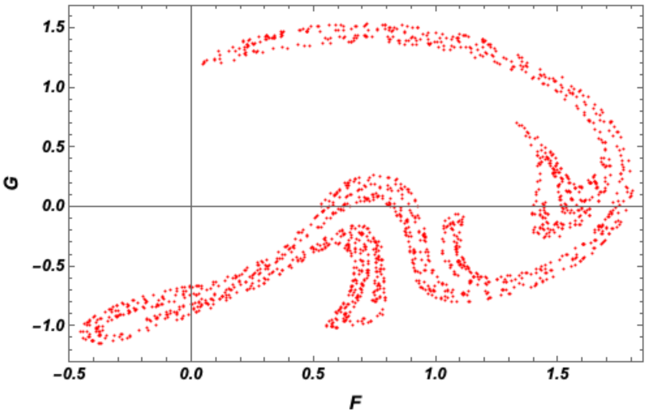

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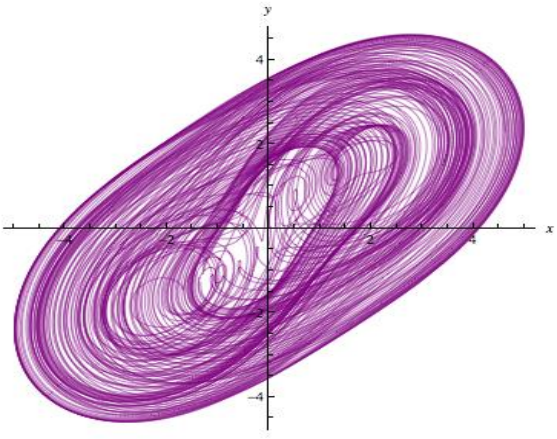
Figure 2: Thirring System (a) Phase Space Example (b) Poincare Section Example



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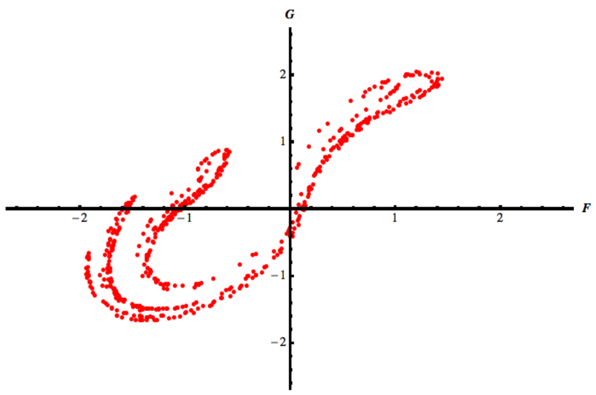

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Figure 3: Dirac-Gursey System (a) Phase Space Example (b) Poincare Section Example

IV. CONCLUSION

The nonlinearity of the equations of motion of the systems we are working with prevents the solutions from being found analytically. Therefore, solutions were found using numerical methods and their dynamics were examined.

The structure of the phase spaces of the Duffing, Thirring and Dirac-Gursey systems and the characteristics of their behavior are very similar. This is a result that should be considered. Because the examined system is different from each other in terms of both dimensions and spinor quantum numbers. This gives the result that for the three systems studied, their behavior in the phase space is independent of the quantum spinor numbers as well as their dimensions [7-10].

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