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Two-Position Synthesis of the Four-Bar Planar Linkage Mechanisms Using Artificial Neural Networks

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Abstract **–** The four-bar linkage mechanism is a fundamental and widely recognized mechanism with diverse applications, including various vehicle components, rehabilitation robotics, and rotary and reciprocating engines. Traditional textbooks introduce graphical and analytical solutions for the kinematic synthesis problem of the four-bar mechanism in different positions, while research articles explore its applications in various fields. Recently, artificial neural network (ANN) methods have gained popularity across different research domains. Researchers have proposed different solution approaches using ANN algorithms for the inverse and forward kinematic analysis problems of these mechanisms. However, the specific use of ANN algorithms for solving the two-position kinematic synthesis problem of the four-bar planar linkage mechanism has not been explored yet. This study aims to address this gap by introducing a solution for the two-position kinematic synthesis problem of the four-bar planar linkage mechanism using an artificial neural network algorithm. The Levenberg-Marquardt backpropagation neural network algorithm is chosen due to its speed, combination of Gauss-Newton training algorithm and steepest descend method, and ability to provide stable convergence of the training error. The neural network algorithm is trained, validated, and tested using a total of 50 randomly split data sets. Additionally, an additional test is conducted using all 50 data sets to evaluate the performance of the trained neural network algorithm. The study presents and discusses the results of the artificial neural network algorithm solution.

Keywords – Artificial Neural Networks, Two-position synthesis, Four-bar mechanisms, Levenberg-Marquardt, Backpropagation

I. Introduction

The four-bar linkage mechanism is one of the fundamental and well-known mechanisms and has different applications such as in various part of the vehicles, rehabilitation robotics or the packaging industry. Consequently, the kinematic synthesis and analysis problem of the four-bar planar linkage mechanism has been extensively studied, and researchers have developed various methods to solve it. With the advent of artificial neural network methods in different research fields, researchers have explored different applications of neural network algorithms for the kinematic analysis and synthesis problems of these mechanisms.

Numerous textbooks and research articles have addressed the various-position kinematic synthesis of four-bar linkage mechanism, highlighting their fundamental nature. Below are a few examples of these scholarly works. In such standard textbooks, the traditional graphical and analytical solutions of the two-position kinematic synthesis are introduced [1-3]. In these textbooks, solution steps of the synthesis problem are explained in detail with the different examples. As an example of four-bar linkage mechanism different applications, the two-position synthesis of the four-bar mechanism is presented by Denizhan and Chew in a few studies

[4-5]. In these studies, the four-bar mechanism is designed and optimized for the automotive engine hood specifically and two-position synthesis of the mechanism is presented in detail. However, the artificial neural networks have not been utilized in these studies. The following articles serve as a few examples of the application of artificial neural networks for the inverse and forward kinematic analysis of these mechanisms: A feedforward nets is used for a four-bar mechanism inverse kinematic problem by Jack et. al [6]. The inverse kinematics problem for a three-degree-of-freedom four-bar mechanism in three-dimensional space is solved by using multi-layer neural networks in this study. Another inverse kinematics solution for a robotic arm based on neural networks is introduced by Duka [7]. In this study, a planar three-link manipulator inverse kinematic solution is introduced by using feed-forward neural network. An artificial neural network solution approach for the kinematics of a parallel manipulator is presented by Khattab et. al [8]. In this study, forward and inverse kinematics solutions of the 3 limbs of prismatic-universal structure is introduced by using two artificial neural network algorithms. Kinematic synthesis of a parallel manipulator via neural network is introduced by Ghasemi et. al [9]. In this study, the inverse kinematic equation of the parallel manipulator is solved by using multi-layer perception neural network algorithm with redial basis function.

As mentioned earlier, the artificial neural network algorithms are commonly used for solving the inverse and forward kinematic analysis problems of various mechanisms. However, there is a lack of research on the specific application of artificial neural network algorithms for the two-position linkage synthesis of four-bar planar mechanism. This study aims to fill this gap by solving the two-position linkage synthesis problem of the four-bar planar mechanism using an artificial neural network algorithm. The chosen algorithm for the solution is the Levenberg-Marquardt backpropagation algorithm. The training, validation, and test results of the algorithm are presented, and an additional test conducted using all available data sets. The results are then compared and discussed.

II. Two-Position Synthesis of the Four-Bar Planar Linkage Mechanism

The four-bar planar linkage mechanism with right- and left-side dyads in two-position is illustrated in Fig. 1. Detailed design parameters and drawings of the four-bar planar linkage mechanism in the two positions are depicted in Fig. 1.

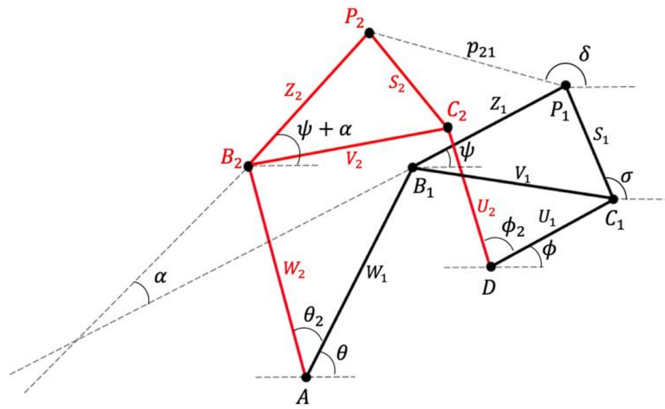

Fig. 1. The four-bar planar linkage mechanism right- and left-side dyads in two-position

According to the Fig. 1, the following two-position kinematic synthesis equations can be written:

Left-side dyad:

$$we^{i\theta} = \frac{ze^{i(\psi+\alpha)} - p_{21}e^{i\delta} - ze^{i\psi}}{1 - e^{i\theta_2}} \tag{1}$$

Right-side dyad:

$$ue^{i\phi} = \frac{se^{i(\sigma+\alpha)} - p_{21}e^{i\delta} - se^{i\sigma}}{1 - e^{i\phi_2}} \tag{2}$$

where w is the length of the $\vec{W}_1$ and $\vec{W}_2$ ($|\vec{W}_1| = |\vec{W}_2|$, rigid link [AB]), z is the length of the $\vec{Z}_1$ and $\vec{Z}_2$ ($|\vec{Z}_1| = |\vec{Z}_2|$), u is the length of the $\vec{U}_1$ and $\vec{U}_2$ ($|\vec{U}_1| = |\vec{U}_2|$, rigid link [DC]), s is the length of the $\vec{S}_1$ and $\vec{S}_2$ ($|\vec{S}_1| = |\vec{S}_2|$), angle θ is the angle of $\vec{W}_1$ (link [AB] at the first position), angle θ_2 is the angle between $\vec{W}_1$ and $\vec{W}_2$ (link [AB] at the first and last positions respectively), angle ϕ is the angle of $\vec{U}_1$ (link [DC] at the first position), angle ϕ_2 is the angle between $\vec{U}_1$ and $\vec{U}_2$ (link [DC] at the first and last positions respectively), angle ψ is the angle of $\vec{Z}_1$ (link [BP] at the first position), angle α is the angle between $\vec{Z}_1$ and $\vec{Z}_2$ (link [BP] at the first and last positions respectively), angle δ is the angle of pivot point P, angle σ is the angle of $\vec{S}_1$ (link [CP] at the first position) and p_{21} is the length between the first and last positions of the pivot point P.

As seen in Eqns. (1) and (2), there are four unknown parameters in the two-position synthesis of the four-bar planar linkage mechanism. According to the solution procedure outlined in textbooks, three parameters should be initially specified: $p_{21} = 2.416$ m., $\alpha = 43.3$ rad. and $\delta = 165.2$ rad. The remaining six parameters are considered free choices, and their respective values are $s = 1.035$ m., $z = 1.298$ m., $\sigma = 104.1$ rad., $\psi = 26.5$ rad., $\theta_2 = 38.4$ rad. and $\phi_2 = 85.6$ rad. [1]. By utilizing the provided parameters in conjunction with Eqns. (1) and (2), the problem of two-position synthesis for the four-bar planar linkage mechanism can be solved.

III. Two-Position Synthesis with Artificial Neural Networks

In this study, a two-layer feedforward network (with hidden and output layers) is investigated for the two-position kinematic synthesis problem of the four-bar planar mechanism. The network consists of with sigmoid hidden neurons and linear output neurons. The artificial neural network algorithm utilized in this study has two input parameters and three output parameters. The input parameters chosen are position of the Joint C, represented by the (x_c, y_c) coordinates. The output parameters selected for the supervised leaning algorithm are the angles (θ, ϕ and ψ) of the four-bar mechanism. The Levenberg-Marquardt backpropagation neural network algorithm is employed for training. This algorithm is selected due to its speed and ability to combine the Gauss-Newton training algorithm and steepest descent method, and ensuring stable convergence of the training error. [10-12].

In this study, the neural network algorithm employs a total of 50 data sets for training, validation, and testing purposes. These data sets are randomly divided into three independent sets. Specifically, 80% of the data set is allocated for training, 10% for validation, and the remaining 10% for testing. Additionally, an extra test is conducted using all 50 data tests to evaluate the performance of the trained neural network algorithm. The hidden layer of the neural network consists of 100 neurons with a sigmoid activation function, while the output layer comprises 3 neurons with a linear activation function.

Figure 2 shows structure of the feedforward neural network. As seen in Fig. 2, artificial neural network has 2 input parameters (x_c, y_c), 100 neurons in hidden layer, 3 neurons in output layer and 3 output parameters (θ, ϕ and ψ). In Fig. 2, w and b refer to the neural network weights and bias respectively. As previously mentioned, the hidden layer activation function is sigmoid function and output layer activation function is the linear function.

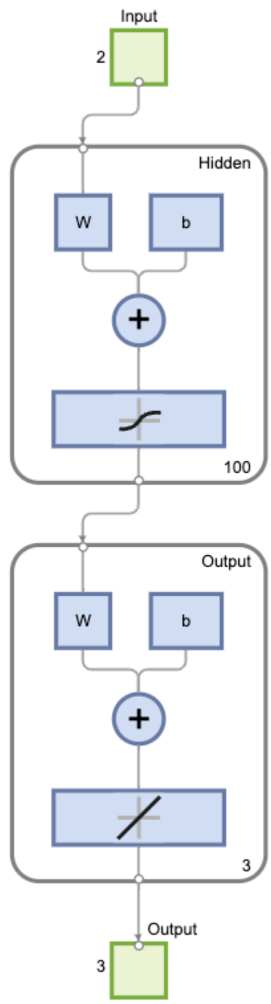

Fig. 2 Structure of the artificial feedforward neural network

By following the traditional analytical solution procedure of the two-position linkage synthesis problem, the values of the four unknown parameters (w, u, θ and ϕ) are determined. According to this traditional solution, the values of these unknown parameters are found to be $w = 2.4669$ m., $u = 1.4856$ m., $\theta = 1.2492$ rad. and $\phi = 0.2691$ rad. Once these unknown parameters are found, a data set is created between these two positions of the four-bar planar linkage mechanism. In this study, 50 different positions of the four-bar planar mechanism are determined between these two positions. For each of these 50 positions, all the parameters of the four-bar mechanism are calculated. As mentioned before, the coordinates of Joint C (x_c, y_c) are chosen input parameters, and the angles of the four-bar planar mechanism $(\theta,\ \phi$ and $\psi)$ are chosen as the output parameters for the Levenberg-Marquardt neural network algorithm in this study.

IV. RESULTS

In this study, MATLAB Neural Net Fitting App is utilized for training, validation, testing and conducting additional test of the neural network algorithm.

Table 1. Levenberg-Marquardt algorithm training progress

Unit	Initial Value	Stopped value	Target value
Epoch	0	13	1000
Elapsed Time	---	00:00:01	---
Performance	5.84	1.43e-07	0
Gradient	8.59	5.79e-05	1e-07
Mu	0.001	1e-06	1e+10
Validation Checks	0	6	6

Table 1 shows a summary of the neural network training process. Training is finished when it met with the validation criterion. As seen in Table 1, Levenberg-Marquardt neural network algorithm reached validation criteria after 13 epochs in 1 second. Mu is the adaptive value in Levenberg-Marquardt backpropagation algorithm. According to the Table 1, stopped values are close to the target values when the training is finished.

Table 2. Levenberg-Marquardt algorithm training results

	Observations	MSE	R
Training	40	0.0000	1.0000
Validation	5	0.0181	0.9696
Test	5	0.0118	0.9627
Additional Test	50	0.0030	0.9927

Table 2 shows the number of observations (data sets), mean squared error (MSE), and regression (R) results for training, validation, testing and additional test. As previously mentioned, 40 data sets of the total of 50 data sets are split for training. For the training phase, the mean squared error is 0, and regression value is 1. A total of 10 data sets of the total of 50 data sets are split for validation and test. According to the Table 2, the mean squared error (MSE) for test data sets is slightly lower and indicating better performance, while the regression value is better for the validation sets. The same 50 data sets are utilized for the additional test, resulting in an almost zero mean squared error.

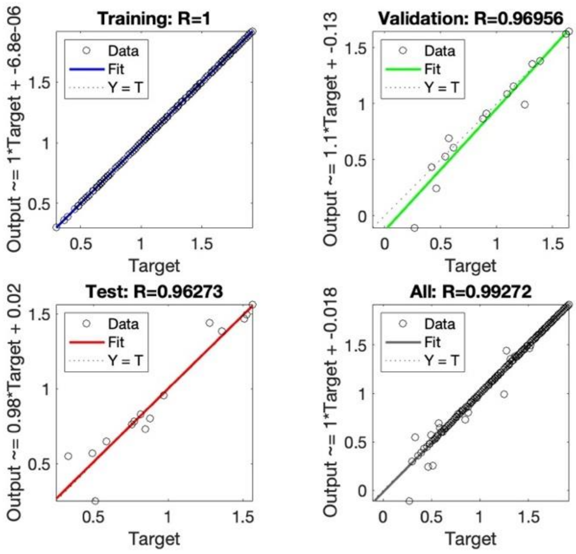

Fig. 3 Regression plots of feedforward neural network

Figure 3 displays regression plots of the neural network algorithm. In Fig. 3, network predictions (output) with respect the response (target) for training, validation and test sets can be seen clearly. According to the Fig. 3, the regression value is 1 and data sets are perfectly fit in the training plot. On the other hand, the regression value is 0.96956 in the validation plot and the validation data set does not fit perfectly. According to the test plot in Fig 3, the regression value is 0.96273 and test data is not perfectly fit. Plot for all in Fig. 3 shows that the regression value is 0.99272 and data is almost fit. Based on Fig. 3, it can be concluded that the fit is good overall for all the data sets.

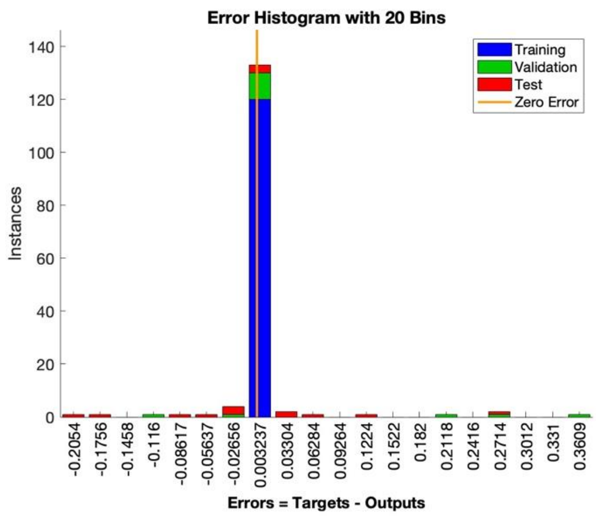

Fig. 4 Error histogram graph of feedforward neural network

Figure 4 shows error histogram graph of the Levenberg-Marquardt neural network algorithm. According to the Fig. 4, the error value is 0.003237 and all of the training data sets have almost zero error. Figure 4 shows some of the test and validation data sets have different error values but overall, the data set is good.

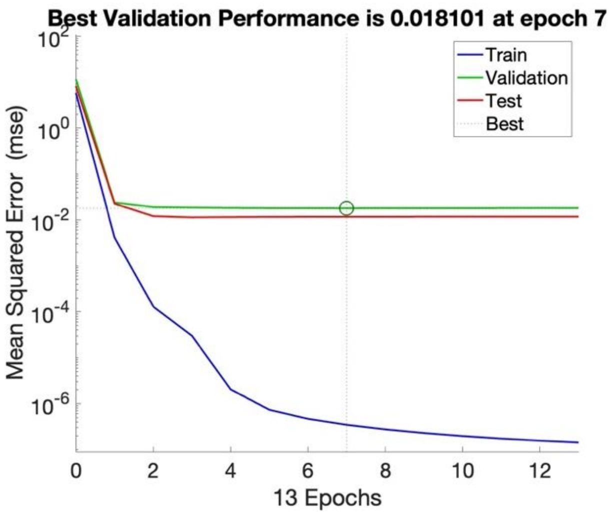

Fig. 5 Validation performance graph of the feedforward neural network

Figure 5 shows training, validation and testing errors for the Levenberg-Marquardt neural network algorithm in this study. As seen in Fig. 5, the mean squared error is small for training, validation and testing data sets. According to the Fig. 5, the best validation performance occurs at epoch 7 and the validation and train data set has similar characteristics.

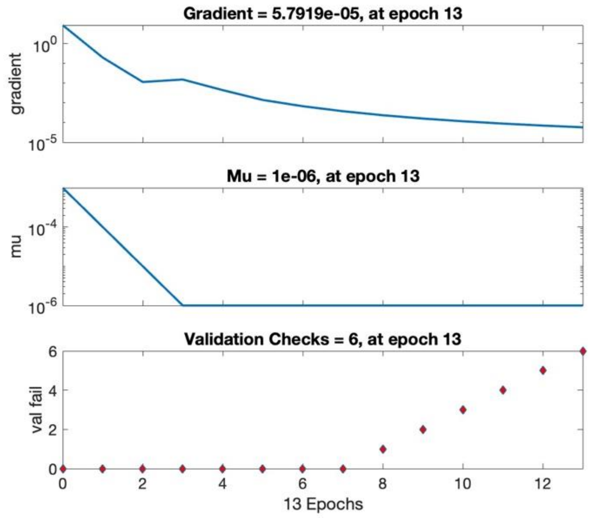

Fig. 6 Feedforward neural network training state plots

Figure 6 displays training state plots for the gradient, Levenberg-Marquardt algorithm adaptive value (Mu) and validation checks. According to the Fig. 6, the Mu value remains constant after epoch 3, while the gradient value decreases over time. Neural network training stop criteria is when the validation check is 6 and Fig. 6 shows that the validation check is 6 at epoch 13.

In addition, an additional test is conducted using the same 50 data sets. It is important to note that 5 data sets are reversed specifically for testing the trained neural network. The main goal of the additional test is to assess the performance of the trained neural network. Figure 7 shows additional test regression plot of the trained neural network algorithm. As observed in Fig. 7, the regression value is 0.99272, indicating that the data set does not fit perfectly but the overall fit is reasonably good for all data sets.

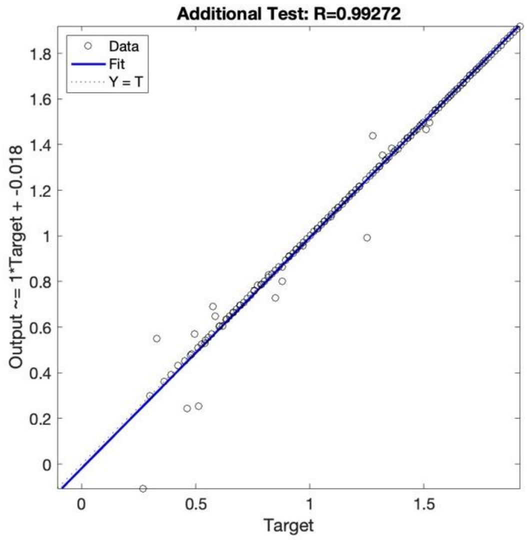

Fig. 7 Regression graph of feedforward neural network for the additional test

Figure 8 displays additional test error histogram graph for the trained neural network algorithm. As depicted in Fig. 8, there are no training and validation data sets in the additional test since all 50 data sets are used for this test. According to the Fig. 8, the additional test error value is the same with the network training error value but certain data sets exhibit different error values.

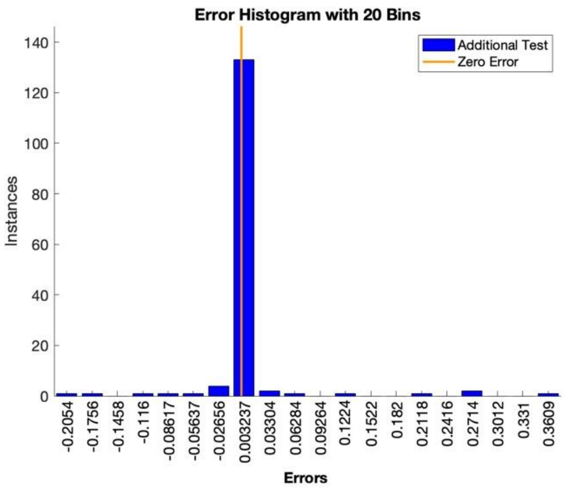

Fig. 8 Error histogram graph of neural network after additional test with 50 data set

V. DISCUSSION

In artificial neural networks, having more data sets significantly contributes to better algorithm training. In other words, a larger number of data sets is desirable for improved results in neural network algorithms. as a potential future research direction, it would be beneficial to incorporate a larger number of data sets for training and analyze the impact of the data set quantity on the results.

As mentioned previously, the two-position synthesis problem involves six "free choices" parameters for the solution, which are selected based on the dimensions of the designed mechanisms. A future research direction could focus on utilizing neural network algorithms to determine these six free choices automatically.

This article presents a solution to the two-position synthesis problem using the Levenberg-Marquardt backpropagation artificial neural network algorithm. However, there are other algorithms available, such as Bayesian Regularization algorithm or Scaled Conjugate Gradient algorithm. Another potential future research direction could involve exploring the application and comparison of these different algorithms to the same four-bar planar linkage mechanism.

VI. CONCLUSION

In this study, a solution to the two-position linkage synthesis problem of the four-bar planar mechanism is presented using artificial neural network algorithm. The training of the neural network is performed using the Levenberg-Marquardt backpropagation algorithm. The input parameters chosen for the neural network are the coordinates of Joint *C*, which consists of 2 inputs. The output parameters of interest are the angles of the four-bar mechanism, resulting in 3 outputs. The training procedure of the neural network is described, and the results demonstrate a perfect fit between the data set and the trained model.

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