All Sciences Proceedings
http://as-proceeding.com/

1st International Conference on Contemporary Academic Research
May 17-19, 2023 : Konya, Turkey

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Analysis of the buckling of magneto-piezoelectric nano-plates by the theory of nonlocal elasticity

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Abstract – Smart materials capable of modifying their physical properties in response to stresses (variation in temperature, electric or magnetic field, mechanical stress, etc.), the material reacts to a stimulus detected outside and adapts its response. Actuators in particular provide a mechanical action or change their appearance (deformation, color change, etc.) to indicate a change in the environment or to provoke a corrective action. Piezoelectric materials belong to this category of material.

Keywords – Piezoelectric Materials, Smart Materials, Mechanical Action.

I. INTRODUCTION

Piezoelectricity attracts the attention of many researchers [1-2]. Han and Pan [3] presented exact solution for the functionally graded multilayer rectangular magneto-piezoelectric plate. Also, and by using the local methods, Sladek et al [4] solved the problems of buckling of the magneto-piezoelectric plate resting on elastic foundation under a stationary load harmonic.

In this chapter, we are interested in the analysis of buckling of magneto-piezoelectric nano-plates resting on elastic foundation by the non-local elasticity theory of Mindlin. Electric and magnetic fields can be ignored in the plane and the equations motion of the magneto-piezoelectric nano-plates are established by applying the variationals principle. The numerical results show effects of electric potentiality, magnetic potentiality, Winkler foundation coefficient and Pasternak foundation coefficient on the buckling load.

II. THEORY AND FORMULATION

A. Non-local theory of magneto-piezoelectric plates

The nonlocal elasticity theory suppose that the state of stress at a point in the body depends not only on state of stress at that point, but as well on the state at all points in the body, The general form of constitutive relation in the nonlocal elasticity type representation involves, over the whole body, an integral which contains a nonlocal function describing the influence of scale effect on the constraint. The nonlocal constitutive equations of an elastic and homogeneous solid are given by Eringen [5].

$$\sigma_{ij}^{nl}(x)=\int_{v}\alpha.\left(\left|x-\bar{x}\right|,\tau\right)\sigma_{ij}^{l}dV(x') \qquad (1)$$

With:

$\sigma_{ij}^{nl},\sigma_{ij}^{l}$: The non-local stress tensor and the local stress tensor; respectively.

$\alpha\left(\left|x-\bar{x}\right|,\tau\right)$: The non-local module.

$\left|x-\bar{x}\right|$: The Euclidean distance.

$\tau=e_0a/L$

Or :

L : External characteristic length of the nanoplate.

e_0 : Appropriate constant for each material.

a : Internal characteristic length of material.

Therefore, e_0a: constant parameter obtained with structural mechanics through molecular and experimental studies. For the magneto-piezoelectric solid, the nonlocal constitutive equation is the equations:

$$D_{ij}^{nl}(x)=\int_v \alpha.(|x-x'|,\tau)D_{ij}^{l}dV(x') \qquad (2)$$

$$B_{ij}^{nl}(x)=\int_v \alpha(|x-x'|,\tau)B_{ij}^{l}dV(x') \qquad (3)$$

With:

D_{ij}^{nl}, D_{ij}^{l}: Components of non-local and local electrical displacement; respectively.

B_{ij}^{nl}, B_{ij}^{l}: Components of local and nonlocal magnetic induction, respectively.

By the assumptions give out by Eringen [5-6], the non-local elasticity equations simplified to the following differentials equations:

$$\begin{cases}(1-(e_0a)^2\nabla^2)\sigma_{ij}^{nl}=\sigma_{ij}^{l}\\(1-(e_0a)^2\nabla^2)D_{ij}^{nl}=D_{ij}^{l}\\(1-(e_0a)^2\nabla^2)B_{ij}^{nl}=B_{ij}^{l}\end{cases} \qquad (4-6)$$

B. MINDLIN PLATE THEORY

For Mindlin's plate theory [7-8], the displacement field can be expressed as:

$$\begin{cases}u_x(x,y,z)=z.\psi_x(x,y)\\u_y(x,y,z)=z.\psi_y(x,y)\\u_z(x,y,z)=w.(x,y)\end{cases} \qquad (7-9)$$

The strain field obtained in the form:

$$\begin{cases}\varepsilon_{xx}=z\dfrac{\partial\psi_x}{\partial x}\\\varepsilon_{yy}=z\dfrac{\partial\psi_y}{\partial y}\\\gamma_{yz}=\dfrac{\partial w}{\partial y}+\psi_y\\\gamma_{zx}=\dfrac{\partial w}{\partial x}+\psi_x\\\gamma_{xy}=z\left(\dfrac{\partial\psi_x}{\partial y}+\dfrac{\partial\psi_y}{\partial x}\right)\end{cases} \qquad (10-14)$$

III. MODELING THE PROBLEM

Figure .1 represents a magneto-piezoelectric nano-plate of length. width and thickness resting on an elastic foundation of the Winkler-Pasternak type. A Cartesian system is used to describe the nano-plate which is subjected to bi-axial compressive load, electrical potentiality and magnetic potentiality.

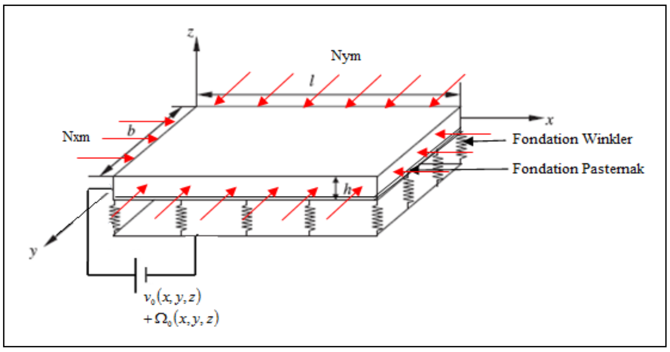

Figure.1 Magneto-piezoelectric nano-plate

A. NON-LOCAL CONSTITUTIVE RELATIONS FOR THE MAGNETO-PIEZOELECTRIC NANO-PLATE

For magneto-piezoelectric isotropic solids, the constitutive relations can be formulated as follows:

$$\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{yz}\\\sigma_{zx}\\\sigma_{xy}\end{Bmatrix}-(e_0a)^2\nabla^2\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{yz}\\\sigma_{zx}\\\sigma_{xy}\end{Bmatrix}=\begin{Bmatrix}c_{11}&c_{12}&0&0&0\\c_{12}&c_{22}&0&0&0\\0&0&c_{44}&0&0\\0&0&0&c_{44}&0\\0&0&0&0&c_{66}\end{Bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\\varepsilon_{yy}\\\gamma_{yz}\\\gamma_{zx}\\\gamma_{xy}\end{Bmatrix}-\begin{Bmatrix}0&0&e_{31}\\0&0&e_{31}\\0&e_{24}&0\\e_{15}&0&0\\0&0&0\end{Bmatrix}\begin{Bmatrix}E_x\\E_y\\E_z\end{Bmatrix}-\begin{Bmatrix}0&0&f_{31}\\0&0&f_{31}\\0&f_{24}&0\\f_{15}&0&0\\0&0&0\end{Bmatrix}\begin{Bmatrix}H_x\\H_y\\H_z\end{Bmatrix}$$

$$\begin{Bmatrix}D_x\\D_y\\D_z\end{Bmatrix}-(e_0a)^2\nabla^2\begin{Bmatrix}D_x\\D_y\\D_z\end{Bmatrix}=\begin{Bmatrix}0&0&0&e_{15}&0\\0&0&e_{24}&0&0\\e_{31}&e_{31}&0&0&0\end{Bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\\varepsilon_{yy}\\\gamma_{yz}\\\gamma_{zx}\\\gamma_{xy}\end{Bmatrix}+\begin{bmatrix}h_{11}&0&0\\0&h_{22}&0\\0&0&h_{33}\end{bmatrix}\begin{Bmatrix}E_x\\E_y\\E_z\end{Bmatrix}+\begin{bmatrix}g_{11}&0&0\\0&g_{22}&0\\0&0&g_{33}\end{bmatrix}\begin{Bmatrix}H_x\\H_y\\H_z\end{Bmatrix} \qquad (15-17)$$

$$\begin{Bmatrix}B_x\\B_y\\B_z\end{Bmatrix}-(e_0a)^2\nabla^2\begin{Bmatrix}B_x\\B_y\\B_z\end{Bmatrix}=\begin{Bmatrix}0&0&0&f_{15}&0\\0&0&f_{24}&0&0\\f_{31}&f_{31}&0&0&0\end{Bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\\varepsilon_{yy}\\\gamma_{yz}\\\gamma_{zx}\\\gamma_{xy}\end{Bmatrix}+\begin{bmatrix}g_{11}&0&0\\0&g_{22}&0\\0&0&g_{33}\end{bmatrix}\begin{Bmatrix}E_x\\E_y\\E_z\end{Bmatrix}+\begin{bmatrix}\mu_{11}&0&0\\0&\mu_{22}&0\\0&0&\mu_{33}\end{bmatrix}\begin{Bmatrix}H_x\\H_y\\H_z\end{Bmatrix}$$

With:

c_{ij}, e_{ij} *and* h_{ij}: The elastic, piezoelectric and dielectric constants.

H_i and E_i : magnetic and electric field strengths.

f_{ij}, g_{ij} *et* μ_{ij} :The piezo-magnetic, electromagnetic and magnetic coefficients; respectively.

$$\begin{cases}E=-\nabla\phi\\H=-\nabla\varphi\end{cases} \qquad (18-19)$$

With :

ϕ and φ: Gradients of electric and magnetic potentialities

A.EQUATIONS OF MOTION

The deformation energy of the magneto-piezoelectric nano-plate can be given by the following relation:

$$U=\frac{1}{2}\int_{\Omega}\int_{-h/2}^{h/2}\left(\sigma_{xx}\varepsilon_{xx}+\sigma_{yy}\varepsilon_{yy}+\sigma_{yz}\gamma_{yz}+\sigma_{zx}\gamma_{zx}+\sigma_{xy}\gamma_{xy}-D_xE_X-D_yE_y-D_zE_z-B_xH_x-B_yH_y-B_zH_z\right)d\Omega dz \quad (20)$$

the stress-displacement relations written as:

$$\{M_{xx},M_{yy},M_{xy}\}=\int_{-h/2}^{h/2}\{\sigma_{xx},\sigma_{yy},\sigma_{xy}\}zdz$$

$$\{Q_{xx},Q_{yy}\}=k\int_{-h/2}^{h/2}\{\sigma_{zx},\sigma_{yz}\}dz \quad (21-22)$$

With k, the shear correction coefficient; we take $k=\pi^2/12$

By replacing the equations (10-14) and (15-17) in the equation (20) we arrive at the following equation:

$$U=\frac{1}{2}\int_{\Omega}\left(M_{xx}\frac{\partial\psi_x}{\partial x}+M_{yy}\frac{\partial\psi_y}{\partial y}+M_{xy}\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right)+Q_{xx}\left(\psi_x+\frac{\partial W}{\partial x}\right)+Q_{yy}\left(\psi_y+\frac{\partial W}{\partial y}\right)\right)dxdy+\frac{1}{2}\int_{\Omega}\int_{-h/2}^{h/2}\left(D_z\frac{\partial\phi}{\partial z}+B_z\frac{\partial\varphi}{\partial z}\right)d\Omega dz \quad (23)$$

Virtual strain energy expressed in the following form:

$$\delta U=\frac{1}{2}\int_{\Omega}\left(M_{xx}\delta\left(\frac{\partial\psi_x}{\partial x}\right)+M_{yy}\delta\left(\frac{\partial\psi_y}{\partial y}\right)+M_{xy}\delta\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right)+Q_{xx}\delta\left(\psi_x+\frac{\partial W}{\partial x}\right)+Q_{yy}\delta\left(\psi_y+\frac{\partial W}{\partial y}\right)\right)d\Omega+\frac{1}{2}\int_{\Omega}\left(D_z\delta\left(\frac{\partial\phi}{\partial z}\right)+B_z\delta\left(\frac{\partial\varphi}{\partial z}\right)\right)d\Omega \quad (24)$$

The elastic surrounding environment external virtual work can be written as:

$$\delta W=\frac{1}{2}\int_{\Omega}q_p\delta w d\Omega \quad (25)$$

With q_p being a force related to Pasternak foundation and the transverse load which expressed in the form:

$$q_p=K_w.w-K_g.\nabla^2w+\left(N_{xm}+N_{xe}+N_{xa}\right)\frac{\partial^2w}{\partial x^2}+\left(N_{ym}+N_{ye}+N_{ya}\right)\frac{\partial^2w}{\partial y^2} \quad (26)$$

With:

K_g and K_w: Pasternak and Winkler elastic foundation coefficients; respectively.

Now, we use the following variational principle:

$$\delta U-\delta W=0. \quad (27)$$

By replacing the equations (24) - (25) in (27), by integrating by parts, we obtain the governing equations:

$$\begin{cases}\dfrac{\partial M_{xx}}{\partial x}+\dfrac{\partial M_{xy}}{\partial y}-Q_{xx}=0\\ \dfrac{\partial M_{xy}}{\partial x}+\dfrac{\partial M_{yy}}{\partial y}-Q_{yy}=0\\ \dfrac{\partial Q_{xx}}{\partial x}+\dfrac{\partial Q_{yy}}{\partial y}+K_w.w-K_g.\nabla^2w+\left(N_{xm}+N_{xe}+N_{xa}\right)\dfrac{\partial^2w}{\partial x^2}+\left(N_{ym}+N_{ye}+N_{ya}\right)\dfrac{\partial^2w}{\partial y^2}=0\\ \dfrac{\partial D_z}{\partial z}=0\\ \dfrac{\partial B_z}{\partial z}=0\end{cases} \quad (28-32)$$

With: $N_{xm}, N_{xe}, N_{xa}, N_{ym}, N_{ye}$ and N_{ya} The mechanical, electrical and magnetic forces for the directions x *et* y.

$$\begin{aligned}N_{xm}&=P\\ N_{xe}&=e_{31}V_0\\ N_{xa}&=f_{31}\Omega_0\\ N_{ym}&=\lambda P\\ N_{ye}&=e_{31}V_0\\ N_{ya}&=f_{31}\Omega_0\end{aligned} \quad (33-38)$$

By substituting the equations (III.16) - (III.17) in the equations (III.31) - (III.32), the following two algebraic equations easily obtained:

$$\begin{cases}e_{31}\dfrac{\partial\psi_x}{\partial x}+e_{31}\dfrac{\partial\psi_y}{\partial y}-h_{33}\dfrac{\partial^2\phi}{\partial z^2}-g_{33}\dfrac{\partial^2\varphi}{\partial z^2}=0\\ f_{31}\dfrac{\partial\psi_x}{\partial x}+f_{31}\dfrac{\partial\psi_y}{\partial y}-g_{33}\dfrac{\partial^2\phi}{\partial z^2}-\mu_{33}\dfrac{\partial^2\varphi}{\partial z^2}=0\end{cases} \quad (39-40)$$

Thus, by adopting Cramer's rule, we can have:

$$\begin{cases} \dfrac{\partial^2 \phi}{\partial z^2} = \dfrac{\mu_{33} e_{31} - g_{33} f_{31}}{h_{33}\mu_{33} - g_{33}^2} \Delta \\ \dfrac{\partial^2 \varphi}{\partial z^2} = \dfrac{h_{33} f_{31} - g_{33} e_{31}}{h_{33}\mu_{33} - g_{33}^2} \Delta \end{cases} \quad (41-42)$$

With :

$$\Delta = \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y} \quad (43)$$

By derivation of equations (41-42), we arrive at:

$$\begin{cases} \dfrac{\partial \phi}{\partial z} = \dfrac{\mu_{33} e_{31} - g_{33} f_{31}}{h_{33}\mu_{33} - g_{33}^2} z\Delta + \phi_0(x,y) \\ \dfrac{\partial \varphi}{\partial z} = \dfrac{h_{33} f_{31} - g_{33} e_{31}}{h_{33}\mu_{33} - g_{33}^2} z\Delta + \varphi_0(x,y) \end{cases} \quad (44-45)$$

So:

$$\begin{cases} \phi = \dfrac{\mu_{33} e_{31} - g_{33} f_{31}}{2(h_{33}\mu_{33} - g_{33}^2)} \Delta\left(z^2 - \dfrac{h^2}{4}\right) + \dfrac{V_0}{h} z + \dfrac{V_0}{2} \\ \varphi = \dfrac{h_{33} f_{31} - g_{33} e_{31}}{2(h_{33}\mu_{33} - g_{33}^2)} \Delta\left(z^2 - \dfrac{h^2}{4}\right) + \dfrac{\Omega_0}{h} z + \dfrac{\Omega_0}{2} \end{cases} \quad (46-47)$$

The magnetic and electric boundary conditions are:

$\phi(h/2). = v_0$; $\phi(-h/2). = 0$

And

$\varphi(h/2) = \Omega_0$, $\varphi(-h/2) = 0$

From equations (21-22), the following relations can be obtained:

$$\begin{cases} \left(1-(e_0 a)^2 \nabla^2\right) M_{xx} = \tilde{c}_{11} \dfrac{h^3}{12} \dfrac{\partial \psi_x}{\partial x} + \tilde{c}_{12} \dfrac{h^3}{12} \dfrac{\partial \psi_y}{\partial y} \\ \left(1-(e_0 a)^2 \nabla^2\right) M_{yy} = \tilde{c}_{12} \dfrac{h^3}{12} \dfrac{\partial \psi_x}{\partial x} + \tilde{c}_{22} \dfrac{h^3}{12} \dfrac{\partial \psi_y}{\partial y} \\ \left(1-(e_0 a)^2 \nabla^2\right) M_{xy} = c_{66} \dfrac{h^3}{12} \left(\dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x}\right) \\ \left(1-(e_0 a)^2 \nabla^2\right) Q_{xx} = c_{44} k h \left(\psi_x + \dfrac{\partial w}{\partial x}\right) \\ \left(1-(e_0 a)^2 \nabla^2\right) Q_{yy} = c_{44} k h \left(\psi_y + \dfrac{\partial w}{\partial y}\right) \end{cases} \quad (48-52)$$

Or :

$$\begin{aligned} \tilde{c}_{11} &= \left(c_{11} + e_{31} M_1 + f_{31} M_2\right) \\ \tilde{c}_{12} &= \left(c_{12} + e_{31} M_1 + f_{31} M_2\right) \quad (53-55) \\ \tilde{c}_{22} &= \left(c_{22} + e_{31} M_1 + f_{31} M_2\right) \end{aligned}$$

With:

$$M_1 = \frac{\mu_{33} e_{31} - g_{33} f_{31}}{h_{33}\mu_{33} - g_{33}^2}$$

$$M_2 = \frac{h_{33} f_{31} - g_{33} e_{31}}{h_{33}\mu_{33} - g_{33}^2} \quad (56-57)$$

By replacing the equations (III.48) - (III.52) by the equations (28-30), we can obtain:

$$\begin{cases} \dfrac{\tilde{c}_{11} h^2}{12} \dfrac{\partial^2 \psi_x}{\partial x^2} + \dfrac{\tilde{c}_{12} h^2}{12} \dfrac{\partial^2 \psi_y}{\partial x \partial y} + \dfrac{c_{66} h^2}{12}\left(\dfrac{\partial^2 \psi_x}{\partial y^2} + \dfrac{\partial^2 \psi_y}{\partial x \partial y}\right) - c_{44} k \left(\psi_x + \dfrac{\partial w}{\partial x}\right) = 0 \\ \dfrac{c_{66} h^2}{12}\left(\dfrac{\partial^2 \psi_y}{\partial x^2} + \dfrac{\partial^2 \psi_x}{\partial x \partial y}\right) + \dfrac{c_{12} h^2}{12} \dfrac{\partial^2 \psi_x}{\partial x \partial y} + \dfrac{c_{22} h^2}{12} \dfrac{\partial^2 \psi_y}{\partial y^2} - c_{44} k \left(\psi_y + \dfrac{\partial w}{\partial y}\right) = 0 \\ c_{55} k h \left(\dfrac{\partial \psi_x}{\partial x} + \dfrac{\partial^2 w}{\partial x^2}\right) + c_{44} k h \left(\dfrac{\partial \psi_y}{\partial y} + \dfrac{\partial^2 w}{\partial y^2}\right) + \left(1-(e_0 a)^2 \nabla^2\right)\Big[K_{w.} w - K_g . \nabla^2 w + \left(N_{xm} + N_{xe} + N_{xa}\right) \dfrac{\partial^2 w}{\partial x^2} \\ + \left(N_{ym} + N_{ye} + N_{ya}\right) \dfrac{\partial^2 w}{\partial y^2}\Big] = 0 \end{cases} \quad (58-60)$$

We introduce the following non-dimensional term:

$$\tilde{w} = \frac{w}{L} \;;\; \bar{x} = \frac{x}{L} \;;\; \bar{y} = \frac{y}{b} \;;\; \delta = \frac{h}{L} \;;\; \eta = \frac{\mathrm{L}}{\mathrm{b}} \;;\; \theta = \frac{k}{\delta^2} \;;\; \bar{c}_{ij} = \frac{\tilde{c}_{ij}}{c_{11}}$$

$$k_w = \frac{K_w L^4}{h^3 c_{11}} \;;\; k_g = \frac{K_g L^2}{h^3 c_{11}} \;;\; \bar{N}_{ij} = \frac{N_{ij} L^2}{h^3 c_{11}} \quad (61)$$

Equations (58-60) written in the following non-dimensional form:

$$\begin{cases} \dfrac{\bar{c}_{11}}{12} \dfrac{\partial^2 \psi_x}{\partial \bar{x}^2} + \dfrac{\bar{c}_{12}\eta}{12} \dfrac{\partial^2 \psi_y}{\partial \bar{x} \partial \bar{y}} + \dfrac{\bar{c}_{66}}{12}\left(\eta^2 \dfrac{\partial^2 \psi_x}{\partial \bar{y}^2} + \eta \dfrac{\partial^2 \psi_y}{\partial \bar{x} \partial \bar{y}}\right) - \bar{c}_{44}\theta\left(\psi_x + \dfrac{\partial \bar{w}}{\partial \bar{x}}\right) = 0 \\ \dfrac{\bar{c}_{66}}{12}\left(\dfrac{\partial^2 \psi_y}{\partial \bar{x}^2} + \eta \dfrac{\partial^2 \psi_x}{\partial \bar{x} \partial \bar{y}}\right) + \dfrac{\bar{c}_{12}\eta}{12} \dfrac{\partial^2 \psi_x}{\partial \bar{x} \partial \bar{y}} + \dfrac{\bar{c}_{22}\eta^2}{12} \dfrac{\partial^2 \psi_y}{\partial \bar{y}^2} - \bar{c}_{44}\theta\left(\psi_y + \eta \dfrac{\partial \bar{w}}{\partial \bar{y}}\right) = 0 \\ \bar{c}_{44}\theta\left(\dfrac{\partial \psi_x}{\partial \bar{x}} + \dfrac{\partial^2 \bar{w}}{\partial \bar{x}^2}\right) + \bar{c}_{44}\theta\left(\eta \dfrac{\partial \psi_y}{\partial \bar{y}} + \eta^2 \dfrac{\partial^2 \bar{w}}{\partial \bar{y}^2}\right) + \bar{k}_w \bar{w} - \bar{k}_g\left(\dfrac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \eta^2 \dfrac{\partial^2 \bar{w}}{\partial \bar{y}^2}\right) + \left(\bar{N}_{xm} + \bar{N}_{xe} + \bar{N}_{xa}\right)\dfrac{\partial^2 \bar{w}}{\partial \bar{x}^2} \\ + \left(\bar{N}_{ym} + \bar{N}_{ye} + \bar{N}_{ya}\right)\eta^2 \dfrac{\partial^2 \bar{w}}{\partial \bar{y}^2} - (e_0 a)^2\Big[k_w\left(\dfrac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \eta^2 \dfrac{\partial^2 \bar{w}}{\partial \bar{y}^2}\right) - k_g\left(\dfrac{\partial^4 \bar{w}}{\partial \bar{x}^4} + 2\eta^2 \dfrac{\partial^4 \bar{w}}{\partial \bar{x}^2 \partial \bar{y}^2} + \eta^4 \dfrac{\partial^4 \bar{w}}{\partial \bar{y}^4}\right) \\ + \left(\bar{N}_{xm} + \bar{N}_{xe} + \bar{N}_{xa}\right)\left(\dfrac{\partial^4 \bar{w}}{\partial \bar{x}^4} + \eta^2 \dfrac{\partial^4 \bar{w}}{\partial \bar{x}^2 \partial \bar{y}^2}\right) + \left(\bar{N}_{ym} + \bar{N}_{ye} + \bar{N}_{ya}\right)\eta^2\left(\dfrac{\partial^4 \bar{w}}{\partial \bar{x}^2 \partial \bar{y}^2} + \eta^2 \dfrac{\partial^4 \bar{w}}{\partial \bar{y}^4}\right)\Big] = 0 \end{cases} \quad (62-64)$$

B. NAVIER SOLUTION OF A SIMPLYS SUPPORTED MAGNETO-PIEZOELECTRIC NANO-PLATE

For a simply supported magneto-piezoelectric nano-plate, the following boundary conditions:

$$\overline{w} = 0 \quad ,. \quad \psi_y = 0 \quad , \quad M_{xx} = 0 \quad , \quad x = 0, L$$

$$\overline{w} = 0 \quad ,. \quad \psi_x = 0 \quad , \quad M_{yy} = 0 \quad , \quad x = 0, b \qquad (65-66)$$

To solve the buckling problem, we assume the following solution:

$$\begin{cases} \psi_x = X_{mn}.\cos(\alpha\overline{x}).\sin(\beta\overline{y}) \\ \psi_y = Y_{mn}.\sin(\alpha\overline{x}).\cos(\beta\overline{y}) \\ \overline{w} = W_{mn}.\sin(\alpha\overline{x}).\cos(\beta\overline{y}) \end{cases} \qquad (67-69)$$

in which

$$\alpha = m\pi$$

and

$$\beta = n\pi$$

With :

m and n : The numbers of the modes.

We therefore obtain the following solution:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} X_{mn} \\ Y_{mn} \\ W_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (70)$$

The matrix parameters are:

$$\begin{cases} L_{11} = -\left(\alpha^2\overline{c}_{11} + \eta^2\beta^2\overline{c}_{66} + 12\theta\overline{c}_{44}\right)/12 \\ L_{12} = L_{21} = -\alpha\beta\eta\left(\overline{c}_{12} + \overline{c}_{66}\right)/12 \\ L_{13} = L_{31} = -\alpha\theta\overline{c}_{44} \\ L_{22} = -\left(\alpha^2\overline{c}_{66} + \eta^2\beta^2\overline{c}_{22} + 12\theta\overline{c}_{44}\right)/12 \\ L_{23} = L_{32} = -\beta\theta\eta\overline{c}_{44} \\ L_{33} = -\alpha^2\theta\overline{c}_{44} - \beta^2\theta\eta^2\overline{c}_{44} + k_w\left[1 + (e_0a)^2\left(\alpha^2 + \beta^2\eta^2\right)\right] + k_g\left[\left(\alpha^2 + \beta^2\eta^2\right) + (e_0a)^2\left(\alpha^4 + 2\alpha^2\beta^2\eta^2 + \beta^4\eta^4\right)\right] \\ \quad -\left(\overline{N}_{xm} + \overline{N}_{xe} + \overline{N}_{xa}\right)\left[\alpha^2 + (e_0a)^2\left(\alpha^4 + \alpha^2\beta^2\eta^2\right)\right] - \left(\overline{N}_{ym} + \overline{N}_{ym} + \overline{N}_{ym}\right)\left[\beta^2\eta^2 + (e_0a)^2\left(\alpha^2\beta^2\eta^2 + \beta^4\eta^4\right)\right] \end{cases} \qquad (71-76)$$

We can write the equation (70) in the form:

$$\left\{ \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & \overline{L}_{33} \end{bmatrix} + P\,L_{mn} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} \begin{Bmatrix} X_{mn} \\ Y_{mn} \\ W_{mn} \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (77)$$

With :

$$\overline{L}_{33} = -\alpha^2\theta\overline{c}_{44} - \beta^2\theta\eta^2\overline{c}_{44} + k_w + k_g\left(\alpha^2 + \beta^2\eta^2\right) - \alpha^2\left(\overline{N}_{xm} + \overline{N}_{xe} + \overline{N}_{xa}\right) - \beta^2\eta^2\left(\overline{N}_{ym} + \overline{N}_{ye} + \overline{N}_{ya}\right)$$

$$L_{mn} = -\alpha^2 - \lambda\beta^2\eta^2$$

- Buckling load:

$$P = \frac{\det\,[A]}{L_{mn}\left(L_{11}L_{22} - L_{12}^2\right)}$$

The expression of the critical buckling load is given in the dimensionless form:

$$P_{cr} = \frac{PL^2}{h^3 c_{11}}$$

IV. CONCLUSION

The behavior of square magneto-piezoelectric nano-plates resting on an elastic medium was obtained using first-order equations of the shear deformation theory. Magnetic and Electric fields are ignored in the plane of the nano-plate. From Maxwell's equation and electromagnetic boundary conditions, the variation of magnetic and electric potentialities along the thickness direction is obtained.

These equations can be useful for the design and analysis of intelligent structures built from piezo-electric materials.

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