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Applications of the statistical convergence and of the ideals in integration

Anita Caushi

Department of Mathematics, Polytechnic University of Tirana, Albania

anita_caushi@yahoo.com

Abstract – In this paper we relativize the concept of statistical Pettis Integration and we propose on type of Pettis integration in concept of ideal convergence. We obtain some properties of ideal Pettis integration which are well known for the statistical Pettis integration on Banach space.

Keywords – *Statistical Pettis Integration, Statistical converges, Ideal Pettis Integration, Ideal converges, Ideal Measurable Function*

INTRODUCTION

In recent years, statistical convergence has increasingly become an attractive area of research. The idea of statistical convergence was initially described by Zigmund [19].The base line concept is the statistical Cauchy convergence of Fridy , [7]. On the Banach space, we adopted the approachfrom the work of Connor , [4] . This paper was inspired by [20] and [21] where the concept of I-conergence of the sequences of real numbers and I-convergence of the function of real valued.We will often quote some results from [20] that can be transferred to function in Banahspace.In [21] it is shown that our I-convergence is, in a sense, equivalent to μ-statistical convergence of J. Connor , [4] . The concept of I-convergence is a generalization of statistical convergence and it is based on the notion of the ideal I of subsets of the set N of positive integers.

I. Preliminaries

Definition 1. Let Y be a set that is not the empty set , Y≠∅. Family$\Im\subset\Pi(Y)$ is called *ideal of the set* Yif and only if,that for A, B∈ℑ it follows that, A∪B∈ℑ and for every A∈ℑand B ⊂ A we will have B ∈ℑ.

(b) The ideal ℑ is called *non-trivial* if and only if,ℑ≠∅and y∉ℑ. A non-trivial ideal is called *acceptable*when it contains the sets with only one point on it.

Let(T, Σ, μ)be a space with probabilistic measure μ, where T is an random set on a line, Σ-Borel'salgebra and μ is a defined measure.

Throughout the paper N will denote the set of positive integers. Let be An a subset of ordered set N. It said to have density δ(A) ,if$\delta(A) = lim_{n\to\infty}\frac{|A_n|}{n}$ where $A_n = \{k < n \,;\, k\in A\}$.

Definition 2 :The vectorial sequence x is statistically convergent to the vector (element) L of a vectorial normed space if for each ε >0

$$lim_{n\to\infty}\frac{1}{n}|\{k \le n : ||x_k - L|| \ge \varepsilon\}| = 0$$

Definition 3.A sequence $x = (x_n)$,n∈N of elements of X is said to be I-convergent to L∈ X if and only if for each $\varepsilon > 0$the set$A(\varepsilon) = \{n\epsilon N : \|x_n - L\| \ge \varepsilon\}$belongs to I. The element L is called the I-limit of the sequence x = {xn},n∈N.I-lim x_n =L.

Definition 4..A sequence $x = (x_n)$,n∈N of elements of X is said to be I-Caushy if for each $\varepsilon > 0$there exists q ∈ N such that $\{n\epsilon N : \left\|x_n - x_q\right\| \ge \varepsilon\} \in I$

Definition 5.A sequence$x = (x_n)$,n∈N is called weackly I-covergentif the sequence $x^*(x_n)$ is I-convergent for every $x^* \in X^*$.

Now, we deals with generalization of Ideal convergence of functions on normed space.
The sequence of functions {f_k} contains the functions with value in vectorial
space.

Definition6 **:**The function$f: T \to X$ is called $\mathfrak{I}$-*measurableonT* , if for every t∈T, $\varepsilon > 0$ and $A \subset \mathfrak{I}$ there is a sequence of simple functions$f_n: T \to X$ for which we have

$\|f_n(t) - f(t)\| < \varepsilon$ for $n \in \mathbb{N} \backslash A$.

Definition 7. The subsequence $(f_{n_k})_{k\in\mathbb{N}}$of the sequence$(f_n)_{n\in\mathbb{N}} \overset{\mathfrak{I}}{\to} f$ is called fundamental iff,for$A' = \{n_1 < n_2 < \cdots < n_k < \cdots\}$; $f_{n_k} \overset{\mathfrak{I}}{\to} f$ for $n \in \mathbb{N}\backslash A'$ where$A' \subset A$.

Definition 8 **.** Let(I, Σ, μ)be a measurable complete space with a non-negative measure**.** The sequence of measured functions $(f_n)_n$in I is $\mathfrak{I}$-convergent according to the measure μ to thefunction f, if for each ε> 0 and σ> 0 there is an essential subsequence$(f_{n_k})_k$of the sequence$(f_n)_n$ such that: $\mu\{t: \|f_{n_k}(t) - f(t)\| \geq \sigma\} < \varepsilon$ for $n_k \in \mathbb{N}\backslash A'$ and t ∈I. We denote $f_n(t) \overset{\mathfrak{I}-\mu}{\longrightarrow} f(t)$.

Definition 9**.** The sequence of measured functions$(f_n)_n$with values in *Banah space* is called $\mathfrak{I}$-fundamental according to the measure μ , $S \subset \mathfrak{I}$,if there is a natural number $(\sigma, S) \subset \mathbb{N}\backslash A$ and there is a subsequence $(f_{n_k})_k$of$(f_n)_n, if\ \forall\varepsilon > 0$ and $\sigma > 0$, $\mu\{t: \|f_{n_k}(t) - f(t)\| \geq \sigma\} < \varepsilon$.

II. Ideal Pettis integration in Banach space.

Lemma 1. [15] (Salat) A sequence x=(x_k) is ideal convergent to p if and only if there exists a set $A' = \{n_1 < n_2 < \cdots < n_k < \cdots\}$,$I - x_{n_k} \to p$, n∈ N/A' ,$A' \subset A$.The $x_{n_k}$is called the essential subsequence of (x_k).
The above lemma can beformulated:
A sequence (x_k) is ideal convergentto p if and only if there exists an essential subsequence (x_{n_k}) which convergesin usual meaning to limes p. We write $I - \lim_K x_k = p$.
We can formulate an immediate corollary of Salat's lemma.

Proposition 1. The sequence $f_k(x)$ where f_n:T→X, (X a vectorial normed space) is ideal-convergent to f(x), if and only if, there exists an essential subsequence $f_{k_n}$of it that is convergent to f(x).
Corollary .The sequence $f_k(x)$is ideal convergent almost everywhere to f(x) on T if there exists an essentialf_{k_n}subsequence of f such that is convergent almost everywhere to f(x).
III . Statistical Pettis integration
Definition 11. [15] A point p is called aideal-sequential accumulation point of the set F if there is a sequence x=(x_k)of points in F\{p} such that I-lim(x_k)=p. The set of all Ideal–sequential accumulation points of F is called Ideal sequential closure of F. We say that a set is Ideal-sequential closed if it contains all the points in its Ideal-closure.
Definition 12. A subset F of X is called Ideal-sequential compact if whenever x=(x_k) is a sequence of points in F there is a subsequence y=(y_{k_n}) of x with I-limy_{k_n}=p∈F.
Proposition 2 **.**A subset F of X is sequentially compact if and only if it is Ideal-sequential compact in it.
Proof. Let F be a subset of Ideal-sequential compact set X. By definition, for every sequence x in F there is asubsequence y_k such that is Ideal convergent to the point p∈F. But the sequence y_k has an essentialsubsequence $(y_{k_n}$)convergent to the same point p. This means that F is sequentially compact.
Let we extend the concept of Pettis integration by means of Ideal convergence. Let (T, Σ,μ) be a measurable space with finite measure μ and X one Banach space.
Definition 13. Let E be a subset of the set T. The function f :T→X is called Ideal Pettis integrable if
a) The function x*f is Ideal Bohner integrable for every x*∈X*
b) There exists an element x_E of X such that
$x*(x_E) = I - \int_E x*(f)\, d\mu$ për çdo $x* \in X*$
The element x_E is called indefinite Ideal Pettis integral and we denote x_E =IP-$\int_E f d\mu$.

Proposition 3. If the function f:T→X is I- Bochner integrable then it is also I- Pettis integrable and the equation holds

$$PI - \int_E f d\mu = BI - \int_E f d\mu$$

Proof**.** Since the function f(s) is I- Bochner integrable there exists a determinant sequence of simple functions $f_n$convergent almost everywhere uniformly and for almost every n to the function f. While the functions x* of X* arecontinuous, we have

$|x^*(f_n)-x^*(f)| \leq \|x^*\|.\|f_n(s)-f(s)\| \to 0$ a.a. n,

So

$$IB\text{-}\int_E |x^*(f_n)-x^*(f)|\,d\mu \leq \|x^*\|\int_E \|f_n - f\|\,d\mu \to 0$$

This means that the sequence of functions x*(fn) is Ideal convergent to x*f. It follows that x*f is I-Bochner integrable as the real function. Considering once more the property of integration of simple functions we have

$x^*\int_E f_n d\mu = \int_E x^* f_n d\mu \to \int_E x^* f d\mu$ for every x* of X*.

On the other hand , the sequence which is Ideal-weakly convergent has a unique limit. It implies that from ideal convergence of the sequence of integrals $\left\{\int_E f_n d\mu\right\}$ to the I-Bochner integral $\int_E f d\mu$ entails the convergence to the I-Bochner integral of the sequence

$$x^*\int_E f_n d\mu \to x^*\int_E f d\mu .$$

Consequently

$$\int_E x^* f d\mu = x^*\int_E f d\mu$$

Because

$$BI - \int_E x*(f_n - f)d\mu|$$
$$\leq BI - \int_E |x*(f_n - f)|d\mu$$
$$\leq ||x*||_{X*}.(BI - \int_E ||f_n - f||_X d\mu$$

And

$$\lim_{n\to\infty} - BI - \int_E ||f_n - f||_X d\mu = 0$$

we proved the existence of Ideal- Pettis integral and its are equal.

III. Conclusion

In this papper we relativized the Pettis integration in the Ideal form and we proved somme properties.

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