All Sciences Proceedings
All Sciences Proceedings
http://as-proceeding.com/

2nd International Conference on Engineering, Natural and Social Sciences
April 4-6, 2023 : Konya, Turkey

ICENSOS
https://www.icensos.com/

Curve Fitting-based Performance Measurement for Decision Making at Noisy Multiobjective Optimization Problems

Tolga Altinoz[1*]

[1]*Department of Electrical and Electronics Engineering, Ankara University, Turkey*

[*]*(taltinoz@ankara.edu.tr) Email of the corresponding author*

Abstract – Engineering problems ae generally contains real life systems that can measure with sensors or contains some factors which can be modelled as noise with probabilistic and statistical properties. In real life, Noise is inherent to engineering problems. As one of the toolsets for engineering problems, the optimization problems like multiobjective optimization problems can be under noise thread. For this reason, to handle the noise, multiobjective optimization algorithms should be improved and additional techniques should be defined/introduced. However, another problem arises at the obtained solution set from the optimization algorithm which is the performance measurement and therefore selecting the solution by decision maker. Since the obtained objective values contains measurement the position of the objectives on the objective space do not represent their true position. For this reason, it is not easy -not possible- for the decision maker to select the proper solution. Also using the conventional the performance measurements on this noisy data is not represent the exact or supportive information for the decision maker. For these reasons in this research a method which is based on curve fitting is proposed. In the proposed method by using the obtained solutions an average -fitted- function is obtained and it is sampled with respect to the position of the obtained solution, and then the shadows of obtained solutions are generated. This shadow set is used to measure the performance and used as indicator for the decision maker.

Keywords – optimization, multiobjective optimization, noise, metric, performance measurement

I. Introduction

The Multiobjective optimization problems has more than one objective so that instead of a single solution a set of solutions are desired to be obtained from the Multiobjective optimization algorithms. The Multiobjective optimization algorithm is defined as

$$\min \quad F(x) = \left(f_1(x) \dots f_M(x)\right) \quad (1)$$
$$subject\ to\ x \in \Omega$$

where Ω is the decision space and the real valued objective functions are defined as F : Ω→R^M [1]. The best solution set at the decision space is called Pareto set and their objective values is called Pareto Front (PF) [2]. The objectives of the Multiobjective optimization problems may be under the additive noise, hence their values are changed with this additive noise. The definition of the problem is changed as

$$F_{noisy}(x) = \overline{F(x)} + \overline{r} \quad (2)$$
$$= \{f_1 + r_1, f_2 + r_2, \dots, f_M + r_M\} \quad (3)$$

where noise vector (r) is added to the each of the objective values. The performance of the Multiobjective optimization algorithms is measured with the pre-definite functions called metrics. A function can be a metric if it represents the difference between two different sets in a sensible manner. That means if the sets with are far away with each other should have different metric

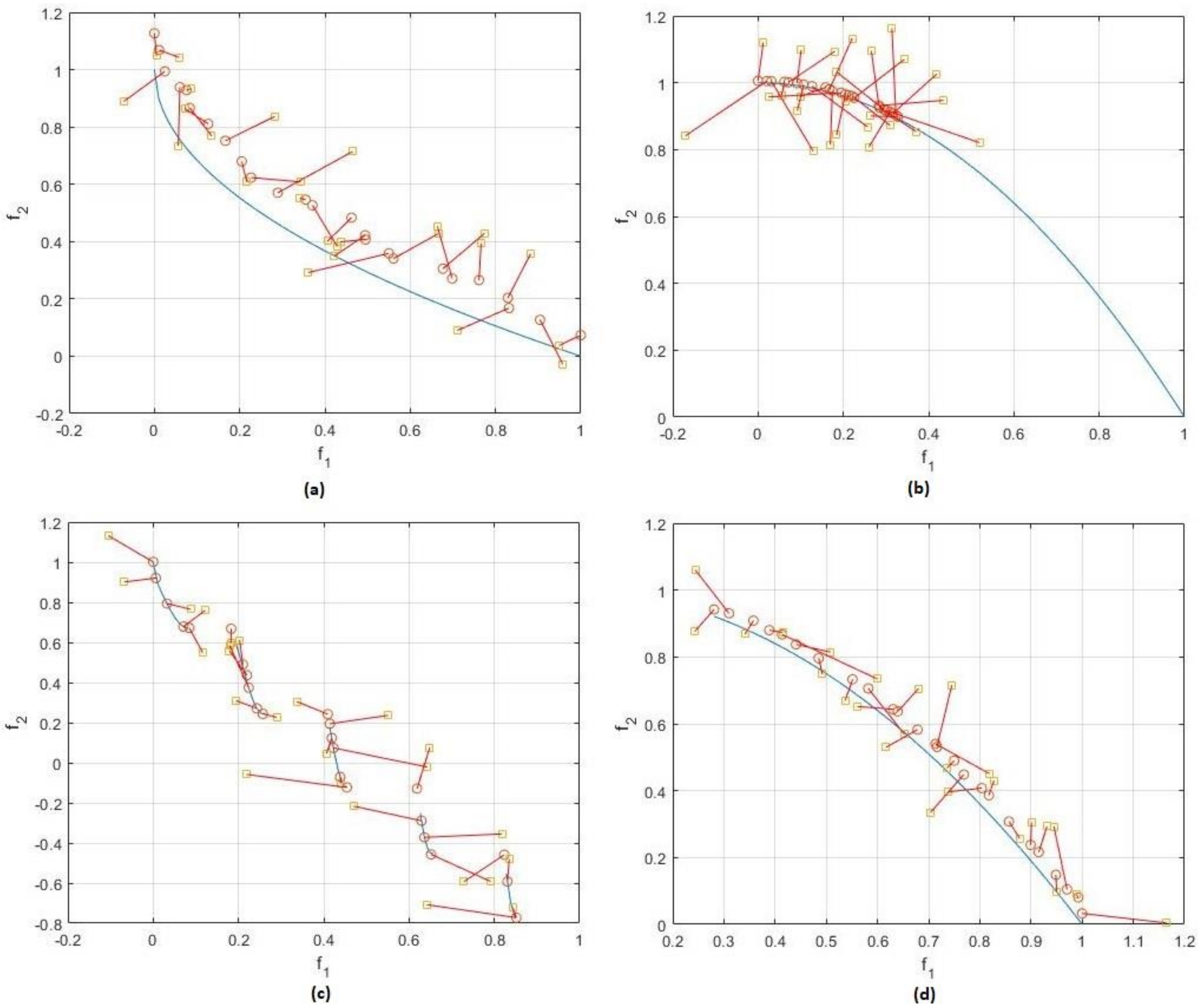

Figure 1. The datasets nose generated from a) ZDT1, b) ZDT2, c) ZDT3, and d) ZDT6 problems and 0.1 standard deviation and zero mean Gaussian noise is added to these datasets. Blue line corresponding to the Pareto Front, circle is the noiseless data and square for the noisy data.

values with a solid comparison between each other. There are many metrics are proposed in literature. One set of metrics are depended on the Pareto front that means to measure the metric Pareto front must be known. These metrics can be divided into three groups. At first group of metrics the closeness of the obtained solution set on objective space to the Pareto front is calculated. The best known and frequently used metric is called inverted generational distance (IGD) [3]. The second group of metrics measures how the solution set distributed on the objective space. This set of metrics are also important because sometimes instead of best solution the number of different solutions are desired by the decision maker (DM). As an example, the spread metric can be givens which measured the average distance of the solutions on the objective space [4]. The last group is the metrics that gives and overall performance indicator. The well-known metric is the hypervolume metric [5]. Hypervolume metric as name indicates the hypervolume of the solution set and a given reference -or inverse- is calculated. This metric gives a value that represents both closeness and distribution of the solutions on objective space with respect to the Pareto front.

These metrics are good for the indicator of the Multiobjective optimization algorithms performance. However, the question remains “Are these metrics can be reliable for noisy problems?” Since the objective values has noise on them, the dominance relations changes among the solutions in the generations. Also at the final solution set, it is not clear which solution can be chosen and the given metric value cannot be reliable since the noise gives un-accurate results. Unfortunate there

Table 1. After the Curve-fitting, obtained model Parameters.

	Dataset-a			Dataset-b			Dataset-c			Dataset-d		
Std.D	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
0.1	1.975	-1.18	1.365	0.9749	0.1841	1.111	1.039	-0.06521	0.2768	0.9434	0.234	0.6045
0.15	0.9667	-0.4718	1.102	1.212	-0.812	2.093	0.6934	0.05426	0.2576	0.8889	0.2569	0.51
0.20	0.9507	-0.2557	0.9002	1.181	-1.278	3.394	0.8646	0.06519	0.2249	0.9233	0.2093	0.6472
0.25	0.9437	-0.4676	1.279	1.094	-0.4518	2.65	0.6558	-0.1967	0.4427	2.201	-754.3	44.93

are not enough study related to the performance indicator and DM for noisy problems. This is indicated in [7] and [8] so that non-dominated solutions are falsely included in the final solution set, that causes undesired selection by DM and wrong metric value. Recently, in [6] Branke proposed two additional metrics for the noisy problems. At each of them instead of the noisy data the unnoisy data is used with respect to the distance to the Pareto front. Also, in [7] the percentage of the true solutions are taken as the metric. Similarly, in [9] a metric called hypervolume difference is used for noisy problems.

In this research, a new method to evaluate the noisy benchmark problems is proposed. In this method by using the curve fitting toolset, the solution set is used to fit a function. Then the function is sampled by using the features of the obtained dataset. This feature is the distance and the distribution of the solution. This new set called shadow set. Each member of the shadow set is assigned one of the decision sets. Finally, by using this new called shadow set, the metric is calculated, and DM can select proper solution among this new set.

This paper is organized as five sections. Section 2 and Section 3 explains the proposed method and presents the discussion of the noise on the objective space and the metric. Section 4 given for the implementation section that gives the proposed benchmark problems and their possible solutions and comparisons. Finally, the conclusion section is given as the final section.

II. Noiseless Objective Functions are known

In this section, the metrics and DM is discussed when both noisy and noiseless objective functions are known. These problems generally valid for benchmark problems. In real life problems the Pareto Front and especially the exact mathematical model may not be known. However, for linear programming case it is possible to model and known the problem where generally the solution is at the border of the search space.

If all the noiseless objective functions are known, the noise easily neglectable and to calculate the performance metrics or to use some indicators at the optimization algorithm, it is possible to evaluate noiseless objective functions. For this case, the noise is not a problem to solve. How about not for all the functions but some function's noiseless objective is known, or noiseless objectives are not known but all statistical properties are known.

A. *Some of the noiseless objective functions are known.*

If the objectives are represented as f_i for the noiseless objective and fir for the noisy objective function, hence $f_{ir}=f_i+r_i$ and r_i is the noise. In this form of the objective functions i is the index of the objectives and $i=1,2,\ldots,m$ where there are m number of objective functions. Among these objectives some of the objectives' noiseless and noisy functions may be known. In this case, by using the noiseless and noisy functions their values; it is possible to extract the statistical properties of the noise may be obtained. The error signal can be defined as $e_i = |f_{ir} - f_i|$. The known objective functions help to gain information about the noise. The fundamental property is the mean of the noise (signal e). If the mean of the noise signal is known easily this bias is subtracted from the noisy objective functions.

B. *Statistical Information are known or extracted.*

As discuss in the previous sub-section, when the noisy signal is known, or its statistical properties are known it is possible to approximate a noiseless objective value. For this purpose, different methods may be used to solve this problem with the aid of digital signal processing methods and machine learning methods. The neural network may be the first method that can be used to remove the noise from a signal. The network can learn the difference between noisy and noiseless data from the objective function known as discussed in the previous sub-section. This network will be

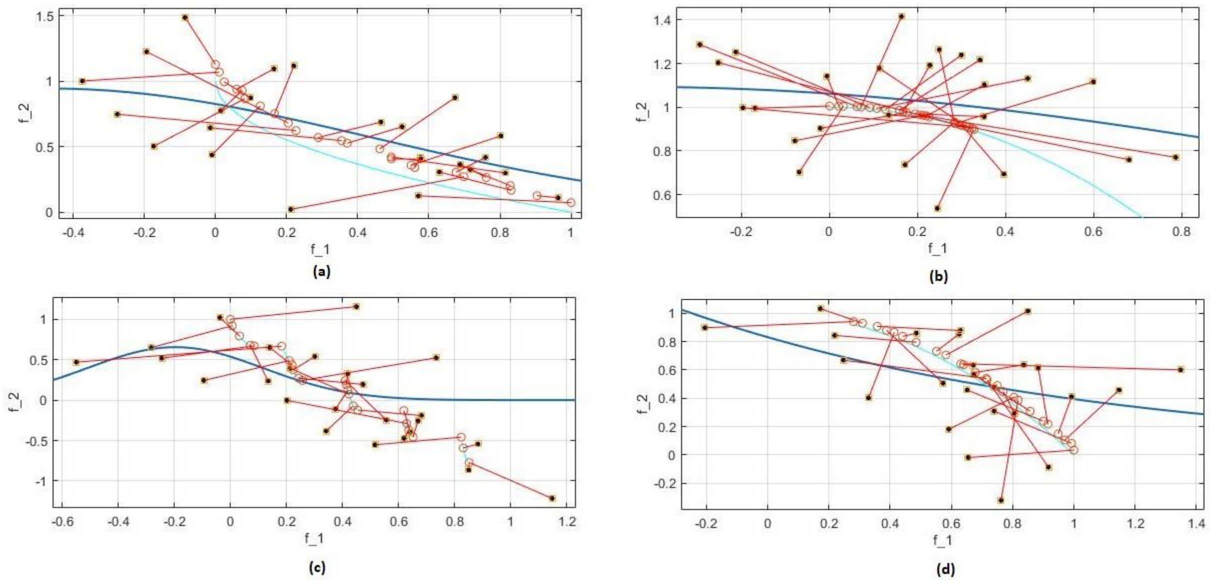

Figure 2. The fitted Gaussian function and corresponding noisy data for standard deviation 0.25

available to use in other objective values. Another example is the well-known Wiener filter. The Wiener filters are preferred for reducing additive noise when the noisy signal is available. This filter retrieves the signal s(n) when the noisy signal is defined as x(n) = s(n) + v(n) where v(n) is the noise. Wiener filter is nothing but an optimization algorithm. It is desired to get the optimal filter weights from linear minimum mean square error estimator. Unfortunately, there isn't enough research in the literature to make research related to these topics however the theoretical background of these methods looks promising, also in [10] it is showed that Wiener filter can be applicable for Multiobjective noisy problems, and in [11] neural networks are applied for noise optimization. There is more research is needed to increase the knowledge related to this topic.

III. Noiseless Objective Functions are not known

This section is the main contribution of this research so that when there is no information about the noiseless objective function and no statistical information exists for the objectives and noise. For this case "How can the performance measured with metrics?" is the main question of this research.

C. *No Information about Noise*

Generally, the noisy objectives are obtained from the noisy decision variables in real life engineering problems. Since the objectives are generally formulated with relatively complex mathematical expressions, it is not easy to extract the statistical information about the noise. Also, since noise exists in the environment it is not possible to get a noiseless objective. For this reason, it is not possible to get the information about noiseless objective. In this case measuring the performance and help the DM process is crucial. In this research a new method based on curve-fitting is proposed. The idea is to map the obtained noisy solution set to a temporary set named as shadow set. This set is generated from sampling of the fitting function. In this research the Gaussian model is preferred, and the model is given in Eq. 4.

$$y(n) = p_1 \exp\left(-\left(\frac{x(n)-p_2}{p_3}\right)^2\right) \quad (4)$$

The model has three parameters to be decided based on obtained solution set -in objective space. Levenberg-Marquart method is selected as the optimizer to the curve fitting process, and three parameters are decided by using this optimizer.

After the parameters of the model given in Eq. 4 obtain, the new function sampled to get the same amount of data with the obtained solution set. In this case there are two different method is proposed: Case 1: Equal division, Case 2: Division with respect to previous set. For Case 1, the function is samples so that there is same amount of data with the equal distance. At Case 2, the data is sampled by using the obtained data so that the point on Eq 4 with the minimum distance to Fr is calculated as became as the shadow set.

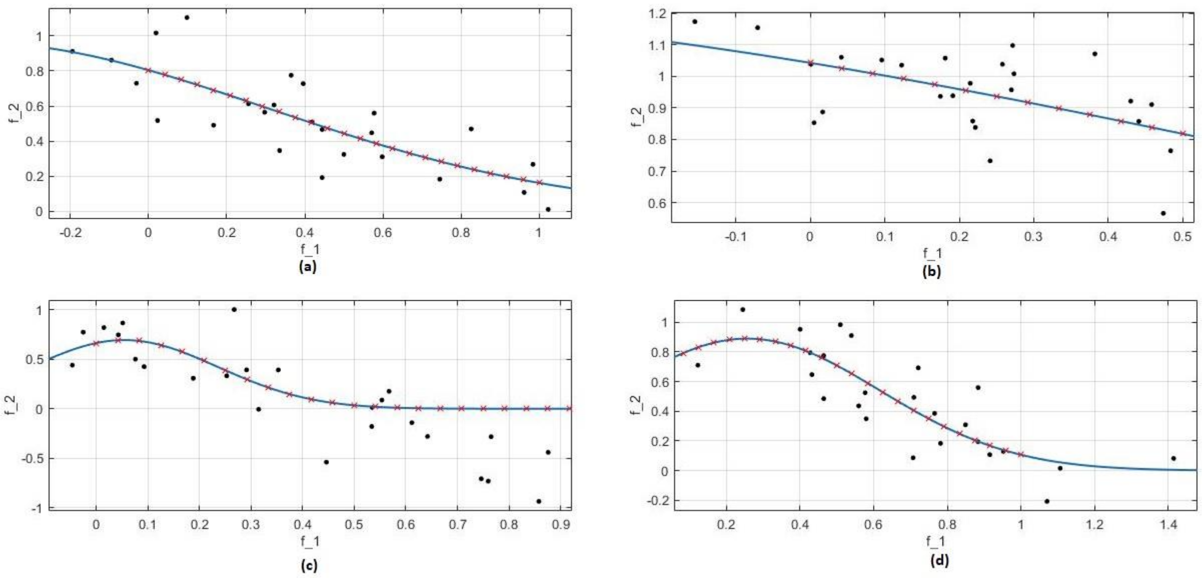

Figure 3. The sampled data with standard deviation 0.15 for metric calculation for Case 1

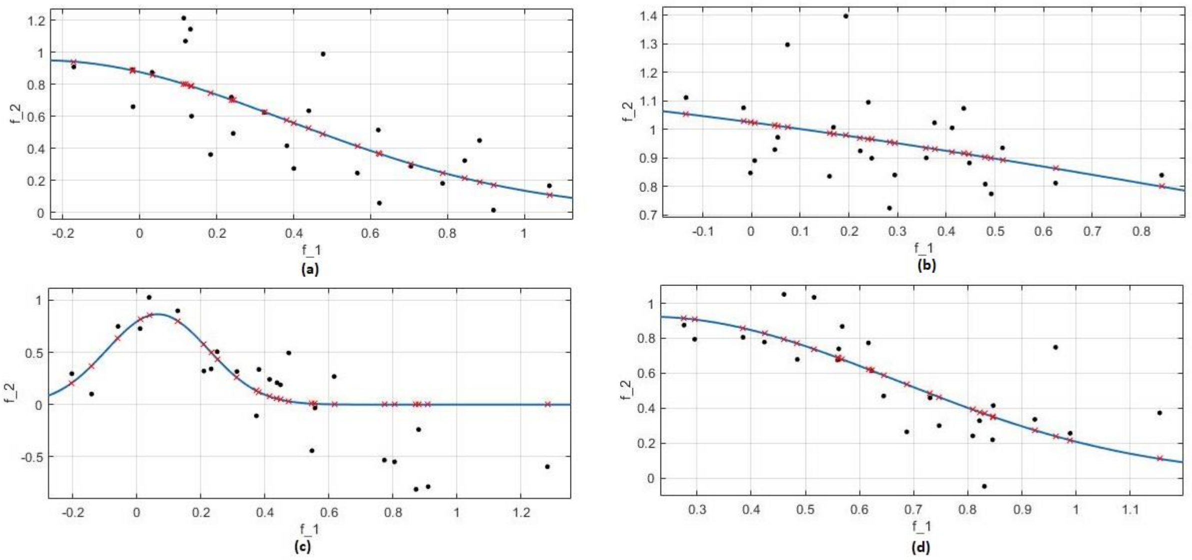

Figure 4. The sampled data with standard deviation 0.2 for metric calculation for Case 2

After the shadow set (s) is generated, the metric is calculated using this set instead of the obtained solution set. Also, DM can make the selection among shadow set (s).

D. *Decision Making and Metric.*

There are many metrics proposed by the researcher can be found in literature. However, among them the most frequently preferred method is the IGD metric that is defined in Eq. 5.

$$f_m = \sqrt{\frac{1}{n}\sum {d_{p,o}}^2} \quad (5)$$

where fm is the metric function, $d_{p,o} = \sqrt{\sum_{k=1}^{M}(p(k) - o(k))^2}$ is the Euclidean distance between Pareto Front and the obtained solution set. The data p and o corresponding to samples set from Pareto Front and the obtained solution set/Shadow set. The metric is calculated after the new set - shadow set- is generated. The DM process can be followed the metric calculation.

IV. BENCHMARK DATA SET AND EXAMPLES

The proposed method and its performance will be demonstrated on the problem set. However, it is not possible to find benchmark dataset for noisy optimization for metric design and other possible r

Table 2. The IGD metric values for all the instances with respect to the given benchmark dataset

	Case 1				Case 2			
Std	Dataset-a	Dataset-b	Dataset-c	Dataset-d	Dataset-a	Dataset-b	Dataset-c	Dataset-d
0.1	0.0951	0.2624	0.1138	0.0984	0.1188	0.2945	0.3013	0.0421
0.15	0.1365	0.2267	0.0755	0.1087	0.1168	0.2593	0.3357	0.0500
0.2	0.0992	0.2099	0.0887	0.0940	0.1256	0.2336	0.2593	0.0397
0.25	0.1229	0.2721	0.1180	0.1420	0.1872	0.2954	0.3182	0.1611
	Noisy Data				Noiseless IGD Metric Values are:			
	Dataset-a	Dataset-b	Dataset-c	Dataset-d	Dataset-a = 0.0866 Dataset-b = 0.3457 Dataset-c = 0.2582 Dataset-d = 0.0303			
0.1	0.0775	0.2552	0.1828	0.0401				
0.15	0.0794	0.1979	0.2493	0.0612				
0.2	0.0607	0.2261	0.1469	0.0564				
0.25	0.1475	0.2283	0.1695	0.0919				

research. Therefore, as the first step of the implementation the datasets are generated. Figure 1 shows these datasets on the objective space.

Four set of datasets are generated for this study. These datasets are graphically demonstrated in Figure 1 and the reader can download the benchmark codes and the data used to generate figures from (10.13140/RG.2.2.34190.84809). These datasets are generated from ZDT1,2,3 and 6 benchmark problems. The main reason is their different Pareto Front shape, convex, concave, and discrete. These three different sets of Pareto shapes make harder and challenging to demonstrate the proposed method's capabilities. In the figures, it is possible to visualize Pareto Front, true position of the obtained solutions and noisy solution set. In this research zero mean additive Gaussian noise is preferred with 0.1, 0.15, 0.2 and 0.25 standard deviations. In figure 1 only 0.1 is demonstrated. The datasets are named as Dataset-a, Dataset-b, Dataset-c and Dataset-d as given in figure, respectively.

Table 1 shows the parameter set of the Fitted Gaussian model for the given noisy datasets. In addition, in Figure 2, the fitted Gaussian function, noisy dataset and obtained solution set is graphically demonstrated for standard deviation 0.25. From Figure 2, it can be observed that the noisy data will not let the fitting curve to follow the Pareto front. Only for concave case (Fig 2b) it is possible to get a concave fit function. For discrete and convex case since the noise is relatively large with respect to the obtained solution a false plot observed. Even these results look undesired, it will be more apparent when the metric values are compared. In addition, the results also support the validation of the proposed benchmark datasets.

Next, the obtained function will be samples to evaluate for metric calculation (IGD). Two different sampling is evaluated in this research. In Case 1: the function is divided into equal number of samples with the obtained solution with equal distance between each other. For Case 1, only the valid section of the fitted function is considered, that means if the Pareto Front is defined in [0,1] that the function samples only inside this range. In Case 2: the data is samples with respect to the obtained solution so that the other objective values are calculated from the value of the first objective. Figure 3 and figure 4 show the sampled data for Case 1 and Case 2, respectively.

As the last step of this research the obtained -shadow- data is applied to the IGD metric and IGD metric values are compared with noiseless obtained data, noisy obtained data, and shadow data on four different data set with four different noise and two different cases. Table 2 gives the metric values for all these test instances.

Table 2 gives the numerical IGD metric results for noisy and noiseless dataset and two shadow datasets as Case 1 and Case 2. The noiseless IGD metric value represents the basis of the solutions. Under the noise, it is expected to get a closer IGD metric value with the noiseless metric values. If the noisy data is compared with the noiseless metric values for all different standard deviation values. For all noise with different standard deviation, as the standard deviation increases the difference between noiseless and noisy metric values increases which demonstrates the impact of the noise and on metric value.

Next the proposed curve-fitting based data preparation method is applied to the problems as model parameters reported in Table 1. It can be observed from the results for Case 1 and 2 that the impact of the noise statistical property -standard deviation- on the metric values decreased greatly. The metric values for all different standard deviation values are close each other. Then when the metric values are compared between Case 1 and Case 2 and noiseless metric values, it can be observed that the results in Case 2 is much like Noiseless metric values than Case 1 and Noisy Data. Hence, it is possible to suggest evaluating the proposed shadow dataset technique with respect to the curve-fitting can be applicable to evaluate the performance of the noisy problems and help the DM process.

V. CONCLUSION

The noise optimization problems are generally assuming to be noiseless and solve the problems. Even they work in experimental studies the real-life systems have noise in them. Therefore, some methods are needed to handle the noise. However, in real like problems the information related to the noise may be not accessible. Therefore, after the Multiobjective optimization algorithm -with or without noise reduction/handle mechanisms-produce a set of solutions with has under the noise influence. After the optimization algorithm completed the performance evaluation and DM process follows. However, since the obtained solution set contains noise the obtained metric values may lead the solutions and selection of the solution misleads the DM. For this reason, in this study a more robust metric calculation methodology is proposed with the ais of curve-fitting tools. The solutions suggested the statistical change on the noise data mostly cancelled by the proposed metho and the obtained metric value is much closer to the noiseless data. Therefore, it is suggested to use the preferred metric method and it is possible to proposed more improved method by following the methodology which is proposed in this research.

REFERENCES

[1] Miettinen K., "Nonlinear Multiobjective Optimization," Norwell, MA: Kluwer, 1999.

[2] Deb K., "Multi-Objective Optimization using Evolutionary Algorithms," New York: Wiley, 2001

[3] C. A. Coello Coello and M. Reyes Sierra, "A study of the parallelization of a coevolutionary multi-objective evolutionary algorithm," Advances in Artificial Intelligence Third Mexican International Conference on Artificial Intelligence, pp. 688–697, 2004.

[4] A. Zhou, Y. Jin, Q. Zhang, B. Sendhoff, E. Tsang, "Combining model-based and genetics-based offspring generation for multi-objective optimization using a convergence criterion," Evolutionary Computation, CEC 2006, pp. 892–899, 2006.

[5] M. Emmerich, A. Deutz, J. Kruisselbrink, P. K. Shukla, "Cone-based hypervolume indicators: construction, properties, and efficient computation," International conference on evolutionary multi-criterion optimization, Springer, pp. 111–127, 2013.

[6] J. Branke, "Performance Metrics for Multi-Objective Optimisation Algorithms Under Noise," arXiv:2302.14179v1, 2023.

[7] Q. Chen, C. Fu, and Q. Zhang, "On performance of decomposition-based moeas in noisy environment," in 2015 IEEE Congress on Evolutionary Computation (CEC). IEEE, 2015, pp. 3412–3417.

[8] S. R. Hunter, E. A. Applegate, V. Arora, B. Chong, K. Cooper, O. Rincon-Guevara, and C. Vivas-Valencia, "An introduction to multi-objective simulation optimization," ACM Transactions on Modeling and Computer Simulation (TOMACS), vol. 29, no. 1, pp. 1–36, 2019.

[9] J. Branke and W. Zhang, "Identifying efficient solutions via simulation: myopic multi-objective budget allocation for the bi-objective case," OR Spectrum, vol. 41, no. 3, pp. 831–865, 2019.

[10] R. Liu, Y. Li, H. Wang, J. Liu, "A noisy multi-objective optimization algorithm based on mean and Wiener filters," Knowledge-Based Systems, Vol. 228, pp. 107215, 2021.

[11] L. Xiao, Z. Zhang, Y. Peng, "Noise Optimization for Artificial Neural Networks," arXiv:2102.04450, pp. 1-11, 2021.