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Study the behavior of nanocomposite beams under bending load

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Abstract – In this work, a trigonometric refined beam theory for the bending analysis of carbon nanotube-reinforced composite (CNTRC) beams resting on elastic foundation is developed. The significant feature of this model is that, in aaddition to including the shear deformation effect, it deals with only 3 unknowns as the Timoshenko beam (TBM) without including a shear correction factor. The single-walled carbon nanotubes (SWCNTs) are aligned and distributed in polymeric matrix with different patterns of reinforcement. The material properties of the CNTRC beams are assessed by employing the rule of mixture. To examine accuracy of the present theory, several comparison studies are investigated. Furthermore, the effects of different parameters of the beam on the bending, buckling responses of CNTRC beam are discussed.

Keywords – CNTRC Beams, Bending, Elastic Foundation, Ship Solution

I. INTRODUCTION

Recently, Carbon nanotubes (CNTs) become a new class of fiber reinforcement in polymer matrix composites due to their superior mechanical, electrical, and thermal properties [1] and have taken a considerable research interests in the materials engineering community. Compared with the classical carbon fiber-reinforced polymer composites, carbon nanotube-reinforced composites (CNTRCs) have the potential of improving increased strength and stiffness. The polymer composites reinforced by aligned CNT arrays were investigated in the first time by [2]. From then, many researchers [3] studied the material characteristics of CNTRCs. [4] studied the bending, buckling and vibration behaviors of carbon nanotube-reinforced composite (CNTRC) beams where several higher-order shear deformation theories are presented and discussed in details. Recently, the stability of FG sandwich plate was studied by [5] using a higher order refined computational models. In literature survey, we can found also some studies dealing about beams resting on elastic foundations such as [6].

In the present work, the bending of the CNTRC beams is investigated using a trigonometric refined beam theory. The simply supported CNTRC beams are supported by the Pasternak elastic foundation, including a shear layer and Winkler spring. Novel analytical solutions of deflections, stresses, are developed and discussed in details. Several aspects of spring parameters, thickness ratios, CNT volume fractions, types of CNT distribution, etc., which have considerable impact on the analytical solutions are also studied.

II. . Functionally Graded Carbon Nanotube-Reinforced Composites Beams

The CNTRC beam under the present study is made from a mixture of the SWCNTs and isotropic polymer matrix. Figure 1a shows a CNTRC beam, having length (L) and thickness (h), supported by the Pasternak elastic foundation. Four different patterns of reinforcement over the cross sections are considered in this study as is indicated in Fig. 1b.

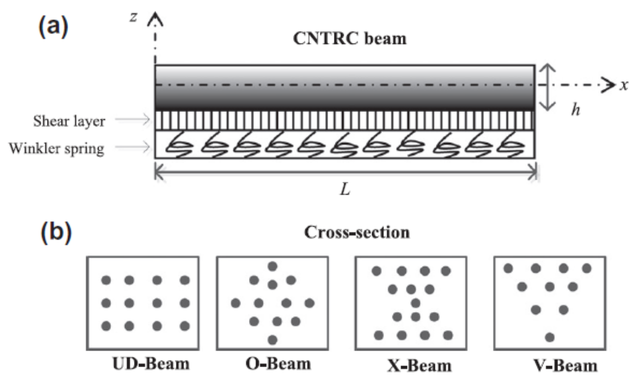

Fig. 1: Geometry of a CNTRC beam on elastic foundation (a) and cross sections of different patterns of reinforcement (b).

The material properties of CNTRC beams can be computed utilizing the rule of mixture which gives the effective Young's modulus and shear modulus of CNTRC beams as [7].

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E_p \qquad (1a)$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E_p} \qquad (1b)$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G_p} \qquad (1c)$$

where E_{11}^{cnt}; E_{22}^{cnt} and G_{12}^{cnt} are the Young's modulus and shear modulus of SWCNT, respectively and E^p and G^p are the corresponding material properties of the polymer matrix. Also, V_{cnt} and V_p are the volume fractions for carbon nanotube and the polymer matrix, respectively, with the relation of $V_{cnt} + V_p = 1$. To introduce the size-dependent material properties of SWCNT, the CNT efficiency parameters, η_i (i = 1, 2, 3), are considered. They can be obtained from matching the elastic moduli of CNTRCs estimated by the MD simulation with the numerical results determined by the rule of mixture [8]. By employing the same rule, Poisson's ratio (ν) and mass density (ρ) of the CNTRC beams are expressed as:

$$\nu = V_{cnt}\nu^{cnt} + V_p \nu^p, \quad \rho = V_{cnt}\rho^{cnt} + V_p \rho^p \qquad (2a)$$

Where ν^{cnt}, ν^p and ρ^{cnt}, ρ^p are the Poisson's ratios and densities of the CNT and polymer matrix respectively.

III. THEORY AND FORMULATIONS

III.1. KINEMATICS AND CONSTITUTIVE EQUATIONS

The displacement field of the present theory, based on [9]. beam theory, can be obtained as:

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \qquad (5a)$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \qquad (5b)$$

Where u_0 is the axial displacement, w_b and w_s are the bending and shear components of transverse displacement along the mid-plane of the beam. In this work, the shape function $f(z)$ is chosen based on a trigonometric function as [10]:

$$f(z) = z - \frac{h}{\pi}\sin\left(\frac{\pi}{h}z\right) \quad (6)$$

$$g(z) = 1 - f'(z) \text{ and } f'(z) = \frac{df(z)}{dz} \quad (7)$$

By assuming that the material of CNTRC beam obeys Hooke's law, the stresses in the beam become:

$$\sigma_x = Q_{11}(z)\,\varepsilon_x \text{ and } \tau_{xz} = Q_{55}(z)\,\gamma_{xz} \quad (8)$$

where

$$Q_{11}(z) = \frac{E_{11}(z)}{1-\nu^2} \text{ and } Q_{55}(z) = G_{12}(z) \quad (9)$$

III.2. EQUATIONS OF MOTION

Hamilton's principle is employed herein to determine the equations of motion:

$$\int_{t_1}^{t_2} (\delta U + \delta V - \delta K)dt = 0 \quad (10)$$

where δU is the virtual variation of the strain energy; δV is the virtual variation of the potential energy; and δK is the virtual variation of the kinetic energy.

The variation of the strain energy of the beam can be stated as

$$\delta U = \int_0^L \int_{-h/2}^{h/2} (\sigma_x \delta\,\varepsilon_x + \tau_{xz}\delta\,\gamma_{xz})dzdx$$
$$= \int_0^L \left(N\frac{d\delta\,u_0}{dx} - M_b\frac{d^2\delta\,w_b}{dx^2} - M_s\frac{d^2\delta\,w_s}{dx^2} + Q\frac{d\delta\,w_s}{dx}\right)dx$$

(11)

where N, M_b, M_s and Q are the stress resultants defined as (6)

$$(N, M_b, M_s) = \int_{-h/2}^{h/2} (1, z, f)\,\sigma_x dz \text{ and } Q = \int_{-h/2}^{h/2} g\,\tau_{xz} dz$$

(12)

The variation of the potential energy by the transverse load q, the axial compressive force N_{x0} and the density of reaction force of foundation f_e can be written :

$$\delta V = -\int_0^L \left[(q + f_e)(\delta\,w_b + \delta\,w_s) + N_{x0}\frac{d(w_b + w_s)}{dx}\frac{d(\delta\,w_b + \delta\,w_s)}{dx}\right]dx$$

(13)

With

$$f_e = K_w w - K_s\frac{\partial^2 w}{\partial x^2}$$

(14)

where K_w and K_s are the Winkler and shearing layer spring constants which can be determined from $K_w = \beta_w A_{110}/L^2$ and $K_s = \beta_s A_{110}$ in which β_w and β_s are the corresponding spring constant factors. It is also defined that A_{110} is the extension stiffness or the value of A_{11} of a homogeneous beam made of pure matrix material.

The variation of the kinetic energy can be expressed as

$$\delta K=\int_0^L\int_{-h/2}^{h/2}\rho(z)\left[\dot{u}\delta\dot{u}+\dot{w}\delta\dot{w}\right]dzdx$$

$$=\int_0^L\left\{I_0\left[\dot{u}_0\delta\dot{u}_0+\left(\dot{w}_b+\dot{w}_s\right)\left(\delta\dot{w}_b+\delta\dot{w}_s\right)\right]-I_1\left(\dot{u}_0\frac{d\delta\dot{w}_b}{dx}+\frac{d\dot{w}_b}{dx}\delta\dot{u}_0\right)\right.$$
$$+I_2\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\dot{w}_b}{dx}\right)-J_1\left(\dot{u}_0\frac{d\delta\dot{w}_s}{dx}+\frac{d\dot{w}_s}{dx}\delta\dot{u}_0\right)+K_2\left(\frac{d\dot{w}_s}{dx}\frac{d\delta\dot{w}_s}{dx}\right)$$
$$\left.+J_2\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\dot{w}_s}{dx}+\frac{d\dot{w}_s}{dx}\frac{d\delta\dot{w}_b}{dx}\right)\right\}dx$$

(15)

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are the mass inertias defined as

$$\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\int_{-h/2}^{h/2}\left(1,z,f,z^2,z\,f,f^2\right)\rho(z)dz$$

(16)

Substituting the expressions for δU, δV, and δK from Eqs. (10), (12), and (14) into Eq.(9) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, δw_b, and δw_s, the following equations of motion of the CNTRC beam are obtained

$$\delta u_0:\ \frac{dN}{dx}=I_0\ddot{u}_0-I_1\frac{d\ddot{w}_b}{dx}-J_1\frac{d\ddot{w}_s}{dx}$$

$$\delta w_b:\ \frac{d^2M_b}{dx^2}+q-f_e+N_{x0}\frac{d^2(w_b+w_s)}{dx^2}=I_0(\ddot{w}_b+\ddot{w}_s)+I_1\frac{d\ddot{u}_0}{dx}-I_2\frac{d^2\ddot{w}_b}{dx^2}-J_2\frac{d^2\ddot{w}_s}{dx^2}$$

$$\delta w_s:\ \frac{d^2M_s}{dx^2}+\frac{dQ}{dx}+q-f_e+N_{x0}\frac{d^2(w_b+w_s)}{dx^2}=I_0(\ddot{w}_b+\ddot{w}_s)+J_1\frac{d\ddot{u}_0}{dx}-J_2\frac{d^2\ddot{w}_b}{dx^2}-K_2\frac{d^2\ddot{w}_s}{dx^2}$$

(17)

By substituting Eq. (7) into Eq. (8) and the subsequent results into Eq. (11), the constitutive equations for the stress resultants are obtained as

$$N=A_{11}\frac{du_0}{dx}-B_{11}\frac{d^2w_b}{dx^2}-B_{11}^s\frac{d^2w_s}{dx^2}$$

$$M_b=B_{11}\frac{du_0}{dx}-D_{11}\frac{d^2w_b}{dx^2}-D_{11}^s\frac{d^2w_s}{dx^2}\quad(18)$$

$$M_s=B_{11}^s\frac{du_0}{dx}-D_{11}^s\frac{d^2w_b}{dx^2}-H_{11}^s\frac{d^2w_s}{dx^2}$$

$$Q=A_{55}^s\frac{dw_s}{dx}$$

where A_{11}, B_{11}, etc., are the beam stiffness, defined by

$$\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right)=\int_{-h/2}^{h/2}Q_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)dz$$

(19)

and

$$A_{55}^s=\int_{-h/2}^{h/2}Q_{55}\left[g(z)\right]^2dz,\qquad(20)$$

Equations (16) can be expressed in terms of displacements (u_0, w_b, w_s) by using Eqs. (17) and (16) as follows:

$$B_{11}^s\frac{\partial^3u_0}{\partial x^3}-D_{11}^s\frac{\partial^4w_b}{\partial x^4}-H_{11}^s\frac{\partial^4w_s}{\partial x^4}+A_{55}^s\frac{\partial^2w_s}{\partial x^2}+q-f_e+N_{x0}\frac{d^2(w_b+w_s)}{dx^2}=I_0(\ddot{w}_b+\ddot{w}_s)$$
$$+J_1\frac{d\ddot{u}_0}{dx}-J_2\frac{d^2\ddot{w}_b}{dx^2}-K_2\frac{d^2\ddot{w}_s}{dx^2}$$

(20)

III.3. ANALYTICAL SOLUTION

The Navier solution method is employed to obtain the analytical solutions for a simply supported

CNTRC beam. The solution is assumed to be of the form

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x) e^{i \omega t} \\ W_{bm} \sin(\lambda x) e^{i \omega t} \\ W_{sm} \sin(\lambda x) e^{i \omega t} \end{Bmatrix} \quad (21)$$

where U_m, W_{bm}, and W_{sm} are arbitrary parameters to be determined, ω is the eigenfrequency associated with mth eigenmode, and $\lambda = m\pi / L$. The transverse load q is also expanded in Fourier series as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda x) \quad (22)$$

where Q_m is the load amplitude calculated from

$$Q_m = \frac{2}{L} \int_0^L q(x) \sin(\lambda x) dx \quad (23)$$

The coefficients Q_m are given below for some typical loads. For the case of a sinusoidally distributed load, we have

$$m = 1 \text{ and } Q_1 = q_0$$

and for the case of uniform distributed load, we have

$$Q_m = \frac{4q_0}{m\pi}, \quad (m = 1,3,5....) \quad (24)$$

Substituting the expansions of u_0, w_b, w_s, and q from Eqs. (20) and (21) into the equations of motion Eq. (19), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \right) \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ Q_m \end{Bmatrix}$$

(25)

Where

$$S_{11} = A_{11}\lambda^2, S_{12} = -B_{11}\lambda^3,$$
$$S_{22} = D_{11}\lambda^4 + K_W + K_S\lambda^2 + N_{X0}\lambda^2,$$
$$S_{23} = D_{11}^s\lambda^4 + K_W + K_S\lambda^2 + N_{X0}\lambda^2,$$
$$S_{33} = H_{11}^s\lambda^4 + A_{55}^s\lambda^2 + K_W + K_S\lambda^2 + N_{X0}\lambda^2$$

(26)

$$m_{11} = I_0, \quad m_{12} = -I_1\lambda, \quad m_{13} = -J_1\lambda,$$
$$m_{22} = I_0 + I_2\lambda^2, \quad m_{23} = I_0 + J_2\lambda^2, \quad m_{33} = I_0 + K_2\lambda^2$$

(27)

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results of bending, behaviors of CNTRC beams are presented and discussed. The effective material characteristics of CNTRC beams at ambient temperature employed throughout this work are given as follows. Poly methyl methacrylate (PMMA) is utilized as the matrix and its material properties are: $\nu^p = 0.3$; $\rho^p = 1190$ kg/m^3 and $E^p = 2.5$ GPa. For reinforcement material, the armchair (10, 10) SWCNTs is chosen with the following properties: $\nu^{cnt} = 0.19$; $\rho^{cnt} = 1400$ kg/m^3; $E_{11}^{cnt} = 600$ GPa; $E_{22}^{cnt} =$ 10 GPa and $G_{12}^{cnt} =$ 17.2 GPa.

For convenience, the following nondimensionalizations are employed:

- For bending analysis: $\bar{w} = 100 \frac{E_p h^3}{q_0 L^4} w$,

$$\bar{\sigma}_x = \frac{h}{q_0 L} \sigma_x \left(\frac{L}{2}, \frac{h}{2} \right), \quad \bar{\tau}_{xz} = \frac{h}{q_0 L} \tau_{xz}(0,0)$$

Where A_{110} and I_{00} are A_{11} I_{00} and I_0 I0 of beam made of pure matrix material, respectively. (24)

IV.1. BENDING ANALYSIS OF CNTRC BEAMS

For bending analysis of UD beams with and without elastic foundations, the present method agree well with the bending results of Wattanasakulpong and Ungbhakorn (2013) using third shear deformation theory as shown in Table 1. It can be observed that the beams supported by elastic foundation have lower displacements and stresses compared to those of the beams without elastic foundation. Moreover, increasing amount of CNTs makes the CNTRC beams stiffer.

Figures 2 and 3 present respectively the effect of both Winkler modulus parameter and the Pasternak shear modulus on the deflection of different types of CNTRC beams under uniform load. It is observed that as the Winkler and the Pasternak shear parameters increase the transverse displacement decreases. This decreasing trend is attributed to the stiffness of the elastic medium. Indeed, it is found from Eq. (24) that the foundation parameters appear in the stiffness matrix [*S*] and at last increase the total stiffness of the CNTRC beam. It can be also observed that the strongest beam is the X-Beam with the smallest deflection, and followed by the UD-, V- and O-Beams, respectively.

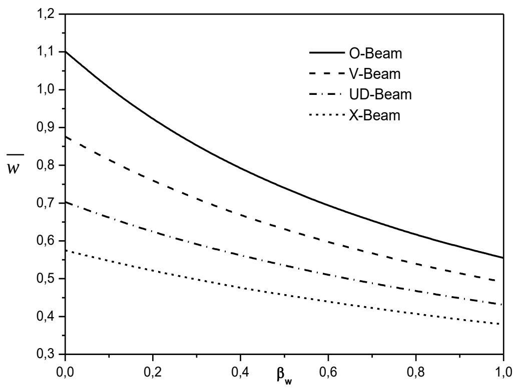

Fig. 2: Effect of Winkler modulus parameter on the dimensionless transverse displacements of CNTRC beams under uniform load ($L/h=10$; $\beta_s=0$; $V_{cnt}^*=0.12$).

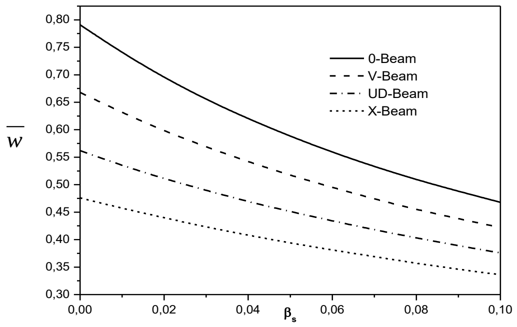

Fig. 3: Effect of Pasternak shear modulus parameter on the dimensionless transverse displacements of CNTRC beams under uniform load ($L/h=10$; $\beta_w=0.4$; $V_{cnt}^*=0.12$).

V. CONCLUSION

In this work, a trigonometric refined beam theory is used to investigate the bending of nanocomposite beams reinforced by single-walled carbon nanotubes resting on Pasternak elastic foundation. The equations of motion have been obtained using the Hamilton's principle. The accuracy of the present theoretical method is numerically checked by comparison with some available results. From the numerical results, it is found that the X-Beam is the strongest among different types of CNTRC beams in supporting the flexure and buckling loads, while the O-Beam is the weakest.

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