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Dynamic Behavior of The Rotor Structure By the Finite Element Method P- Version

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Abstract – This work concerning the vibration behavior analysis of Rotor. Using Finite Element Method p- version with trigonometric shape functions is used to redefine the equation of motion. The beam theory of Timoshenko using for modulization of the rotor system. Through Kinetic and strain energies of shaft, using Euler-Lagrange's equation for determination of equation of motion. The transverse shear deformation, rotating inertia, and gyroscopic effects is incorporating. System of equation resolved by program of calculus developed in MATLAB software for obtention the natural frequencies and eigenvalues. FEM p- version convergence presented with three boundary conditions and three various materials. The validation of numerical method and our program devised in two parts, the first for natural frequencies we validate with result viable in literature, and the last part for associated eigenvalue we use three boundary conditions, Simply Supported (Pinned-Pinned), Clamped-pinned and Pinned-free validate with exact values. In the result we study four fist mode of natural frequency with the objective of show the influence of various boundary condition (we take a same precedent boundary conditions) for three materials Stainless Steel, Nickel and Zirconia, after that a result of the first mode of natural frequency in function of rotating speed and the critical rotating speed for model of shaft used (Campbell's graph).

Vibration Analysis, Rotor, FEM P- Version, Boundary Conditions, Campbell's Graph

I. Introduction

The rotors structures are used in most modern technologies, such as in the aero propulsion systems, in helicopter drive applications, and in industrial machines, such as steam and gas turbines. So, they create mechanical damage with their vibrations.

The importance of this structure and their problems, recommended us to study and analysis the vibration behavior. [1] in Mechanical Vibrations all concepts fully explained and the derivations presented in complete details. [2] studied the beam on the dynamic stability of a rotating cantilever beam subjected to base excitation; using the Euler beam theory and the assumed mode method. [3] analyzed the frequency response for a rotating cantilever beam, using the assumed mode method; the effects of rotating angular speed were studied through numerical study.[4] investigated the nonlinear dynamic responses of a rotating blade subjected to various rotating speeds and under high-temperature supersonic gas. [5] examined the effect of type materials, on the natural frequencies of a simply supported model of shell, the governing equation was obtained using Rayleigh Ritz method. [6] studied the effect of boundary conditions and type materials on the natural frequencies of a spinning shaft using the p-version of the finite element method.

II. EQUATION OF MOTION

A. *Displacement*

The shaft modeled by first order shear deformation theory, considering as a Timoshenko beam, and the following kinematic assumptions are adopted:

$$\begin{cases} U(x,y,z,t) = U_0(x,t) + z\beta_x(x,t) - y\beta_y(x,t) \\ V(x,y,z,t) = V_0(x,t) + z\phi(x,t) \\ W(x,y,z,t) = W_0(x,t) + y\phi(x,t) \end{cases} \quad (1)$$

Where:
$$\begin{cases} U_0(x,t) = U(x,0,0,t) \\ V_0(x,t) = V(x,0,0,t) \\ W_0(x,t) = W(x,0,0,t) \end{cases}$$

With ϕ is the angular displacement, the β_x and β_y are the rotation angles about y and z axis respectively.

B. *Stress tensor*

Cylindrical coordinate system of the stain components:

$$\begin{cases} \varepsilon_{xx} = \frac{\partial U_0}{\partial x} + r\sin\theta\frac{\partial \beta_x}{\partial x} - r\cos\theta\frac{\partial \beta_y}{\partial x} \\ \varepsilon_{rr} = \varepsilon_{\theta\theta} = \varepsilon_{r\theta} = 0 \\ \varepsilon_{x\theta} = \varepsilon_{\theta x} = \frac{1}{2}\left(\beta_y\sin\theta + \beta_x\cos\theta - \sin\theta\frac{\partial V_0}{\partial x} + \cos\theta\frac{\partial W_0}{\partial x} + r\frac{\partial \phi}{\partial x}\right) \\ \varepsilon_{xr} = \varepsilon_{rx} = \frac{1}{2}\left(\beta_x\sin\theta - \beta_y\cos\theta - \sin\theta\frac{\partial W_0}{\partial x} + \cos\theta\frac{V_0}{\partial x}\right) \end{cases} \quad (2)$$

The stress-strain relation:

$$\begin{cases} \sigma_{xx} = Q_{11}\varepsilon_{xx} \\ \tau_{x\theta} = \tau_{\theta x} = k_s Q_{66}\gamma_{x\theta} \\ \tau_{xr} = \tau_{rx} = k_s Q_{55}\gamma_{xr} \end{cases} \quad (3)$$

Where: $Q_{11} = \frac{E}{1-\upsilon^2}$; $Q_{55} = Q_{66} = \frac{E}{2(1+\nu)}$

C. *Strain and kinetic energies*

The following are the equations of Strain and kinetic energies used in our study:

$$E_d = \frac{1}{2}A_{11}\int_0^L\left(\frac{\partial U_0}{\partial x}\right)^2 dx + \frac{1}{2}B_{11}\left[\int_0^L\left(\frac{\partial \beta_x}{\partial x}\right)^2 dx + \int_0^L\left(\frac{\partial \beta_x}{\partial x}\right)^2 dx\right] + \frac{1}{2}k_s B_{66}\int_0^L\left(\frac{\partial \phi}{\partial x}\right)^2 dx + \frac{1}{2}k_s(A_{55} + A_{66})\left[\int_0^L\left(\frac{\partial V_0}{\partial x}\right)^2 dx + \int_0^L\left(\frac{\partial W_0}{\partial x}\right)^2 dx + \int_0^L \beta_x^2\, dx + \int_0^L \beta_y^2\, dx + 2\int_0^L \beta_x\frac{\partial W_0}{\partial x}dx - \int_0^L \beta_y\frac{\partial V_0}{\partial x}dx\right] \quad (4)$$

$$E_c = \frac{1}{2}\int_0^L\left[I_m\left(\dot{U}_0^2 + \dot{V}_0^2 + \dot{W}_0^2\right) + I_d\left(\dot{\beta}_x^2 + \dot{\beta}_y^2\right) - 2\Omega I_p\beta_x\dot{\beta}_y + 2\Omega I_p\dot{\phi} + I_p\dot{\phi}^2 + \Omega^2 I_p + \Omega^2 I_d\left(\beta_x^2 + \beta_y^2\right)\right]dx \quad (5)$$

Where:

$$A_{11} = 2\pi\int_{R_i}^{R_o} Q_{11}(r)\, r\, dr\ ;\ A_{55} = A_{66} = \pi\int_{R_i}^{R_o} Q_{55}(r)\, r\, dr$$

$$B_{66} = 2\pi\int_{R_i}^{R_o} Q_{66}(r)\, r^3\, dr\ ;\quad B_{11} = \pi\int_{R_i}^{R_o} Q_{11}(r)\, r^3\, dr \quad (6)$$

$$I_m = 2\pi\int_{R_i}^{R_o} \rho(r)\, r\, dr\ ;\ I_d = \pi\int_{R_i}^{R_o} \rho(r)\, r^3\, dr$$

$$I_d = 2\pi\int_{R_i}^{R_o} \rho(r)\, r^3\, dr \quad (7)$$

D. *Hierarchical Beam element formulation*

The spinning flexible shaft is discretized by hierarchical beam element (p- element) with two nodes 1 and 2 is shown in figure 5. The element's nodal d.o.f. at each node are U_0 , V_0, W_0 , β_x , β_y and ϕ .
The local and non-dimensional co-ordinates are related by:

$$\xi = x/L \qquad \text{with } (0 \leq \xi \geq 1)$$

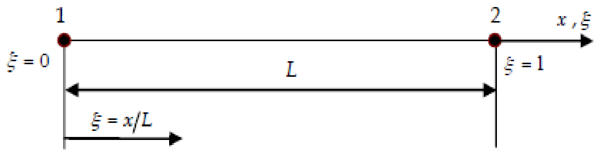

Fig 1. Beam element with two nodes.

We can rewrite the U_0 , V_0, W_0 , β_x , β_y and ϕ as:

$$\begin{cases} U_0 = [N_U]\{q_U\} = \sum_{m=1}^{pU} x_m(t).f_m(\xi) \\ V_0 = [N_V]\{q_V\} = \sum_{m=1}^{pV} y_m(t).f_m(\xi) \\ W_0 = [N_W]\{q_W\} = \sum_{m=1}^{pW} z_m(t).f_m(\xi) \\ \beta_x = [N_{\beta_x}]\{q_{\beta_x}\} = \sum_{m=1}^{p\beta_x} \beta_{x_m}(t).f_m(\xi) \\ \beta_y = [N_{\beta_y}]\{q_{\beta_y}\} = \sum_{m=1}^{p\beta_y} \beta_{y_m}(t).f_m(\xi) \\ \phi = [N_\phi]\{q_\phi\} = \sum_{m=1}^{p\phi} \phi_m(t).f_m(\xi) \end{cases} \quad (9)$$

With $[N]$ is the matrix of shape:

$$\left[N_{U,V,W,\beta_x,\beta_y,\phi}\right] = \left[f_1 f_2 \dots \dots f_{pU,pV,pW,p\beta_x,p\beta_y,p\phi}\right] \quad (10)$$

In this work $q_U = q_V = q_W = q_{\beta_x} = q_{\beta_y} = q_\phi = p$ (are the number of shape functions of displacements).
The group of the shape functions used in this study (see [7]) is given by

$$\begin{cases} f_1 = 1 - \xi \\ f_2 = \xi \\ f_{r+2} = \sin(\delta_r\,\xi), \quad \delta_r = r\pi\ ;\ r = 1, 2, 3, \dots \end{cases} \quad (11)$$

After all that and the application of Euler-Lagrange's equations, we obtained the following motion's equation of free vibration of spinning:

$$[M]\{\ddot{q}\} + [G]\{\dot{q}\} + [K]\{q\} = \{0\} \qquad (12)$$

III. Results

A. Convergence

The convergence results are for three first bending mode for various boundary conditions, the mechanical properties: E=2e11 N/m^2; ν =0,3; ρ = 7800 kg/m^3 and the geometric parameters: D=0,05 m; L=0,9 m. The shear correction factor: k_s=6/7. as a function of the number of hierarchical terms p are shown in Figure 1.

The results are converged where the number of shafts increased, p should be up to 7 for get a great result for third mode, but in of very small frequencies p=5 sufficient.

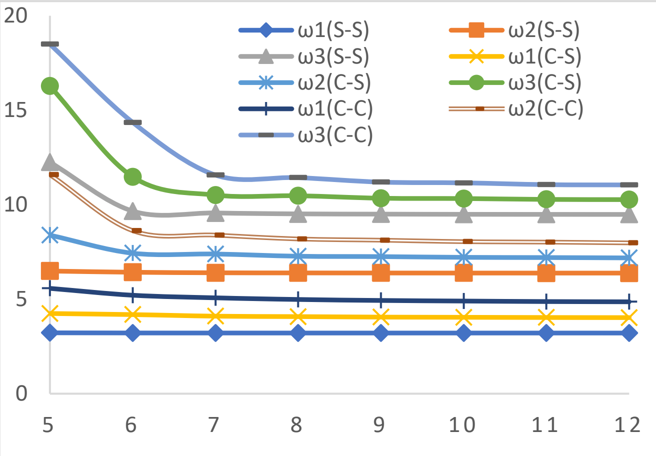

Fig 2. Convergence of the frequency ω for the 3 bending modes for different boundary conditions (S: simply supported; C: clamped) as a function of the number of hierarchical terms p

B. Validation

In the first validation (Table 1), the natural frequency ω for the 3 bending modes of a simply supported homogeneous
shaft for the stationary case (Ω) and other with rotating speed Ω = 10^4 rad/s, is compared with those available in the literature to verify the present program. With same mechanical properties and geometric parameters used in convergence study. The shaft is modeled by one element of length L. The shaft is simply-supported at the ends, Compared with [6]. In this validation, p =10.

The last (Table 2) with exact values of associated eigenvalue used the parameter of frequency, for fourth normal mode. Compared with [1]. In this validation, p =15.

C. Results and interpretation

In this study we conserve the same precedent geometric parameters for all trying and we varied the materials and the boundary conditions. Table 3 shows the mechanical properties for materials used in this work.

Table 1. The mechanical properties for Stainless Steel, Nickel and Zirconia.

Material	E[N/m^2]	ν	ρ[kg/m^3]
Stainless Steel (SS)	2.07788 e11	0.317756	8166
Nickel (Ni)	2.05098 e11	0.31	8900
Zirconia (Zi)	1.68063 e11	0.297996	5700

D. The boundary conditions influence

The graphs as under (Fig3) illustrate the fourth first natural frequency for various materials of shaft and different boundary conditions.

Table 2. The frequency ω for the 3 bending modes of a simply supported homogeneous shaft (D= 0.05 m, L= 0.9 m, E =2·1011 N/m2, ν= 0.3, ρ= 7800 kg/m3, ks = 6/7).

Rotating speed Ω [rad/s]	**0**			**10^4**					
				Backward mode			Forward mode		
Bending Mode ω [Hz]	Present	Boukhelfa 2014	% Error	Present	Boukhelfa 2014	% Error	Present	Boukhelfa 2014	% Error
1	128,1875	128,1837	0,003%	122,3503	122,3467	0,003%	134,2957	134,2918	0,003%
2	507,0955	507,0806	0,003%	484,6671	484,6529	0,003%	530,4533	530,4377	0,003%
3	1120,9267	1104,8937	1,451%	1073,6138	1073,5822	0,003%	1169,8442	1169,8098	0,003%

Table 3. The associated eigenvalue for fourth normal mode

Boundary Conditions	**Simply Supported (Pinned-Pinned)**			**Clamped-pinned**			**Pinned-free**		
Normal mode	FEM-p values	Exact values	% Error	FEM-p values	Exact values	% Error	FEM-p values	Exact values	% Error
1	3,210058	3,141593	2,179%	4,013746	3,926991	2,209%	4,034264	3,926991	2,732%
2	6,382146	6,283185	1,575%	7,167774	7,068583	1,403%	7,232795	7,068583	2,323%
3	9,483184	9,424778	0,620%	10,232749	10,210176	0,221%	10,362474	10,210176	1,492%
4	12,487160	12,566371	0,630%	13,196601	13,351769	1,162%	13,406167	13,351769	0,407%

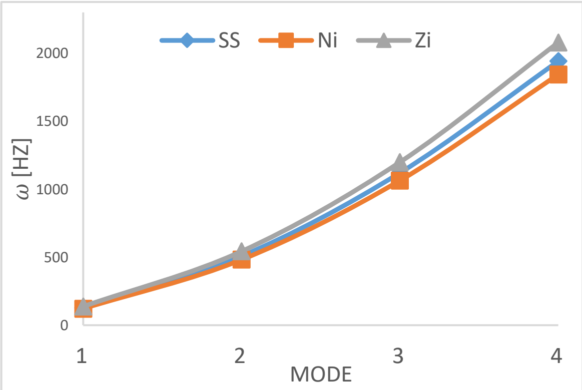

Fig 3a

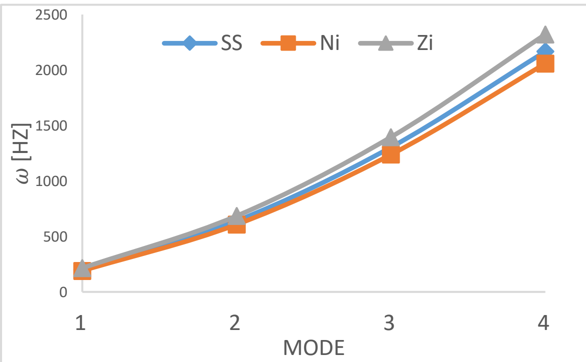

Fig 3b

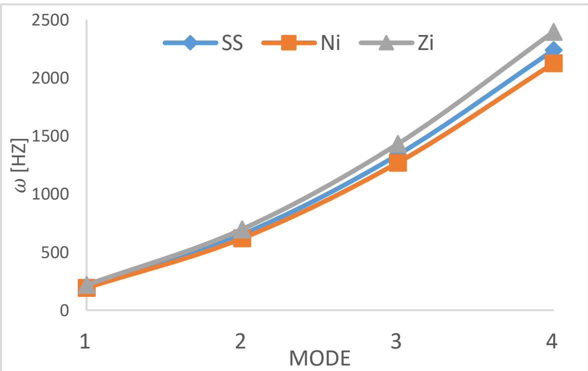

Fig 3c

Fig 3. The fourth first natural frequency for various materials of shaft and different boundary conditions, a) Simply supported, b) Clamped-pinned, c) Pinned-free

The Campbell's graph. The graph follows (Fig 4) shows the difference first backward and forward mode of frequencies as a function of rotating speed (Ω) between precedent materials and shows the critical speeds.

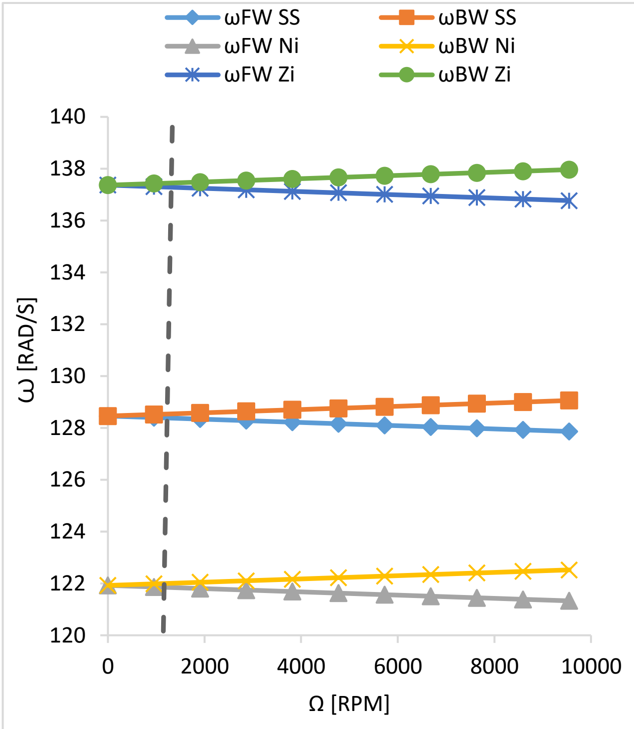

Fig 4. The Campbell's graph for difference shaft materials, using Simply-supported boundary conditions.

Interpretation. The change between boundary condition Simply-supported and anther create difference but between Clamped -Pinned and Pinned-Free we haven't difference. The critical speed for various situations of our model shaft in Table 4.

Table 4. Critical speeds as a function with materials of shaft

Materials	**Stainless Steel**		**Nickel**		**Zirconia**	
Modes	Backward	Forward	Backward	Forward	Backward	Forward
Critical speeds [Rpm]	1227,4345	1225,9698	1164,9887	1163,5985	1312,5409	1310,9733

IV. CONCLUSIONS

The analysis of the free vibrations of the homogeneous spinning shafts using the p-version of the finite element method with trigonometric shape functions is presented in this work. The results obtained agree with those available in the literature. Several examples were treated to determine the influence of the various boundary condition and various materials of the spinning shafts. This work enabled us to arrive at the following conclusions:

1. Number of function shaft influence to get best results.
2. The differences between boundary conditions and the mechanical properties creates a difference in natural frequency
3. The change of materials changes the critical speed of shaft, and vary the forward and backward frequencies.

The objective of this works validated our calculation program for used in advanced studies, Fortunately the objective is reached.

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