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# SOME NEW GENERALIZED CLASSES OF SEQUENCES OF FUZZY NUMBERS DEFINED BY A ORLICZ FUNCTION I

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***Abstract*** – In this paper we introduce the concept of strongly $A_\sigma^{\lambda(p)}-$ convergence of fuzzy numbers with respect to an Orlicz function and examine some properties of the resulting sequence spaces and $\lambda(\sigma)$-statistical convergence. It also shown that if a sequence of fuzzy numbers is strong $\lambda(\sigma)$ convergent with respect to an Orlicz function then it is $\lambda(\sigma)$-statistically convergent.

*Keywords – Orlicz Function,Paranorm, De La Vallee-Poussin Means, Fuzzy Number.*

## I. INTRODUCTION

The concept of paranorm is closely related to linear metric spaces.It is a generalization of that of absolute value. Let $X$ be a linear space. A function $g : X \to R$ is called paranorm, if

(i) $g(0)=0$ ,(ii) $g(x)\geq 0$, for all $x\in X$, (iii) $g(-x)=g(x)$, for all $x\in X$, (iv) $g(x+y)\leq g(x)+g(y)$, for all $x,y\in X$, and (v) if $(\alpha_n)$ is a sequence of scalars with $\alpha_n\to\alpha$ $(n\to\infty)$ and $(x_n)$ a sequence of vectors with $g(x_n-x)\to 0$ $(n\to\infty)$, then $g(\alpha_n x_n-\alpha x)\to 0$ $(n\to\infty)$.This property is called continuity of multiplication by scalars.The space $X$ is called the paranormed space with the paranorm g.

Let $\sigma$ be a one-to-one mapping of the set of positive integers into itself such that $\sigma^m(n)=\sigma(\sigma^{m-1}(n)), m=1,2,3,\ldots$ . A continuous linear functional $\varphi$ on $l_\infty$ , the set of all bounded sequences, is said to be an invariant mean or a $\sigma$-mean if and only if

(i) $\varphi(x)\geq 0$ when the sequence $x=(x_n)$ has $x_n\geq 0$ for all n,

(ii) $\varphi(e)=1$, where e=(1,1,1,…) and

(iii) $\varphi(\{x_{\sigma(n)}\})=\varphi(\{x_n\})$ $all\ x=(x_n)\in l_\infty$.

For certain kinds of mappings $\sigma$, every invariant mean $\varphi$ extends the limit functional on the space c,the set of all convergent sequences, in the sense that $\varphi(x)=lim x$ for all $x=(x_n)\in c$. Consequently, $c\subset V_\sigma$, where $V_\sigma$ is the set of bounded sequences all of whose $\sigma$-means are equal.

An Orlicz function is a function M: $[0,\infty[\to[0,\infty[$ , which is continuous, non-decreasing and convex with M(0)=0, M(x) > 0 for x > 0 and M(x)$\to\infty$ $as\ x\to\infty$ .

An Orlicz function is said to satisfy $\Delta_2$-condition for all values of u, if there exists a constant K > 0, such that M(2u) ≤ KM(u), u ≥ 0.

Lindenstrauss and Tzafriri [8] used the idea of Orlicz function to construct the sequence space

$$l_M = \left\{ x = (x_k): \quad \sum_k M\left(\frac{|x_k|}{\rho}\right) < \infty, \quad for\ some\ \rho > 0 \right\}.$$

The space $l_M$ with the norm

$$\|x\| = Inf\left\{ \rho > 0: \quad \sum_k M\left(\frac{|x_k|}{\rho}\right) \le 1 \right\}$$

becomes a Banach space which is called an Orlicz sequence space. The space $l_M$ is closely related to the space $l_p$ which is an Orlicz sequence space with $M(x) = x^p, 1 \le p < \infty.$

In the later stage different Orlicz sequence spaces were introduced and studied by Parashar and Choudhary [9], Esi,Isik and Esi [10] , Esi [11], Esi and Et [12], Esi[13] and many others.

The purpose of this paper is to introduce and study the concepts of strong $A_\sigma^{\lambda(p)}$ convergence of fuzzy numbers with respect to an Orlicz function and $\lambda(\sigma)$-istatistical convergence and some relations between them.

Let $p=(p_k) \in \ell_\infty$ , then the following well-known inequality will be used the paper:
For sequences $(a_k)$ and $(b_k)$ of complex numbers we have

$$|a_k + b_k|^{p_k} \le K(|a_k|^{p_k} + |b_k|^{p_k})$$

where $K = max\,(1,2^{H-1})$ and $H = \sup_k p_k$.

We now give here a brief introduction about the sequences of fuzzy numbers (see [1] and [6])

Let $D$ denote the set of all bounded intervals $A = [\underline{A}, \overline{A}]$ on the real line R. For $A, B \in D$, define

$$A \le B \quad if\ and\ only\ if\ \underline{A} \le \underline{B} \quad and \quad \overline{A} \le \overline{B},$$

$$d(A,B) = \max\left\{ |\underline{A} - \underline{B}|,\ |\overline{A} - \overline{B}| \right\}.$$

Then it can be easily seen that d defines a metric on $D$ and $(D,d)$ is a complete metric space [1]. Also the relation $\le$ is a partial order on $D$.

A fuzzy number is a fuzzy subset of the real line R which is bounded, convex and normal.Let $L$(R) have compact support, i.e. if $X \in L$(R) then for any $\alpha \in [0,1]$, $X^\alpha$ is compact,where

$$X^\alpha = \{ t: X(t) \ge \alpha \ \ if \ \ \alpha \in (0,1] \}$$

and

$$X^0 = cl\left( \{ t \in R: X(t) > \alpha \ \ if \ \ \alpha = 0 \} \right),$$

where $cl(A)$ is the closure of A.

The set R of real numbers can be embedded in $L$(R) if we define $\bar{r} \in L$(R) by

$$\bar{r}(t) = \begin{matrix} 1, & if\ t = r \\ 0, & if\ t \ne r \end{matrix}.$$

The additive identity and multiplicative identity of $L$(R) are denoted by $\bar{0}$ and $\bar{1}$, respectively. Then the arithmetic operations on $L$(R) are defined as follows:

$$(X \oplus Y)(t) = \sup\{ X(s) \wedge Y(t-s) \}, t \in R;$$

$$(X \Theta Y)(t) = \sup\{ X(s) \wedge Y(s-t) \}, t \in R;$$

$$(X \otimes Y)(t) = \sup\{ X(s) \wedge Y(t/s) \}, t \in R;$$

$$(X / Y)(t) = \sup\{ X(st) \wedge Y(s) \}, t \in R.$$

These operations can be defined in terms of $\alpha$-level sets as follows:

$$[X \oplus Y]^\alpha = \left[ a_1^\alpha + b_1^\alpha, a_2^\alpha + b_2^\alpha \right],$$

$$[X \Theta Y]^\alpha = \left[ a_1^\alpha - b_1^\alpha, a_2^\alpha - b_2^\alpha \right];$$

$$[X \otimes Y]^{\alpha} = \left[\min_{i\in\{1,2\}} a_i^{\alpha} b_i^{\alpha}, \max_{i\in\{1,2\}} a_i^{\alpha} b_i^{\alpha}\right],$$

$$\left[X^{-1}\right]^{\alpha} = \left[\left(a_2^{\alpha}\right)^{-1}, \left(a_1^{\alpha}\right)^{-1}\right], \quad a_i^{\alpha} > 0$$

for each $0 < \alpha \leq 1$.

For $r$ in R and X in $L$(R), the product rX is defined as follows:

$$rX(t) = \begin{cases} X\left(r^{-1}t\right) & if \ r \neq 0 \\ 0 & if \quad r = 0 \end{cases}.$$

Define a map $\bar{d} : L(R) x L(R) \to R_+ \cup \{0\}$ by $\bar{d}(X,Y) = \sup_{0\leq\alpha\leq1} d\left(X^{\alpha}, Y^{\alpha}\right)$. For $X, Y \in L$(R) define $X \leq Y$ if and only if $X^{\alpha} \leq Y^{\alpha}$ for any $\alpha \in [0,1]$. It is known that ($L$(R), $\bar{d}$ ) is a complete metric space [7].

A metric on $L$(R) is said to be a translation invariant if $\bar{d}(X+Z, Y+Z) = \bar{d}(X,Y)$ for $X, Y, Z \in L$(R).

**LEMMA**. [2]. If $\bar{d}$ is a translation invariant metric on $L$(R), then

(i) $\bar{d}\left(X+Y, \bar{0}\right) \leq \bar{d}\left(X, \bar{0}\right) + \bar{d}\left(Y, \bar{0}\right)$,

(ii) $\bar{d}\left(\lambda X, \bar{0}\right) \leq |\lambda| \bar{d}\left(X, \bar{0}\right), |\lambda| > 1$.

A sequence $X = (X_k)$ of fuzzy numbers is a function $X$ from the set $N$ of natural numbers into $L(R)$. The fuzzy number $X_k$ denotes the value of the function at $k \in N$.

A sequence $X = (X_k)$ of fuzzy numbers is said to be bounded if the set $\{X_k : k \in N\}$ of fuzzy numbers is bounded.

A sequence $X = (X_k)$ of fuzzy numbers is said to be converge to a fuzzy number $X_o$ if for every $\varepsilon > 0$ there is a positive integer $n_o$ such that $\bar{d}(X_k, X_o) < \varepsilon$ for $k > n_o$.

## II. RESULTS

Let $\Lambda = (\lambda_n)$ be a non-decreasing sequence of positive real numbers tending to infinity and $\lambda_1 = 1$ and $\lambda_{n+1} \leq \lambda_n + 1$ , $I_n = [n - \lambda_n + 1, n]$. Let M be an Orlicz function, $p = (p_k)$ be any sequence of strictly positive real numbers and $X = (X_k)$ be sequence of fuzzy numbers, then for some $\rho > 0$;

Esi [13] has defined the following classes of sequences of fuzzy numbers:

$$F_o[M,\lambda,p] = \left\{(X_k): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} \left[M\left(\frac{\bar{d}(X_k,\bar{0})}{\rho}\right)\right]^{p_k} = 0\right\},$$

$$F[M,\lambda,p] = \left\{(X_k): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} \left[M\left(\frac{\bar{d}(X_k,X_0)}{\rho}\right)\right]^{p_k} = 0\right\},$$

$$F_\infty[M,\lambda,p] = \left\{(X_k): \sup_n \frac{1}{\lambda_n} \sum_{k\in I_n} \left[M\left(\frac{\bar{d}(X_k,\bar{0})}{\rho}\right)\right]^{p_k} < \infty\right\}$$

and in this paper, we define the following new classes of sequences of fuzzy numbers:

$$F_o[A_\sigma,M,\lambda,p] = \left\{X = (X_k): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{mk} \left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{\rho}\right)\right]^{p_k} = 0, \ uniformly \ in \ m\right\},$$

$$F[A_\sigma,M,\lambda,p] = \left\{X = (X_k): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{mk} \left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},X_0\right)}{\rho}\right)\right]^{p_k} = 0, uniformly \ in \ m\right\},$$

$F_\infty[A_\sigma, M, \lambda, p] = \left\{ X = (X_k): \sup_{n,m} \frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \left[ M\left( \frac{\bar{d}\left(X_{\sigma^k(m)}, \bar{0}\right)}{\rho} \right) \right]^{p_k} < \infty \right\}.$

We denote $F[A_\sigma, M, \lambda, p]$, $F_o[A_\sigma, M, \lambda, p]$ and $F_\infty[A_\sigma, M, \lambda, p]$ as $F[A_\sigma, M, \lambda]$, $F_o[A_\sigma, M, \lambda]$ and $F_\infty[A_\sigma, M, \lambda]$, when $p_k = 1$ for all k. If $X = (X_k) \in F[A_\sigma, M, \lambda, p]$ we say that $X = (X_k)$ is of strongly $A_\sigma^{\lambda(p)}$- convergent to fuzzy number $X_0$ with respect to the Orlicz function M. If ,M(x)=x, A=(C,1) matrix order 1, $\lambda_n = n$ for all n, and $\sigma(n) = n$ , then $F[A_\sigma, M, \lambda, p]$= $F[p]$, $F_o[A_\sigma, M, \lambda, p]$=$F_o[p]$ and $F_\infty[A_\sigma, M, \lambda, p]$=$F_\infty[p]$, which were defined by Mursaleen and Basarir [2]. If $X = (X_k) \in F[p]$, we say that $X = (X_k)$ strongly convergent to fuzzy number $X_0$.

In this section we examine some topological properties of $F[A_\sigma, M, \lambda, p]$, $F_o[A_\sigma, M, \lambda, p]$ and $F_\infty[A_\sigma, M, \lambda, p]$ classes.

If $\bar{d}$ is a translation invariant, we have the following theorem.

**THEOREM 1.** For any Orlicz function M and any sequence $p = (p_k)$ of strictly positive real numbers, then $F[A_\sigma, M, \lambda, p]$, $F_o[A_\sigma, M, \lambda, p]$ and $F_\infty[A_\sigma, M, \lambda, p]$, are linear spaces over the set of complex numbers.

**PROOF.** We shall prove only for $F_o[A_\sigma, M, \lambda, p]$. The others can be treated similarly. Let $X = (X_k)$ , $Y = (Y_k) \in F_o[A_\sigma, M, \lambda, p]$ and $\alpha, \beta \in C$ . In order to prove the result we need to find some $\rho_3 > 0$ such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \left[ M\left( \frac{\bar{d}\left(\alpha X_{\sigma^k(m)} + \beta Y_{\sigma^k(m)}, \bar{0}\right)}{\rho_3} \right) \right]^{p_k} = 0, \ uniformly\ in\ m.$$

$\alpha$

Since $X = (X_k)$ , $Y = (Y_k) \in F_o[A_\sigma, M, \lambda, p]$, there exist some $\rho_1 > 0$ and $\rho_2 > 0$ such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \left[ M\left( \frac{\bar{d}\left(X_{\sigma^k(m)}, \bar{0}\right)}{\rho_1} \right) \right]^{p_k} = 0, \ uniformly\ in\ m$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \left[ M\left( \frac{\bar{d}\left(Y_{\sigma^k(m)}, \bar{0}\right)}{\rho_2} \right) \right]^{p_k} = 0, \ uniformly\ in\ m.$$

Define $\rho_3 = \max(2|\alpha|\rho_1, 2|\beta|\rho_2)$. Since M is non-decreasing and convex

$$\frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \left[ M\left( \frac{\bar{d}\left(\alpha X_{\sigma^k(m)} + \beta Y_{\sigma^k(m)}, \bar{0}\right)}{\rho_3} \right) \right]^{p_k} \leq \frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \left[ M\left( \frac{\bar{d}\left(\alpha X_{\sigma^k(m)}, \bar{0}\right)}{\rho_3} + \frac{\bar{d}\left(\beta Y_{\sigma^k(m)}, \bar{0}\right)}{\rho_3} \right) \right]^{p_k}$$

$$\frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \frac{1}{2^{p_k}} \left[ M\left( \frac{\bar{d}\left(X_{\sigma^k(m)}, \bar{0}\right)}{\rho_1} \right) + M\left( \frac{\bar{d}\left(Y_{\sigma^k(m)}, \bar{0}\right)}{\rho_2} \right) \right]^{p_k} \leq \frac{1}{\lambda_n} \sum_{k \in I_n} a_{mk} \left[ M\left( \frac{\bar{d}\left(X_{\sigma^k(m)}, \bar{0}\right)}{\rho_1} \right) + M\left( \frac{\bar{d}\left(Y_{\sigma^k(m)}, \bar{0}\right)}{\rho_2} \right) \right]^{p_k}$$

$$\leq \frac{K}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{\rho_1}\right)\right]^{p_k}$$
$$+\frac{K}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(Y_{\sigma^k(m)},\bar{0}\right)}{\rho_2}\right)\right]^{p_k}$$
$$\to 0, \qquad as\ n\to\infty\ uniformly\ in\ m.$$

where $K = max\,(1,2^{H-1})$, $H = \sup_k p_k$, so that $\alpha X + \beta\, Y \in F_o[A_\sigma, M, \lambda, p]$. *This completes* the proof.

**THEOREM 2.** For any Orlicz function M and a bounded sequence $p = (p_k)$ of strictly positive real numbers, $F_o[A_\sigma, M, \lambda, p]$ and $F[A_\sigma, M, \lambda, p]$ are paranormed spaces with

$$g(X) = inf\left\{\rho^{p_n/H} : \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{\rho}\right)\right]^{p_k}\right)^{1/M} \leq 1,\ \begin{matrix} n = 1,2,3,\ldots \\ m = 1,2,3,\ldots\end{matrix}\right\}$$

where $M = \max(1, H)$.

**PROOF.** Clearly $g(\bar{0}) = 0$ and $g(X) = g(-X)$. It can be seen easily that $g(X+Y) \leq g(X) + g(Y)$ for $X = (X_k)$, $Y = (Y_k) \in F_o[A_\sigma, M, \lambda, p]$, since $\bar{d}$ is a translation invariant.Since $M(0) = 0$, we get $Inf\left\{\rho^{p_n/H}\right\} = 0$ for $X = \bar{0}$.Conversely, suppose that $g(X) = 0$, then

$$inf\left\{\rho^{p_n/H} : \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{\rho}\right)\right]^{p_k}\right)^{1/M} \leq 1,\ \begin{matrix} n = 1,2,3,\ldots \\ m = 1,2,3,\ldots\end{matrix}\right\} = 0.$$

This implies that for a given $\varepsilon > 0$, there exists some $\rho_\varepsilon\ (0 < \rho_\varepsilon < \varepsilon)$ such that

$$\left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{\rho_\varepsilon}\right)\right]^{p_k}\right)^{1/M} \leq 1.$$

Thus,

$$\left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{\varepsilon}\right)\right]^{p_k}\right)^{1/M}$$
$$\leq \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{\rho_\varepsilon}\right)\right]^{p_k}\right)^{1/M} \leq 1$$

for each n and m.

Suppose that $\bar{d}\left(X_{\sigma^{k_s}(m)},\bar{0}\right) \neq 0$ for some $s \in I_n$. Let $\varepsilon \to 0$, then $\frac{\bar{d}\left(X_{\sigma^{k_s}(m)},\bar{0}\right)}{\varepsilon} \to \infty$. It follows that

$$\left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^{k_s}(m)},\bar{0}\right)}{\varepsilon}\right)\right]^{p_k}\right)^{1/M} \to \infty,$$

which is a contradiction. Therefore $X_{\sigma^{k_s}(m)} \neq \bar{0}$.

Finally, we prove that scalar multiplication is continuous. Let $\gamma$ be any complex number. By definition

$$g(\gamma X) = \inf\left\{\rho^{p_n/H} : \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(\gamma X_{\sigma^k(m)},\bar{0}\right)}{\rho}\right)\right]^{p_k}\right)^{1/M} \leq 1, \begin{matrix} n = 1,2,3,\dots \\ m = 1,2,3,\dots \end{matrix}\right\}.$$

Then

$$g(\gamma X) = \inf\left\{(|\gamma| t)^{p_n/H} : \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{t}\right)\right]^{p_k}\right)^{1/M} \leq 1, \begin{matrix} n = 1,2,3,\dots \\ m = 1,2,3,\dots \end{matrix}\right\}$$

where $t = \frac{\rho}{|\gamma|}$. Since $|\gamma|^{p_k} \leq \max\left(1,|\gamma|^H\right)$, we have

$$g(\gamma X) \leq \left(\max\left(1,|\gamma|^H\right)\right)^{1/M} \inf\left\{t^{p_n/H} : \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{mk}\left[M\left(\frac{\bar{d}\left(X_{\sigma^k(m)},\bar{0}\right)}{t}\right)\right]^{p_k}\right)^{1/M} \leq 1, \begin{matrix} n = 1,2,3,\dots \\ m = 1,2,3,\dots \end{matrix}\right\}.$$

So, the scalar multiplication is continuous follows from the above inequality.

## III. CONCLUSION

In this conference paper, we studied some sequence spaces of fuzzy numbers defined by Orlicz function. Some topological properties were argued.

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