

Integration of Discrete-Event Simulation and Statistical Process Control Methods

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Abstract – This study aimed to integrate the discrete-event simulation system with the statistical process control method to test whether the discrete-event simulation system is under control using the discrete-event simulation method. This study created a three-dimensional discrete-event simulation model of the emergency service unit. Using the data of patient stay (or length of stay) and waiting times obtained in the simulation model, analyses were made with Xbar-R and Xbar-S control charts in statistical process control methods. Determining which values of the simulation data are out of control was carried out with statistical process control charts. For Xbar-R and Xbar-S graphics, 901 data were analyzed, with 30 subgroups. This study determined that the patient was under control according to Xbar-R and Xbar-S control charts of patient stay and waiting times, but the system was out of control according to R and S control charts. As a result, it has been determined that the system in the 3D simulation model is out of control according to the statistical process control charts. It is recommended to take the system under control by observing the data that is out of control. This study emphasized that the results obtained from the systems performed in the computer environment should be integrated with statistical process control charts to verify their validity.

Keywords –Discrete-Event Simulation; Statistical Process Control; Xbar-R Chart; Xbar-S Chart

I. INTRODUCTION

In studies in the field of healthcare, the problems in the health sector are generally solved by considering the human factor [1]. Most of the issues in the healthcare sector have recently been applied to engineering applications such as statistics (descriptive statistics, regression, ANOVA, t-test, z-test, correlation, statistical process control, time series, forecasting, design of experiments, quality tools, etc.), optimization (linear programming, nonlinear programming, integer model, dynamic programming, etc.), artificial intelligence (machine learning algorithms, deep learning model, etc.), and simulation (discrete-event simulation) to increase

the quality of service [2], [3]. In such applications, two variable types are dependent and independent [4]. Dependent variables are affected by many independent variables, such as environmental, economic, social, structural, and resource, depending on the problems [5].

The discrete-event simulation method is generally expressed as computer-based programs for systems with dynamic structures [6], [7]. This method is used in many areas, such as health, production, transportation, and logistics [8]–[10]. In this study, a discrete-event simulation model has been developed for the health system emergency service unit, where the dynamic and human factor is intense. The discrete-event simulation technique provides concrete results in solving many problems,

especially in health [11]–[13]. Since the results of changes in location and human-based health resources require a long time and high cost in real life, discrete-event simulation models can obtain results in a short time and with low cost [14], [15]. Discrete event simulation models are preferred to solve many health-related problems such as patient stay, patient waiting times, staff productivity, and the number of patients treated [16], [17].

In one study, the effects of changes in the number of health resources on patient waiting and length of stay were analyzed using a discrete-event simulation model [18]. In another study, optimum health resource numbers were calculated for the minimum waiting time of patients using the simulation technique and statistical experiment design method [19]. In another study, discrete-event simulation model outputs were integrated with discrete factorial experimental design to investigate the relationship between health facility size and the number of physicians [20]. Atalan et al. have investigated by integrating the discrete-event simulation model with machine learning algorithms, the effects of health resource costs on the number of patients treated and patient waiting times [14].

A second technique is needed to verify the validity of the results obtained in the simulation models. Statistical methods are generally used to analyze the results obtained in simulation models [20]–[23]. In a study, the results obtained in the simulation model were analyzed with a statistical experimental design to obtain optimum results [11]. In another study, the discrete-event simulation model was preferred, and the correlations of the variables were tested using

correlation test tools, which are among the descriptive statistics methods [24]. In this study, statistical process control charts were used to test whether the results obtained in the simulation model were under the control of the model system. Statistical process control charts test whether the systems are under control by considering the mean and standard deviation values of the data [25], [26]. This method is used in many fields [27], [28]. In this study, statistical process control charts were used for the data of the simulation model developed for the health field.

This study consists of four main parts. In the first part, there is a literature review on the subject of the study. Theoretical information about the discrete-event simulation model and statistical control charts proposed in the study are included in the second part of the study. Numerical and statistical data of the study are shared in the third part. The general information about the applicability of the method used is given in the last part of the study.

II. MATERIALS AND METHOD

This study tested the validity of the results obtained by integrating two methods. Using the first method, the discrete-event simulation technique, data on the duration of hospitalization and waiting time of patients in an emergency department were obtained. The simulation model developed for the emergency service was created using the 3-dimensional Flexsim HC computer program. The screenshot of the simulation model is shared in **Figure 1**.

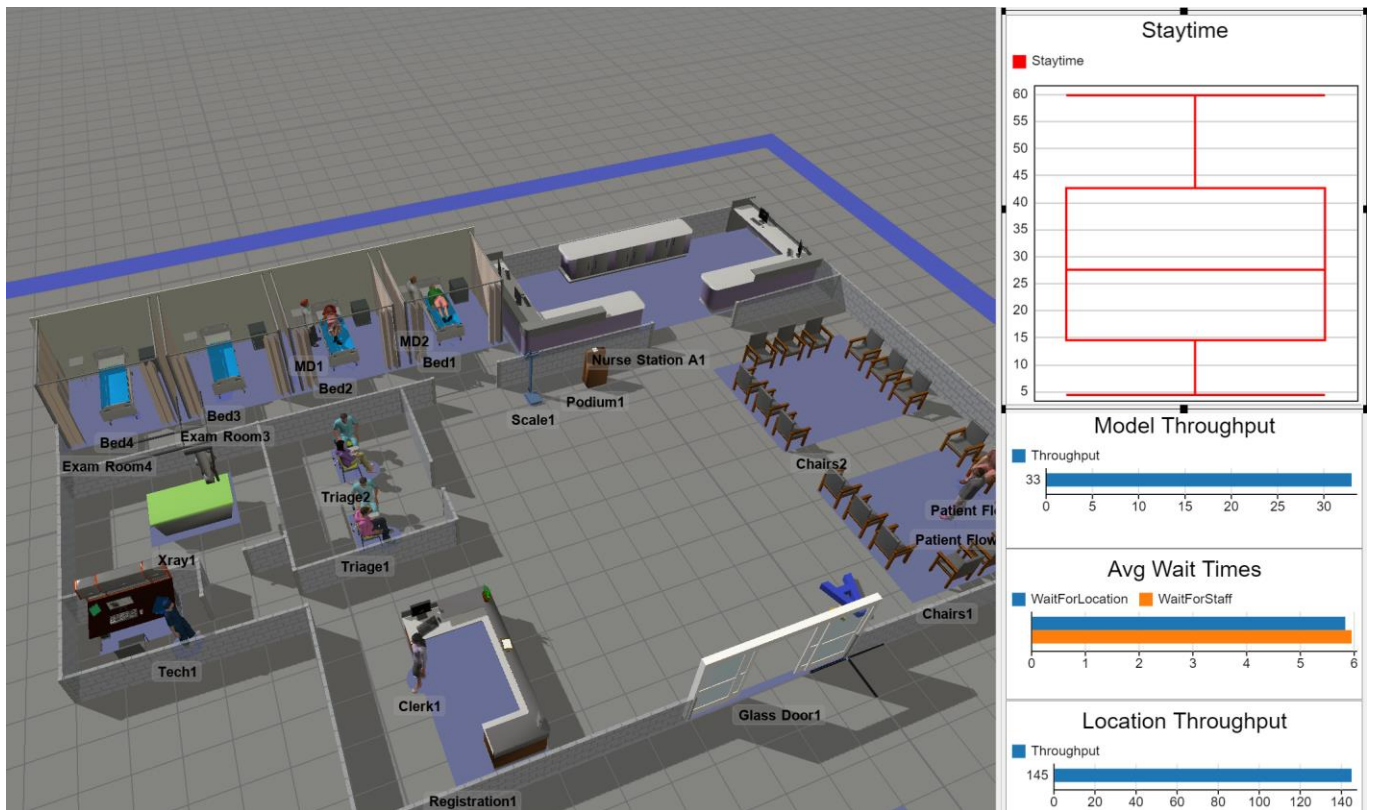


Figure 1. Screenshot of the discrete-event simulation model

The discrete-event simulation model used data from 901 patients' mean waiting and length of stay. The length of stay is based on the time between a patient entering and leaving the hospital. Patient length of stay was defined as the sum of the time a patient spent in contact with health resources for waiting and treatment. On the other hand, patient waiting time is calculated not only as the time until the patients' availability of resources, such as doctors and nurses, but also as the sum of the times until the availability of location-based health resources, such as treatment rooms and triage areas. The descriptive statistics data, which are the mean, standard deviation, variance, kurtosis, skewness, and quartile values of the patient stay and waiting times obtained in the discrete-event simulation model, are given in Table 1.

Table 1. Descriptive statistics of variables

Variable	LOS	WT
Total Sample	901.000	901.00
Mean	17.4420	2.4110
Standard Deviation	13.9700	6.5200
Variance	195.174	42.509
Minimum	1.26700	0.0000

Q1	7.23100	0.0000
Median	13.4190	0.0000
Q3	23.8160	0.8700
Maximum	101.624	57.752
Range	100.357	57.752
Inter quartile range	16.5850	0.8700
Mode	11.9542	0.0000
N for Mode	4.00000	615.00
Skewness	1.65000	3.8500
Kurtosis	3.71000	17.620

*LOS, length of stay; WT, waiting time

Statistical process control charts were used to test whether the data obtained about the patient flow system proposed in the discrete-event simulation model is under control. In this study, the system is controlled by using X-R and X-S control charts. These two control chart types are created using the mean and standard deviation data of the data. Although X-R control charts are the most commonly used charts for measurable variables, it is more appropriate for some systems to choose sample standard deviation (s) as a measure of subgroup dispersion. For this reason, statistical results were obtained using both graphic types in

this study. The following formulas are used for the \bar{x} -control chart of the X-R chart:

For the upper control limit

$$UCL = \bar{\bar{x}} + A_2\bar{R} \quad (1)$$

For the central control limit

$$CL = \bar{\bar{x}} \quad (2)$$

For the central control limit

$$LCL = \bar{\bar{x}} - A_2\bar{R} \quad (3)$$

where, UCL, CL, and LCL are the notations for upper limit, central limit, and lower limit expressions, respectively. The mean value $\bar{\bar{x}}$ of the mean of the subgroup samples are expressed. Expressions A_2 and A_3 represent a constant term depending on the number of subgroups used in the control limits. The \bar{R} value is defined as the difference between the maximum and minimum values of the data in the subgroups. The following formulas are used for the \bar{R} control chart of the X-R chart:

For the upper control limit

$$UCL = D_4\bar{R} \quad (4)$$

For the central control limit

$$CL = \bar{R} \quad (5)$$

For the central control limit

$$LCL = D_3\bar{R} \quad (6)$$

where, the expressions D_3 and D_4 represent a constant term depending on the number of subgroups used in the control limits. The equations suggested for the \bar{x} control chart of the X-S chart is given below:

For the upper control limit

$$UCL = \bar{\bar{x}} + D_3\bar{s} \quad (7)$$

For the central control limit

$$CL = \bar{s} \quad (8)$$

For the central control limit

$$UCL = \bar{\bar{x}} - D_4\bar{s} \quad (9)$$

where, the \bar{s} value is expressed as the mean of the standard deviation values of the data sets belonging to its subgroups. The expressions D_3 and D_4 represent a constant term depending on the number of subgroups used in the control limits. The equations suggested for the \bar{s} control plot of the X-S plot is given below:

For the upper control limit

$$UCL = B_4\bar{s} \quad (10)$$

For the central control limit

$$CL = \bar{s} \quad (11)$$

For the central control limit

$$UCL = B_3\bar{s} \quad (12)$$

If the lower limits of the control charts are negative, the lower limit values are accepted as 0. In this study, the data that is out of control is determined by integrating simulation and statistical process control.

III. RESULTS

In this study, 30-day data on patient stay and patient waiting times, which are defined as output variables, were obtained. According to the statistical process control charts of these data, the data out of control was determined. Four different Xbar-R and Xbar-S control graphs were created for each output variable. The Xbar-R graph of the first variable, length of stay, is shown in Figure 2.

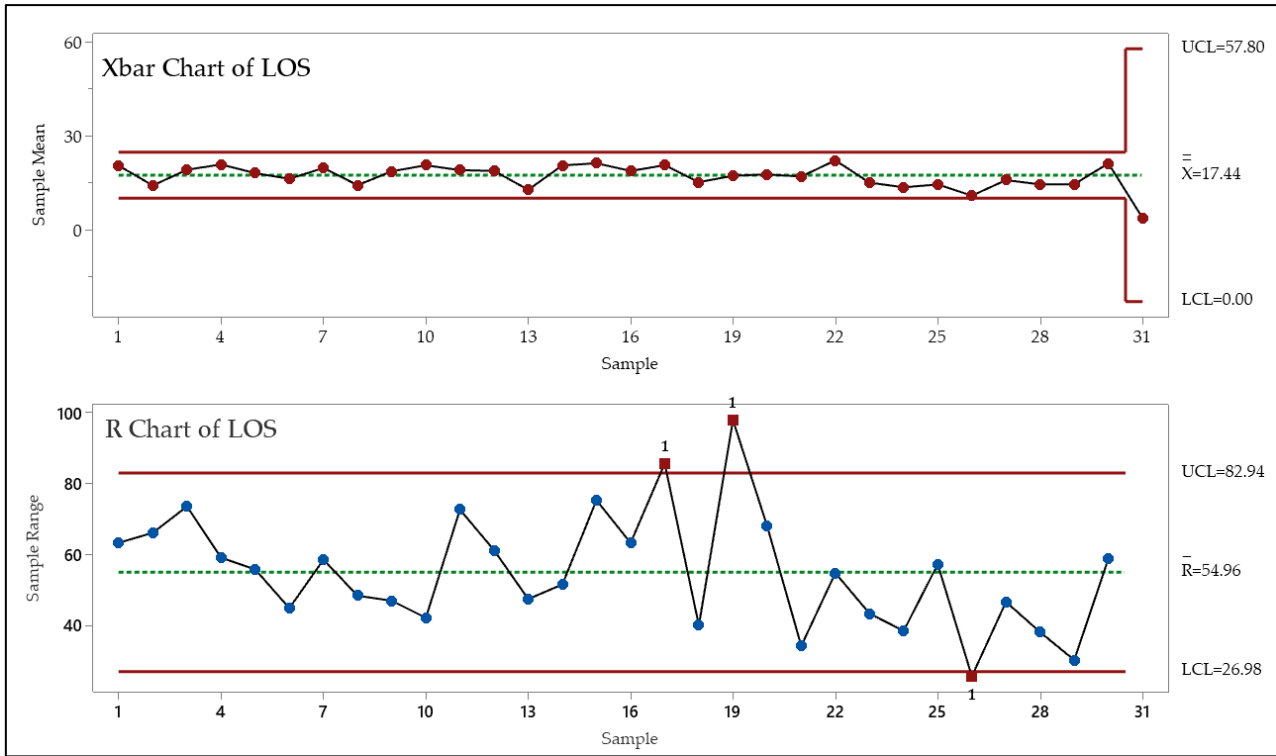


Figure 2. X-bar-R control chart of patient stay

It has been observed that the simulation model system is under control, according to the Xbar graph of the Xbar-R graph of the patient's length of stay. However, according to the R chart, it was observed

that 3 data were out of control and not under control. Data 17, 19, and 26 of the R graphs cause the system to be out of control. The Xbar-R graph of the second variable, length of stay, is shown in Figure 3.

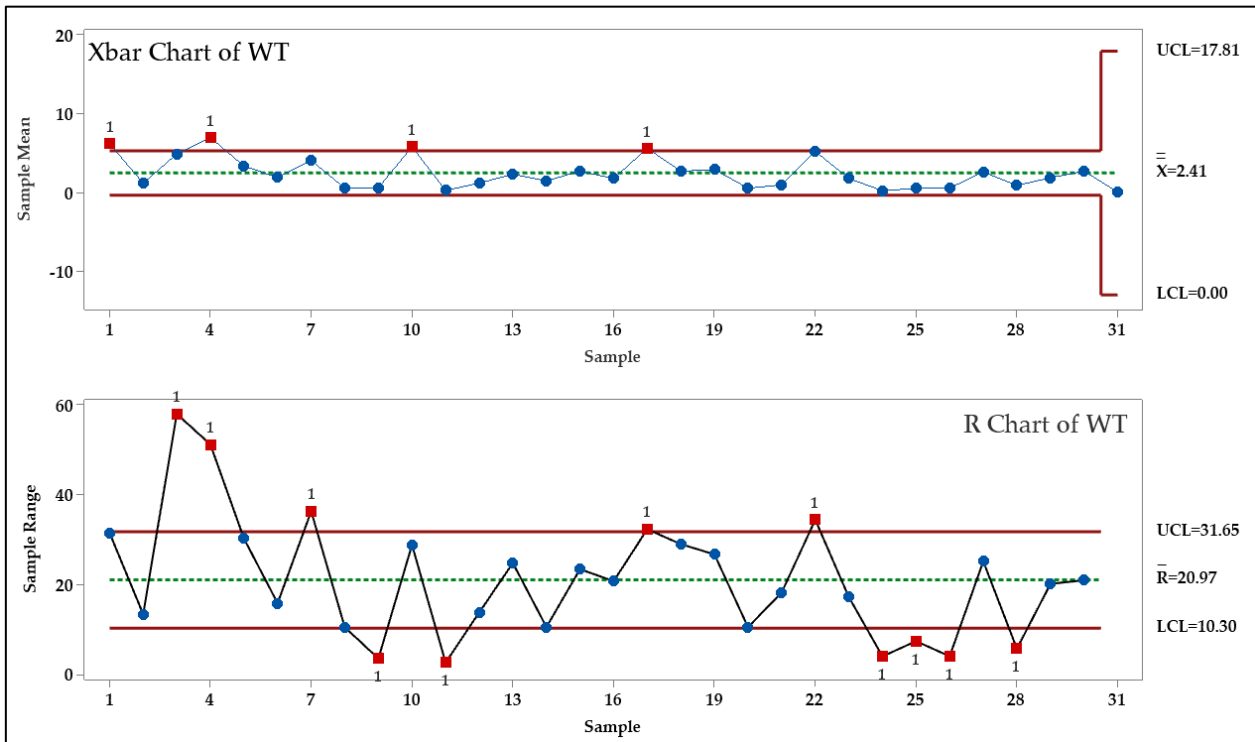


Figure 3. X-bar-R control chart of patient waiting time

According to the Xbar-R control chart of the simulation data of patient waiting times, the system is out of control. It was observed that 4 data, according to the Xbar control chart, and 11 data,

according to the R control chart, were out of control. Data out of control according to patient stay and waiting times are given in Table 3.

Table 3: Out-of-control data according to Xbar-R control charts of patient stay and waiting times

Output Variables	Control Charts	Data Out of Control	Explanation
LOS	X-bar Graph	---	in control
	R Graph	17, 19, 26	out of control
WT	X-bar Graph	1, 10, 14, 17	out of control
	R Graph	3, 4, 7, 9, 11, 17, 22, 24, 25, 26, 28	out of control

Out-of-control points indicate more than 3.00 standard deviations from the centerline. The Xbar

control chart from the Xbar-S control chart of the duration of patient stay, which is the first variable of the study, is shown in Figure 4.

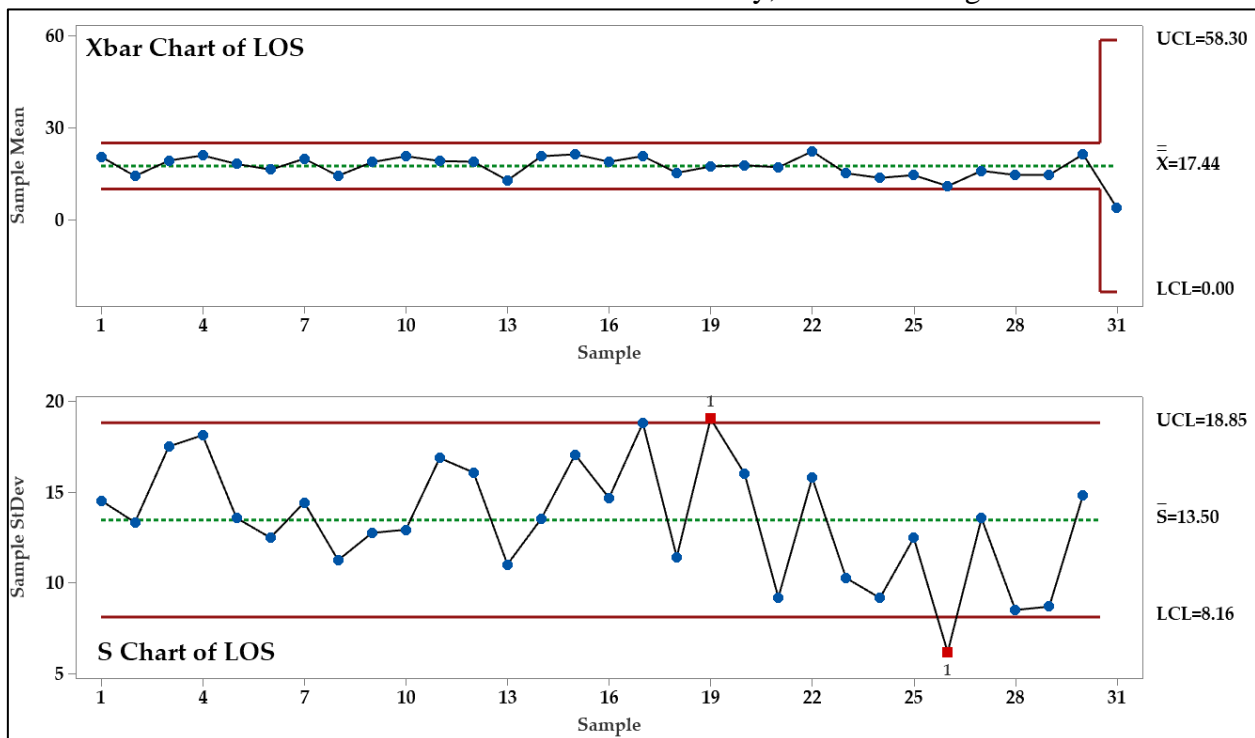


Figure 4. Xbar-S control chart of patient stay

According to the system of the simulation model, although the patient waiting times were under control according to the Xbar control chart, it was observed that 2 data were out of control according to the S control chart. The Xbar-S control chart of

the second variable, length of stay, is shown in Figure 5.

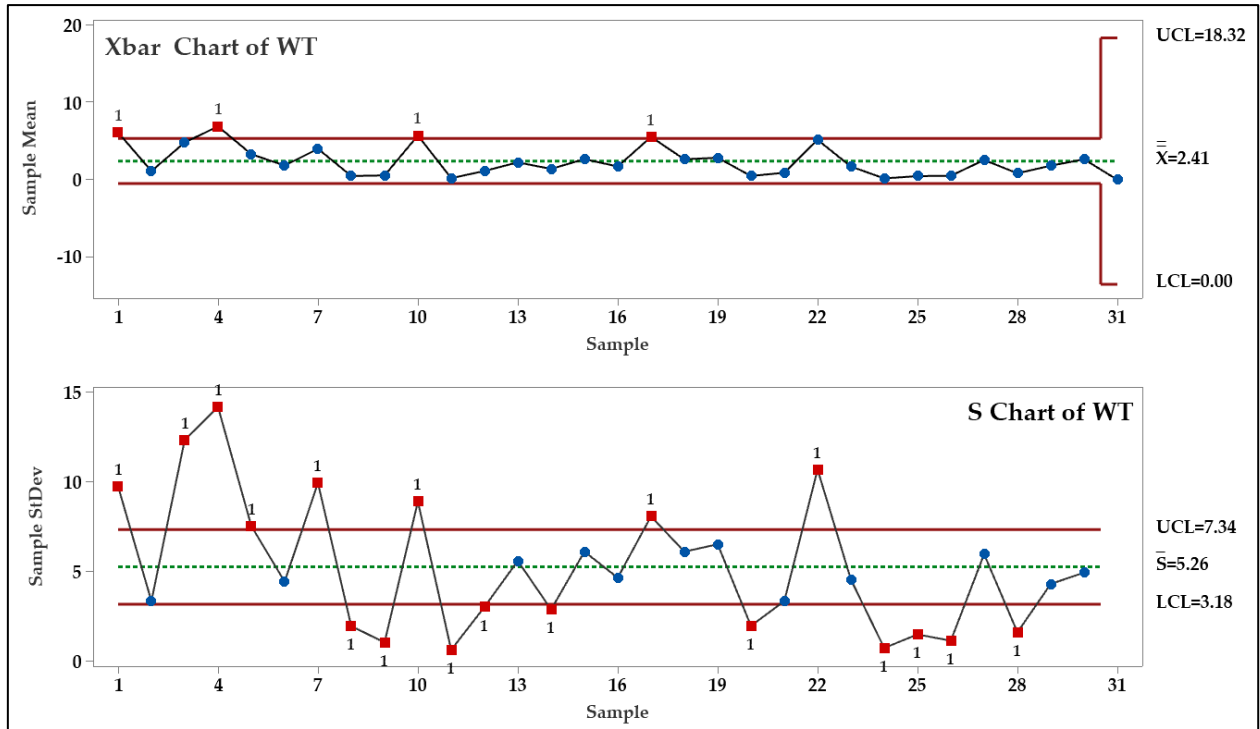


Figure 5. X-bar S control chart of patient waiting time

According to the Xbar-S control chart of the simulation data of patient waiting times, the system is out of control. It was observed that 4 data, according to the Xbar control chart, and 18 data, according to the S control chart, were out of control.

Data out of control according to patient stay and waiting times are given in Table 4.

Table 4: Out-of-control data according to X-bar S control charts of patient stay and waiting times

Output Variables	Control Charts	Data Out of Control	Explanation
LOS	X-bar Graph	---	in control
	R Graph	19, 26	out of control
WT	X-bar Graph	1, 4, 10, 17	out of control
	R Graph	1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 17, 20, 22, 24, 25, 26, 28	out of control

As a result, the accuracy of the results obtained by integrating the discrete-event simulation method and statistical control graphic technique proposed in this study was tested. The method used in this study has some assumptions. The 3D discrete-event simulation model created is according to the emergency department of a hospital, and the results obtained according to other simulation models may

also vary. The result data were derived according to the statistical distributions of the process data belonging to the simulation model. For this reason, derived data were used in the statistical control chart. Finally, based on the results obtained, this study is an example of using the proposed method in many areas such as health, transportation, economy, and production.

IV. CONCLUSION

This study proposes integrating 3D discrete-event simulation with statistical process control charts. A discrete-event simulation model has been developed for the emergency department, which has a dynamic

and dense structure of hospitals. There is variability in the results obtained in the model since the durations of the processes in this model are created according to statistical distributions. Data on two output variables, patient stay and waiting times, were obtained in the simulation model. These data were tested using Xbar-R and Xbar-S control charts from statistical process control charts to test whether the simulation model was under control. This study determined that both output variables were under control in Xbar charts but out of control according to R and S control charts. In addition, the data out of control was determined. This study ensures that the systems developed for simulation models are under control in a short time and at a low cost.

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