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APPROXIMATION OF FUZZY NUMBERS BY FAVARD-SZASZ-MIRAKYAN OPERATORS OF MAX-PRODUCT

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Abstract – In this study, the definition of the generalized Favard-Szasz-Mirakyan operators of max-product kind is extended to an arbitrary compact interval, by proving that their order of uniform approximation is the same as in the particular case of the unit interval. We will show that if fuzzy numbers are presented in parametric form, the generalized Favard-Szasz-Mirakyan maximum product operator produces a set of fuzzy numbers such that it is approximately the same as the uncertainty and value of the fuzzy number.

Keywords – Favard-Szasz-Mirakyan Operators, Max-Product Kind, Uniform Approximation.

I. INTRODUCTION

Most issues in daily life deal with complex ambiguous forms of information that are continuous transitions. Zadeh presented a fuzzy set (FS) in 1965 to interest in some information which has a wide scope of applications in many research fields.

The representation of fuzzy numbers by suitable intervals that depend mainly on the shape of their membership functions is an interesting and important problem and has many applications in various fields. And it is known that dealing with fuzzy numbers is often difficult because of the very complex representation of the shapes of their membership functions. That is to say, the interpretation and expression of fuzzy numbers are more intuitive and more natural whenever the shapes of their membership functions are simpler. In (Ban, 2009; Ban & Coroianu, 2009), authors made investigations on the approximation of fuzzy numbers by trapezoidal or triangular fuzzy members.

Nonlinear operators are extensively researched in approximation theory and these approximations may produce better outcomes than their linear counterparts. There are numerous approximation results utilizing max-product operators in the literature (see (Coroianu, 2014; Coroianu, 2019). Max-product versions of Bernstein, Shepard, and Favard-Szasz-Mirakjan operators (see Bede, 2010; Bede, 2009), have greater orders of approximation and a faster rate of approximation than their classical counterparts in specific circumstances.

In this study, we initially extend to an arbitrary compact interval the definition of the generalized Favard-Szasz-Mirakyan operators of max-product kind, by proving that their order of uniform approximation is the same as in the particular case of the unit interval.

II. MATERIALS AND METHOD

Definition 2.1. (Wu & Gong, 2001) **(Fuzzy numbers)**

A fuzzy subset u of the real line $\mathbb{R}$ with membership function $\mu_u(x):\mathbb{R}\to[0,1]$

is called a fuzzy number if:

1. μ_u is normal, i.e. $\exists x_0$ such that $\mu_u(x)=0$;

2. μ_u is a fuzzy convex subset, i.e. $\mu_u(\lambda x+(1-\lambda)y)\ge\min\{\mu_u(x),\mu_u(y)\}, \forall x,y\in\mathbb{R}, \forall\lambda\in[0,1]$;

3. μ_u is upper semicontinuous on $\mathbb{R}$;and

4. $supp(u)$ is bounded interval, where $supp(u)=cl\{x\in\mathbb{R} \mid \mu_u(x)>0\}$, where (cl) is the closure operator.

It is clear, for any fuzzy number u there exist four numbers $t_1,t_2,t_3,t_4\in\mathbb{R}$ and the functions $l_u,r_u:\mathbb{R}\to[0,1]$ such that we can define a membership function μ_u as follows:

$$\mu_u(x)=\begin{cases} 0 & if \quad x<t_1 \\ l_u(x) & if \quad t_1\le x\le t_2 \\ 1 & if \quad t_2\le x\le t_3 \\ r_u(x) & if \quad t_3\le x\le t_4 \\ 0 & if \quad t_4<x \end{cases}$$

where $l_u:[t_1,t_2]\to[0,1]$ is nondecreasing called the left side of a fuzzy number u and $r_u:[t_3,t_4]\to[0,1]$ is nonincreasing called the right side of a fuzzy number u .

The set of all fuzzy real numbers is denoted by $\mathbb{R}_F$.

Definition 2.2. (Lee, 2004) (α-cut) The α-cut of a fuzzy number u is the crisp set which contains all elements $x\in\mathbb{R}$ of degree higher than or equal to (α) , For $\alpha\in(0,1]$, and denoted by $[u]_\alpha$ i.e. $[u]_\alpha=\{x\in\mathbb{R} \mid \mu_u(x)\ge\alpha\}$.

It is explicit that $[u]_\alpha=[u^-(\alpha),u^+(\alpha)]$, $\alpha\in(0,1]$ where

$u^-(\alpha)=\inf\{x\in\mathbb{R} \mid \mu_u(x)\ge\alpha\}$,
$u^+(\alpha)=\sup\{x\in\mathbb{R} \mid \mu_u(x)\ge\alpha\}$.

Notation

1. if $\alpha=1$ then , $[u]_1=[u^-(1),u^+(1)]$ is called the core of u which will be denoted by core (u)

2. if $\alpha=0$ then , $[u]_0=[u^-(0),u^+(0)]=cl\{x\in\mathbb{R} \mid \mu_u(x)>0\}$, is called the support of u where (cl) is the closure operator,which will be denoted by $supp(u)$ i.e., $supp(u)=[u^-(0),u^+(0)]=\overline{\bigcup_{\alpha\in(0,1]}[u]_\alpha}$

this is why $supp(u)$ is also called the $0-cut$ of u.

III. MAX-PRODUCT GENERALIZED FAVARD-SZASZ-MIRAKYAN OPERATORS

The purpose of this study is to use the generalized Favard-Szasz-Mirakyan operator of max-product kind, firstly introduced in (Bede, 2010), for using continuous membership functions to approximate fuzzy numbers.

These operators are piecewise rational and nonlinear, defined by

$$F_n^{(*M)}(f)(\varsigma)=\frac{\bigvee_{k=0}^{n}s_{n,k}(\varsigma)f(\frac{k}{a_n})}{\bigvee_{k=0}^{n}s_{n,k}(\varsigma)}, a_n\to\infty$$

where $s_{n,k}(\varsigma) = \frac{(n\varsigma)^k}{k!}$ and $f: [0,1] \to \mathbb{R}_+$. (see Bede, 2010)

Construction of Max-Product the generalized Favard-Szasz-Mirakyan operators defined on compact intervals.

In this section, we describe the corresponding max-product the generalized Favard-Szasz-Mirakjan operator on $[\varphi,\theta]$ by

$$F_n^{(*M)}(f;[\varphi,\theta])(\varsigma) = \frac{\bigvee_{k=0}^{n} s_{n,k}(\varsigma) f(\varphi + (\theta-\varphi)\frac{k}{a_n})}{\bigvee_{k=0}^{n} s_{n,k}(\varsigma)}, \varsigma \in [\varphi,\theta], p > 1$$

where $s_{n,k}(\varsigma) = \frac{n^k}{k!}\left(\frac{\varsigma-\theta}{\theta-\varphi}\right)^k$ for a function $f \in C_+([\varphi,\theta])$. Also, we can obtain $F_n^{(M)}(f;[\varphi,\theta])(\theta) = f(\theta)$ and $F_n^{(M)}(f;[\varphi,\theta])(\varphi) = f(\varphi)$.

Theorem 3.1. Let take $\varphi, \theta \in$ and φ<θ , for all $n, \varsigma \in [\varphi,\theta]$,

[i] If the function $f: [\varphi,\theta] \to \mathbb{R}_+$ is continuous, then we get the estimate

$$\left|F_n^{(*M)}(f;[\varphi,\theta])(\varsigma) - f(\varsigma)\right| \le 6([\varphi-\theta]+1)\omega_1(f,\frac{1}{\sqrt{a_n}})_{[\varphi,\theta]}$$

[ii] If $f: [\varphi,\theta] \to \mathbb{R}_+$ is nondecreasing concave function on $[\varphi,\theta]$, then we get the estimate

$$\left|F_n^{(*M)}(f;[\varphi,\theta])(\varsigma) - f(\varsigma)\right| \le ([\varphi-\theta]+1)\omega_1(f,\frac{1}{a_n})_{[\varphi,\theta]}$$

Proof. (i) Let take the function $h: [0,1] \to \mathbb{R}_+$, $h(\sigma) = f(\varphi + (\theta-\varphi)\sigma)$. It is simple to verify that $h\left(\frac{k}{a_n}\right) = f(\varphi + (\theta-\varphi)\frac{k}{a_n})$ for all $k \in \{0,1,...\}$.

Now, let choose arbitrary $(\varsigma \in [\varphi,\theta])$ and let $\sigma \in [0,1]$ be such that $\varsigma = \varphi + (\theta-\varphi)\sigma$. This implies $\sigma = (\varsigma - \varphi)/(\theta-\varphi)$ and $1-\sigma = (\varphi-\varsigma)/(\theta-\varphi)$. Taking into account these equalities and the expressions for $h\left(\frac{k}{a_n}\right)$, we obtain $F_n^{(*M)}(f;[\varphi,\theta])(\varsigma) = F_n^{(*M)}(h;[0,1])(\sigma)$.

By Theorem , we obtain

$$\left|F_n^{(*M)}(f;[\varphi,\theta])(\varsigma) - f(\varsigma)\right| = \left|F_n^{(*M)}(h;[0,1])(\sigma) - h(\sigma)\right| \le 6\omega_1(h,\frac{1}{\sqrt{a_n}})_{[0,1].}$$

From the property , then we get

$\omega_1(h;1/\sqrt{n})_{[0,1]}$ [illegible] $1)\omega_1(f;1/\sqrt{n})$[illegible]

(ii) Keeping the notation from the above point (i), we obtain $F_n^{(*M)}(f;[\varphi,\theta])(\varsigma) = F_n^{(*M)}(h;[0,1])(\sigma)$, which , $h(\sigma) = f(\varphi + (\theta-\varphi)\sigma)$ for all $\sigma \in [0,1]$. The last equality is equivalent to $f(u) = h(\frac{u-\varphi}{\theta-\varphi})$ for all $\sigma \in [\varphi,\theta]$. Now the concavity property of the function f is

[illegible] u_1 [illegible] u_2 [illegible] u_1 [illegible] u_2 [illegible], for all [illegible] $,1]$, u_1, u_2 [illegible] which can be written in terms of g,

[illegible]

Denoting [illegible] u_1 [illegible] $,1]$ and [illegible] u_2 [illegible] $,1]$ this immediately implies the concavity of g on $[0,1]$. Then, we get

$$\left|F_n^{(*M)}(f;[\varphi,\theta])(\varsigma) - f(\varsigma)\right| = \left|F_n^{(*M)}(h;[0,1])(\sigma) - h(\sigma)\right| \le \omega_1(h,\frac{1}{a_n})_{[0,1].}$$

Reasoning now exactly as in the above point (i), we get the desired conclusion.

IV. DISCUSSION AND CONCLUSION

In the present paper, nonlinear maximum product type the generalized Favard-Szasz-Mirakyan operators are introduced. Some new theorems regarding the approximation of these maximum product type operators are given. Later, the approximation properties of the operators for $F_n^{(*M)}$ is investigated using the multivariate moduli of continuity. In addition, the approximation of functions for these maximum product operators under differentiability are investigated.

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