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M_λ Method of Triple Sequence Space

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Abstract – In this paper, we introduce the M_λ −method using by triple sequence $\lambda = (\lambda_{mnk})$ and discuss general topological properties of this method.

Keywords –Regular Matrix, İnfinite Matrix, Silverman-Toeplitz Theorem, Triple Sequence.

I. Introduction

We introduce a new definition of limit of a triple sequence and a triple series on convergent triple sequences and Silverman-Toeplitz theorem for triple sequences and triple series.

A triple sequence (real or complex) can be defined as a function $x: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by *Aiyub et al. [1]*, *Esi et al. [2-5]*, *Bharathi et al. [7]*, *Subramanian et al. [8-18]*, *Debnath et al. [6]* and many others.

2.Definitions and Preliminaries

2.1 Definition

Let (x_{mnk}) be a triple sequence. We say that $lim_{m,n,k\to\infty} x_{mnk} = x$, if for every $\epsilon > 0$, the set $\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - x| \geq \epsilon\}$ is finite, $\mathbb{N}$ being the set of positive integers. In such a case, x is unique and x is called the limit of (x_{mnk}).

2.2 Definition

Let (x_{mnk}) be a triple sequence. We say that $s = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} x_{mnk}$ if $s = lim_{m.n,k\to\infty} s_{mnk}$, where $s_{mnk} = \sum_{r=0}^{m} \sum_{s=0}^{n} \sum_{t=0}^{k} x_{rst}, m, n, k = 0,1,2,\cdots$.

2.3 Definition

The triple series $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} x_{mnk}$ is said to converge absolutely if $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} |x_{mnk}|$ converges.

2.4 Definition

Let $A = (a_{mnk}^{rst})$ be a six dimensional infinite matrix and $x = (x_{mnk})$ a triple sequence. Then the transformation sequence is $A(x) = ((Ax)_{mnk})$, where

$$(Ax)_{m,n,k} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{mnk}^{rst} x_{rst}.$$

If $lim_{m,n,k\to\infty} (Ax)_{m,n,k} = s$, we say that the triple sequence $x = (x_{mnk})$ is $A-$ summable or summable A to s, written as $x_{mnk} \to s(A)$. If $lim_{m,n,k\to\infty} (Ax)_{m,n,k} = s$, whenever $lim_{m,n,k\to\infty} x_{mnk} = s$, we say that the six dimensional infinite matrix $A = (a_{mnk}^{rst})$ is regular.

2.5 Theorem

$lim_{m,n,k\to\infty} x_{mnk} = x$ if and only if

(i) $lim_{m\to\infty} x_{mnk} = x, n, k = 0,1,2,\cdots$,

(ii) $lim_{n\to\infty} x_{mnk} = x, m, k = 0,1,2,\cdots$,

(iii) $lim_{k\to\infty} x_{mnk} = x, m, n = 0,1,2,\cdots$,

(iv) for any $\epsilon > 0$, there exists and $N \in \mathbb{N}$ such that $|x_{mnk} - x| < \epsilon, \forall \quad m,n,k \geq N$. (Note that this is Prinsheims definition of limit of a triple sequence)

2.6 Theorem

If the triple series $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} x_{mnk}$ converges, then

$lim_{m,n,k\to\infty} x_{mnk} = 0$ but the converse is not true.

2.7 Theorem

The six dimensional infinite matrix $A = (a_{mnk}^{rst})$ is regular if and only if (i)$sup_{m,n,k} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} |a_{mnk}^{rst}| < \infty$,

(ii)$lim_{m,n,k\to\infty} a_{mnk}^{rst} = 0, r,s,t = 0,1,2,\cdots$,

(iii)$lim_{m,n,k\to\infty} \sum_{r=0}^{m} \sum_{s=0}^{n} \sum_{t=0}^{k} a_{mnk}^{rst} = 1$,

(iv)$lim_{m,n,k\to\infty} \sum_{r=0}^{\infty} |a_{mnk}^{rst}| = 0, s,t = 0,1,2,\cdots$,

(v)$lim_{m,n,k\to\infty} \sum_{s=0}^{\infty} |a_{mnk}^{rst}| = 0, r,t = 0,1,2,\cdots$,

(vi)$lim_{m,n,k\to\infty} \sum_{t=0}^{\infty} |a_{mnk}^{rst}| = 0, r,s = 0,1,2,\cdots$.

3. The summability method of M_λ

In this section, we introduce the M_λ −method for triple sequences.

3.1 Definition

Let (λ_{mnk}) be a triple sequence such that $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} |\lambda_{mnk}| < \infty$. The method M_λ is defined by the six dimensional infinite matrix (a_{mnk}^{rst}), where

$$(a_{mnk}^{rst}) = \begin{cases} \lambda_{m-r,n-s,k-t} & \text{if } r \leq m, n \leq s, t \leq k \\ 0, & \text{otherwise} \end{cases}$$

3.2 Definition

The methods M_λ and M_μ are said to be consistent if $s_{rst} \to \sigma(M_\lambda)$ and $s_{rst} \to \sigma'(M_\mu) \Rightarrow \sigma = \sigma'$.

3.3 Definition

We say that M_λ is included in M_μ written as $M_\lambda \subseteq M_\mu$ if $s_{rst} \to \sigma(M_\lambda) \Rightarrow s_{rst} \to \sigma(M_\mu)$. The two methods M_λ and M_μ are said to be equivalent if $M_\lambda \subseteq M_\mu$ and $M_\mu \subseteq M_\lambda$.

IV. Discussion and Results

4.1 Theorem

The method M_λ is regular if and only if

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \lambda_{mnk} = 1.$$

In the sequel, let M_λ and M_μ be regular methods such that each row and each column of the infinite matrices $\lambda = (\lambda_{mnk})$ and $\mu = (\mu_{mnk})$ are regular.

4.2 Theorem

If M_λ and M_μ be two regular methods then $\sigma = \sigma'$.

4.3 Theorem

If M_λ and M_μ are regular, then $M_\lambda \subseteq M_\mu$ if and only if $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} |g_{mnk}| < \infty$ and $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} g_{mnk} = 1$.

4.4 Theorem

The regular methods M_λ and M_μ are equivalent if and only if $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} |g_{mnk}| < \infty, \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} g_{mnk} = 1$ and

$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} |h_{mnk}| < \infty, \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} h_{mnk} = 1$.

4.5 Theorem

If $lim_{m,n,k\to\infty} a_{mnk} = 0$ and $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} |b_{mnk}| < \infty$, then $lim_{m,n,k\to\infty} c_{mnk} = 0$, where $c_{mnk} =$

$\sum_{r=0}^{m}\sum_{s=0}^{n}\sum_{t=0}^{k} a_{m-r,n-s,k-t}b_{rst}, m,n,k = 0,1,2,\cdots$.

We now have the following results on the Cauchy multiplication of $M_\lambda -$ summable triple sequences and triple series.

4.6 Theorem

If $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} |a_{mnk}| < \infty$ and (b_{mnk}) is $M_\lambda -$ summable to B, then (c_{mnk}) is $M_\lambda -$ summable to AB, where $c_{mnk} = \sum_{r=0}^{m}\sum_{s=0}^{n}\sum_{t=0}^{k} a_{m-r,n-s,k-t}b_{rst}, m,n,k = 0,1,2,\cdots,\infty$. and $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{mnk} = A$.

4.7 Theorem

If $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} |a_{mnk}| < \infty$ and $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} b_{mnk}$ is $M_\lambda -$ summable to B, then $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} c_{mnk}$ is $M_\lambda -$ summable to AB,

where $c_{mnk} = \sum_{r=0}^{m}\sum_{s=0}^{n}\sum_{t=0}^{k} a_{m-r,n-s,k-t}b_{rst}, m,n,k = 0,1,2,\cdots,\infty$. and

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{mnk} = A.$$

We define we define $(M_\lambda M_\mu)(x) = M_\lambda\left(M_\mu(x)\right)$ for triple sequence $x = (x_{mnk})$.

4.8 Theorem

Let M_λ, M_μ be regular methods. Then, $\left(M_\lambda \cdot M_\mu\right)$ is also regular.

4.9 Theorem

Let M_λ, M_μ and $M_\mathfrak{J}$ are regular methods, $M_\lambda \subseteq M_\mu$ if and only if $(M_\mathfrak{J})(M_\lambda) \subseteq (M_\mathfrak{J})M_\mu$.

4.10 Theorem

Let M_λ, M_μ and $M_\mathfrak{J}$ are regular methods, then the following statements are equivalent:

(i) $M_\lambda \subseteq M_\mu$;

(ii) $(M_\mathfrak{J})(M_\lambda) \subseteq (M_\mathfrak{J})\left(M_\mu\right)$; and

(iii) $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} |g_{mnk}| < \infty$ and $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} g_{mnk} = 1$, where

$\frac{\mu(x)}{\lambda(x)} = g(x) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} g_{mnk}x^m y^n z^k; \lambda(x) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \lambda_{mnk}x^m y^n z^k; \mu(x) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \mu_{mnk}x^m y^n z^k.$

V. CONCLUSION

In this study we introduced the $M_\lambda -$method using by triple sequence $\lambda = (\lambda_{mnk})$ and discussed general topological properties of this method.

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