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Analytical modeling of the behavior of nano-plates by the theory of non-local elasticity

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Abstract – The analysis of mechanical buckling of nano-plates by the non-local theory of Reissner Mindlin which takes into account the effects of small scales is made in this work. A particular effort is made on the various parameters which influence the buckling load of the nano-plates, such as: the number of modes, the geometric parameters and the mechanical parameters.

Keywords – Buckling, Nano-Plates, Non-Local Theory, Number Of Modes, Geometric Parameters.

I. INTRODUCTION

Nanostructures have attracted considerable attention in the scientific communities of researchers for micro-electromechanical (MEMS) and nano-electromechanical (NEMS) systems.

Several analytical and numerical analyzes for the mechanical behaviors of nanostructures have been published in the literature as molecular and mechanical dynamic simulations of continuous media. To cite, Sakhaee-pour et al [1] who, using atomistic modeling, studied the frequency characteristic of a single layer of graphene sheet with different boundary conditions.

Moreover, the elastic buckling behavior of a single layer of graphene sheet is studied by the same modeling [2].

In addition, Behfar and Naghdabadi [3] used the continuum model based on the study of vibrational behavior of multilayer graphene sheets embedded in an elastic medium.

The calculations of molecular mechanics simulations are very demanding, and classical theories do not admit the intrinsic size dependence in the elastic solutions of micro-nanometric materials and structures.

Therefore, the nonlocal elasticity theory is formulated to modify the model of classical elasticity theory by considering the small-scale effect of nanostructures of materials by Eringen [4].

A non-local plate continuum model was formulated for the first investigation of the small-scale influence on micro-nanometer circular plate buckling by Duan and Wang [5] who found that the deflection became larger than the classic plate continuum model.

Among the many challenges, buckling analysis of nanoplates is more important for understanding the stability response under compressive loads for nanoscale plates than micro-electromechanical and nano-electromechanical components.

The buckling behavior of biaxially compressed single-layer graphene sheets is investigated based on the nonlocal plate continuum model [6]. The results showed that they have a depreciating effect on the buckling load.

Pradhan [7] studied the high-order theory of shear strain using Eringen's nonlocal constitutive differential relations.

II. Theoretical formulations

1 Analysis of nano-plates by the theory of non-local elasticity

Unlike the local theory, the non-local theory assumes that the stress at a point depends not only on the strain at that point, but also on the strains at all other points in the body. Eringen [8] proposed a differential form of the nonlocal constitutive relation as follows [9]:

$$\left(1-(e_0 a)^2\nabla^2\right)\sigma_{ij}^{nl} = c_{ij}\varepsilon_{ij} \qquad (01)$$

With:

$(e_0 a)^2$: Parameter not local.

σ_{ij}^{nl} : Tensor of non-local stresses.

c_{ij} : Constant elastic

ε_{ij} : Strain tensor.

∇^2 : Laplacian operator.

In this work the graphene sheet nano-plate is modeled as a nano-plate of length L , width b and thickness h, subjected to bi-axial compressive loads in a coordinate system (x, y, z) , see figure1.

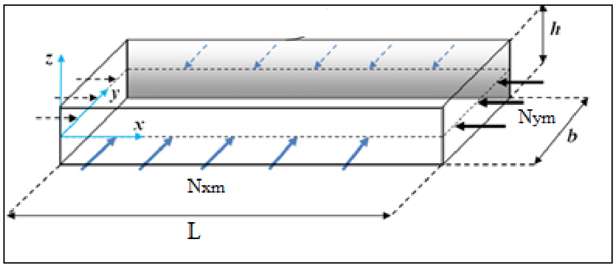

Figure 1 Mindlin nano-plate geometry and properties.

2 Elasticity of a solid

A solid is said to be elastic if it returns to its original state when the external forces that deformed it are removed. This return to the starting state is due to internal constraints.

For small deformations, it has been observed experimentally that the deformations of a solid are linearly proportional to the stresses applied to it. When the deformations are more important, this relation becomes nonlinear but the solid returns to its initial state when the constraints are removed. On the other hand, when the deformations increase and exceed a certain limit, these deformations are no longer elastic.

After this elastic limit, the solid deforms in a permanent way (plastic deformation) and finally it breaks. Figure II.2 shows the interdependence between stresses and strains as a function of stress intensity.

In the hypothesis of small strains, there is a one-to-one relationship between stress and strain [10].

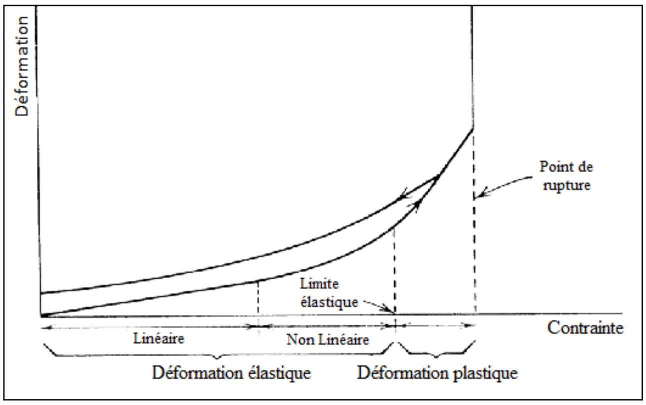

Figure 2 Typical relationship between stresses and strains in a solid [10].

3 Displacement field

3.1 First order shear strain theory (FSDT)

The first-order shear deformation theory extended the classical plate theory by taking into account the transverse shear effect. In this case, the stresses and strains are constant through the thickness of the plate, which requires the introduction of a shear correction factor [11].

2.3.2 Reissner-Mindlin assumptions [12]

The behavior of the material is elastic. The relationship between the stress tensor and the strain tensor is given by Hooke's law. This translates a state of stress constantly proportional to the state of deformation.

Reissner Mindlin's assumptions are as follows:

1- A point of the mean plane has a movement in this plane; membrane stress may therefore appear in the middle sheet.

2- The state of stress is a state of plane stress; stresses normal to the mean sheet are neglected.

3- A straight section, normal to the average sheet in the initial configuration, is not necessarily normal after deformation.

4- The rotational inertia of the straight sections is taken into account.

To introduce the effect of transverse shearing, the kinematic assumption is adopted: the normal remains straight but not perpendicular to the average surface (because of the effect of transverse shearing) in the deformed configuration (Figure 3)

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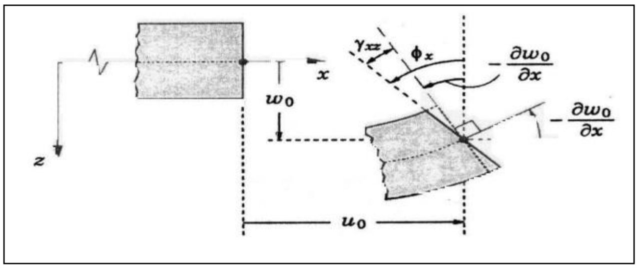

Figure 3 Reissner-Mindlin kinematics [12]

The field of displacement of the plate of Reissner-Mindlin is written in the following way:

$$\begin{cases} u_x(x,y,z) = u_0(x,y) + z\psi_x(x,y) \\ u_y(x,y,z) = v_0(x,y) + z\psi_y(x,y) \\ u_z(x,y,z) = w\ (x,y) \end{cases} \quad (02)$$

With :

u_x, u_y and u_Z : Displacement components.

u_0, v_0, w : Membrane displacements of the normal of the mean plane (z = 0) of the nano-plate.

$\psi_x(x,y), \psi_y(x,y)$: The transverse rotations around the axes x et y; respectively.

4 Deformation field

The general form of the deformations according to displacements can be expressed as follows:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + Z \begin{Bmatrix} \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \psi_y}{\partial y} \\ \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \psi_y + \frac{\partial w}{\partial y} \\ \psi_x + \frac{\partial w}{\partial x} \end{Bmatrix} \quad (03)$$

Or:

$\varepsilon_{xx}, \varepsilon_{yy}$: The components of normal deformations.

$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$: Components of shear strains.

5 Non-local constitutive equations

By the use of the theory of nonlocal elasticity; Equation (1) gives the non-local constitutive relations of the graphene nano-sheet as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy}^{nl} \end{Bmatrix} - (e_0 a)^2 \nabla^2 \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} E/(1-\nu^2) & \nu E/(1-\nu^2) & 0 \\ \nu E/(1-\nu^2) & E/(1-\nu^2) & 0 \\ 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} - (e_0 a)^2 \nabla^2 \begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} 2G & 0 \\ 0 & 2G \end{bmatrix} \begin{Bmatrix} \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix} \quad (04)$$

With :

$G = E / 2(1+\nu)$

E : Young's modulus.

G : Shear modulus.

ν : Poisson coefficient.

6 Equations of motion

Let us use Hamilton's principle to obtain the equations of motion. This principle is given by the following analytical form [13]:

$$\int_0^t \delta(U+W)dt = 0 \quad (05)$$

Or :

δ: Index of variation with respect to x and y; respectively.

U : Strain energy

W : Potential energy.

• The deformation energy of the nano-plate can be given by:

$$U=\frac{1}{2}\int_v \sigma_{ij}^{nl}\varepsilon_{ij}dV=\frac{1}{2}\int_v\left(\sigma_{xx}^{nl}\varepsilon_{xx}+\sigma_{yy}^{nl}\varepsilon_{yy}+\sigma_{xy}^{nl}\gamma_{xy}+\sigma_{yz}^{nl}\gamma_{yz}+\sigma_{xz}^{nl}\gamma_{xz}\right)dV \quad (06)$$

By replacing equations (3) and (4) in equation (6) and integrating through the thickness of the plate, the strain energy is written as follows:

$$U=\frac{1}{2}\int_\Omega\left(N_{xx}\frac{\partial u_0}{\partial x}+N_{yy}\frac{\partial v_0}{\partial y}+N_{xy}\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)+M_{xx}\frac{\partial \psi_x}{\partial x}+M_{yy}\frac{\partial \psi_y}{\partial y}\right.$$
$$\left.+M_{xy}\left(\frac{\partial \psi_x}{\partial y}+\frac{\partial \psi_y}{\partial x}\right)+Q_{xx}\left(\psi_x+\frac{\partial w}{\partial x}\right)+Q_{yy}\left(\psi_y+\frac{\partial w}{\partial y}\right)\right)d\Omega \quad (07)$$

The resulting forces and moments are defined by:

$$N_{xx}=\int_{-h/2}^{h/2}\sigma_{xx}^{nl}dz \quad ; \quad N_{yy}=\int_{-h/2}^{h/2}\sigma_{yy}^{nl}dz \quad ; \quad N_{xy}=\int_{-h/2}^{h/2}\sigma_{xy}^{nl}dz$$
$$M_{xx}=\int_{-h/2}^{h/2}z\sigma_{xx}^{nl}dz \quad ; \quad M_{yy}=\int_{-h/2}^{h/2}z\sigma_{yy}^{nl}dz \quad ; \quad M_{xy}=\int_{-h/2}^{h/2}z\sigma_{xy}^{nl}dz$$
$$Q_{xx}=\int_{-h/2}^{h/2}\sigma_{xz}^{nl}dz \quad ; \quad Q_{yy}=\int_{-h/2}^{h/2}\sigma_{yz}^{nl}dz \quad (08)$$

With :

N_{xx}, N_{yy} and N_{xy} : The normal forces.

M_{xx} and M_{yy} : Bending moments.

M_{xy} : The torque.

Q_{xx} and: Shear forces.

By replacing the equations (3) and (4) in the equation (8) and by integrating through the thickness of the plate, one obtains the expressions of the forces and the moments according to displacements:

$$\begin{cases}
N_{xx}-(e_0a)^2\nabla^2N_{xx}=\dfrac{Eh}{(1-\nu^2)}\left(\dfrac{\partial u_0}{\partial x}+\nu\dfrac{\partial v_0}{\partial y}\right)\\
N_{yy}-(e_0a)^2\nabla^2N_{yy}=\dfrac{Eh}{(1-\nu^2)}\left(\dfrac{\partial v_0}{\partial y}+\nu\dfrac{\partial u_0}{\partial x}\right)\\
N_{xy}-(e_0a)^2\nabla^2N_{xy}=Gh\left(\dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\right)\\
M_{xx}-(e_0a)^2\nabla^2M_{xx}=\dfrac{Eh^3}{12(1-\nu^2)}\left(\dfrac{\partial \psi_x}{\partial x}+\nu\dfrac{\partial \psi_y}{\partial y}\right)\\
M_{yy}-(e_0a)^2\nabla^2M_{yy}=\dfrac{Eh^3}{12(1-\nu^2)}\left(\dfrac{\partial \psi_y}{\partial y}+\nu\dfrac{\partial \psi_x}{\partial x}\right)\\
M_{xy}-(e_0a)^2\nabla^2M_{xy}=\dfrac{Gh^3}{12}\left(\dfrac{\partial \psi_x}{\partial y}+\dfrac{\partial \psi_y}{\partial x}\right)\\
Q_{xx}-(e_0a)^2\nabla^2Q_{xx}=Gh\left(\psi_x+\dfrac{\partial w}{\partial x}\right)\\
Q_{yy}-(e_0a)^2\nabla^2Q_{yy}=Gh\left(\psi_y+\dfrac{\partial w}{\partial y}\right)
\end{cases} \quad (09)$$

• The potential energy of the applied charges can be expressed by:

$$W=-\int_A qw\,d\Omega+\frac{1}{2}\int_A\left[N_{xm}\frac{\partial^2 w}{\partial x^2}+N_{ym}\frac{\partial^2 w}{\partial y^2}+2N_{xym}\frac{\partial^2 w}{\partial x\partial y}\right]d\Omega \quad (10)$$

Or :

q : The transverse distribution effort.

N_{xm}, N_{ym}, N_{xym} : Compressive loads.

By replacing the expressions of equations (7) and (10) in equation (5) and then integrating by parts, the equations of motion of the nano-plate theory are obtained:

$$\begin{cases}
\dfrac{\partial N_{xx}}{\partial x}+\dfrac{\partial N_{xy}}{\partial y}=0\\
\dfrac{\partial N_{xy}}{\partial x}+\dfrac{\partial N_{yy}}{\partial y}=0\\
\dfrac{\partial Q_{xx}}{\partial x}+\dfrac{\partial Q_{yy}}{\partial y}+\dfrac{\partial}{\partial x}\left(N_{xm}\dfrac{\partial w}{\partial x}+N_{xym}\dfrac{\partial w}{\partial y}\right)+\dfrac{\partial}{\partial y}\left(N_{xym}\dfrac{\partial w}{\partial x}+N_{ym}\dfrac{\partial w}{\partial y}\right)=0\\
\dfrac{\partial M_{xx}}{\partial x}+\dfrac{\partial M_{xy}}{\partial y}-Q_{xx}=0\\
\dfrac{\partial M_{xy}}{\partial x}+\dfrac{\partial M_{yy}}{\partial y}-Q_{yy}=0
\end{cases} \quad (11)$$

For this analysis, the nano-plate is subjected to a bi-axial compressive load defined as follows:

$$N_{xm}=P \;,\; N_{ym}=\lambda P \;,\; N_{xym}=q=0 \quad (12)$$

With :

P : Loading in the plane per unit length.
λ : Lateral load parameter.

By using the relations (9) and (11), we obtain the following equations of motion:

$$\begin{cases} Gh.\left(\frac{\partial\psi_x}{\partial x}+\frac{\partial\psi_y}{\partial y}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=\left(1-(e_0a)^2\nabla^2\right).\left[-\frac{\partial}{\partial x}\left(N_{xm}\frac{\partial w}{\partial y}+N_{xym}\frac{\partial w}{\partial y}\right)-\frac{\partial}{\partial y}\left(N_{xym}\frac{\partial w}{\partial x}+N_{ym}\frac{\partial w}{\partial y}\right)\right] \\ \left(\frac{Eh^3}{12(1-\nu^2)}\right).\left(\frac{\partial^2\psi_x}{\partial x^2}+\nu\frac{\partial^2\psi_y}{\partial x\partial y}\right)+\frac{Gh^3}{12}.\left(\frac{\partial^2\psi_x}{\partial y^2}+\frac{\partial^2\psi_y}{\partial x\partial y}\right)-Gh\left(\psi_x+\frac{\partial w}{\partial x}\right)=0 \\ \frac{Eh^3}{12(1-\nu^2)}.\left(\frac{\partial^2\psi_y}{\partial y^2}+\nu\frac{\partial^2\psi_x}{\partial x\partial y}\right)+\frac{Gh^3}{12}.\left(\frac{\partial^2\psi_y}{\partial x^2}+\frac{\partial^2\psi_x}{\partial x\partial y}\right)-Gh\left(\psi_y+\frac{\partial w}{\partial y}\right)=0 \end{cases} \quad (13)$$

7 Solution by the Navier approach

For the resolution of the differential equations (13) we use the approach of Navier.

The boundary conditions of a simply supported nano-plate are written as follows:

$$Du \quad x=0 \quad au \quad x=L;$$

$$u_x=0\ ,\ u_y=0\ ,\ \psi_y=0\ ,\ M_{xx}=0\ ,\ N_{xx}=0 \qquad (14)$$

$$u_x=0\ ,\ u_y=0\ ,\ \psi_x=0\ ,\ M_{yy}=0\ ,\ N_{yy}=0$$

The expressions of displacements are chosen to satisfy the boundary conditions:

$$\begin{cases} w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(\alpha x)\sin(\beta y) \\ \psi_x=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}\cos(\alpha x)\sin(\beta y) \\ \psi_y=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}\sin(\alpha x)\cos(\beta y) \end{cases} \qquad (15)$$

With :

$\alpha=\frac{m\pi}{L}$ and $\beta=\frac{n\pi}{b}$.

By substituting the equations (15) in the equations (13) we obtain:

$$\left\{\begin{bmatrix}\gamma_1 & \gamma_2 & \gamma_3\\ \gamma_2 & \gamma_4 & \gamma_5\\ \gamma_3 & \gamma_5 & \gamma_6\end{bmatrix}+P\,L_{mn}\lambda_{mn}\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\right\}\begin{bmatrix}W_{mn}\\X_{mn}\\Y_{mn}\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad (16)$$

With :

$$\lambda_{mn}=1+(e_0a)^2\left(\alpha^2+\beta^2\right) \quad ; \quad L_{mn}=\alpha^2+\lambda\beta^2$$

$$\begin{cases} \gamma_1=-Gh\left(\alpha^2+\beta^2\right) \\ \gamma_2=-Gh\alpha \\ \gamma_3=-Gh\beta \\ \gamma_4=-\frac{Eh^3}{12(1-\nu^2)}\alpha^2-\frac{Gh^3}{12}\beta^2-Gh \\ \gamma_5=-\frac{Eh^3\nu}{12(1-\nu^2)}\alpha\beta-\frac{Gh^3}{12}\alpha\beta \\ \gamma_6=-\frac{Eh^3}{12(1-\nu^2)}\beta^2-\frac{Gh^3}{12}\alpha^2-Gh \end{cases} \qquad (17)$$

By solving the equation (16) and we obtain the following critical buckling load:

$$P_{cr}=\frac{\gamma_1\left(\gamma_4\gamma_6-\gamma_5^{\,2}\right)-\gamma_2\left(\gamma_2\gamma_6-\gamma_3\gamma_5\right)+\gamma_3\left(\gamma_2\gamma_5-\gamma_3\gamma_4\right)}{L_{mn}\lambda_{mn}\left(\gamma_5^{\,2}-\gamma_4\gamma_6\right)} \qquad (18)$$

III. Analysis of results

We have studied the buckling behavior of square nano-plates simply supported by Mindlin's non-local theory. The differential equations of motion of this theory are given in equations (13).

When we put $(e_0a)^2=0$ in the equations (13), we obtain the expressions of the local Mindlin plate theory. These differential equations are the same expressions given by Hashemi et al [13].

1 Physical parameter of the model

The material used for the present study is an isotropic material (graphene sheet); The following table shows the geometric and mechanical characteristics:

Table 1: Geometric and mechanical characteristics of the material used [7]:

Features	
Nano plate thickness (h)	0.34 nm
Modulus of elasticity (E)	1.02 TPa
Poisson coefficient (ν)	0.3

The buckling load ratio is defined as the ratio of the buckling loads obtained by the non-local elasticity theory P_{cr} to those obtained by the local elasticity theory P_0 when $(e_0a)^2=0$.

So we have:

$$Rap = P_{cr} / P_0 \quad (19)$$

2 Effects of mode number on nano-plate buckling load ratio of graphene sheet

First, and based on the mathematical formulations, a computer program is developed to study the buckling behavior of the

nano-plates using non-local first-order shear strain theory.

Figure 4a represents the buckling load ratio variation of a nano-plate subjected to bi-axial compression for different buckling modes ($m = n$) and for a single non-local parameter value $(e_0 a)^2 = 2\ nm^2$, while for Figure 4b, we have chosen different buckling modes of type ($m \neq n$). It is found that the buckling load ratio decreases with increasing buckling modes and increases with increasing nanoplate length (L).

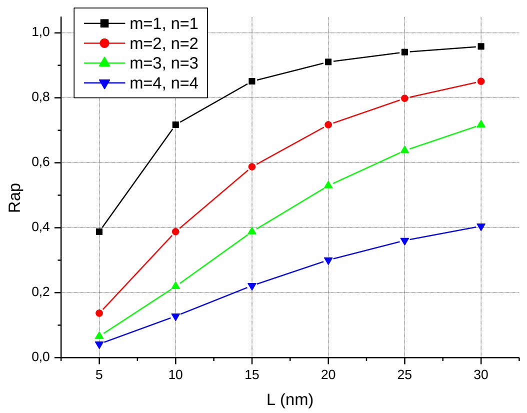

Figure 4a Variation of the buckling load ratio as a function of the length of a square nano-plate for different buckling modes $m = n$.

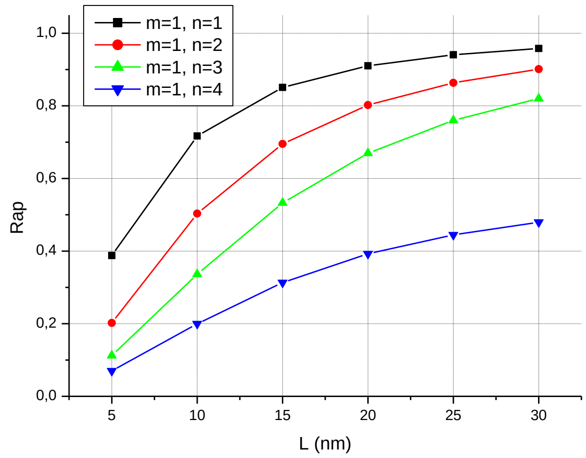

Figure 4b Variation of the buckling load ratio as a function of the length of a square nanoplate for different buckling modes $m \neq n$.

3 Effects of different parameters on nano-plate buckling load ratio of graphene sheet

The buckling load ratio variations for the first mode ($m = n = 1$), the third mode ($m = n = 3$) and the fifth mode ($m = n = 5$) are presented respectively in figures 5a, 5b and 5c as a function of variations in the percentages of different parameters such as graphene sheet length, thickness, non-local parameter and modulus of elasticity. These figures show that the buckling load ratio decreases with increasing nonlocal parameter and increases with increasing nano-plate length. Otherwise, there is an insignificant effect of the modulus of elasticity and the thickness on this ratio. The comparison between the three figures shows that the increase in the number of modes causes a decrease in the buckling load ratio.

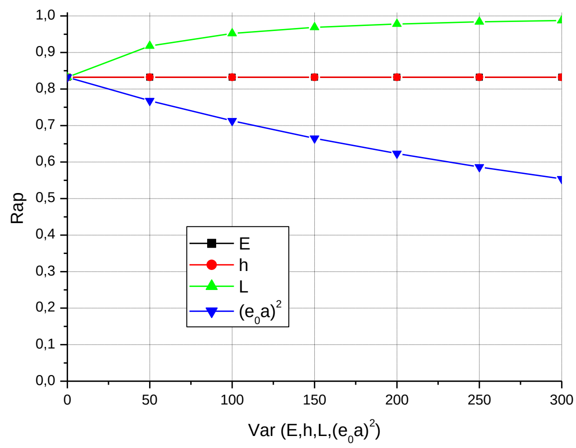

Figure 5a Variation of the buckling load ratio as a function of the percentage variation of the different parameters $(E, h, L, (e_0 a)^2)$ for $m = n = 1$.

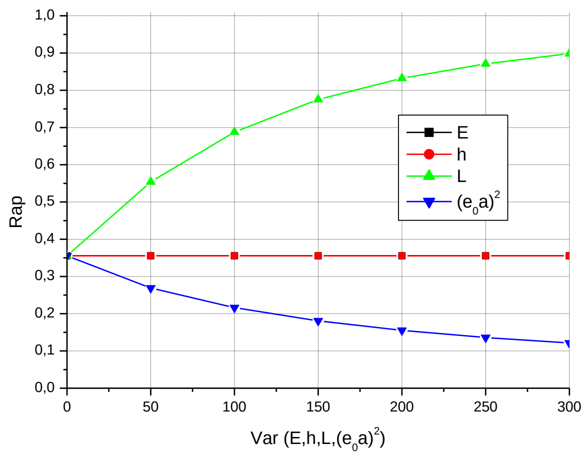

Figure 5b Variation of the buckling load ratio as a function of the percentage variation of the different parameters $(E, h, L, (e_0 a)^2)$ for $m = n = 3$.

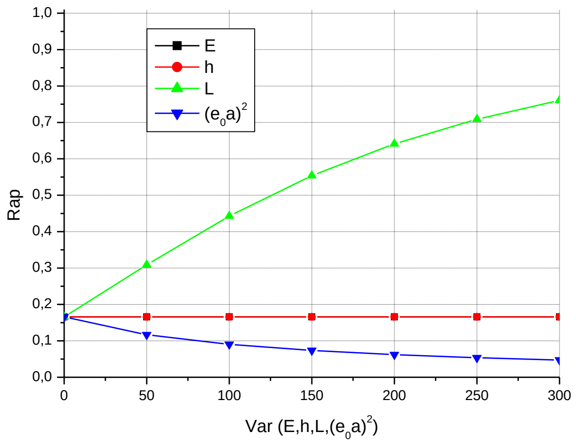

Figure 5c Variation of the buckling load ratio as a function of the percentage variation of the different parameters ($E, h, L, (e_0 a)^2$) for $m = n = 5$.

VI. CONCLUSION

The non-local theory of first-order deformation shear is used for the buckling analysis of nano-plates.

The present theory takes into consideration the effect of the scale which is based on the differential equations of nonlocal and the constitutive relation of Eringen, which led to obtain the equations of motion; using Hamilton's principle.

Analytical buckling load solutions are developed for simply supported plates.

The numerical examples show the effects of different parameters which influence the buckling load such as the effect of the scale, the number of modes, the geometric parameters and the mechanical parameters.

This study can be useful for the design of electronic nano-devices such as atomic dust detectors and biological probes.

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