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An Investigation on a Planar 3 Degree-of-Freedom Underactuated Mechanism

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Abstract – The fully-actuated mechanisms ensure control and feedback linearization within the defined workspace but they are required additional design considerations such as control complexity, design or cost. On the other hand, the underactuated mechanisms have advantages in terms of control complexity, design and cost. In this study, a planar three degree-of-freedom parallel underactuated mechanism is introduced. In this design, only one actuator is used and the particularity of this study resides in the configuration of mechanism, which uses non-zero length compression springs instead of prismatic joints at each of its three links. This particular design allows for three equilibrium positions are determined by solving static Newton's equation of resulting forces. The objective of designed control is determining accuracy of under actuation in reaching a desired equilibrium goal position from given equilibrium starting position. Results show that more than one actuator is needed to control the mechanism and reach the desired position.

Keywords – Planar Mechanisms, Underactuated Mechanisms, Static Equilibrium.

I. Introduction

The parallel actuated mechanisms are an ingenious class of articulated mechanism which rely on the principle that an end-effector is connected to a fixed base via a number of connectors with multiple degrees of freedom [1]. The parallel actuated mechanism entails a tradeoff between complexity of design due to the numerous bodies and redundant links, and freedom in joint control. Indeed, redundancy in link degree of freedom enables a wide range of combination of actuators. In particular, parallel actuated mechanisms allow for actuation though motors placed on the fixed base, which entails a simpler design and more accurate control. For instance, the Delta Robot is a six degree-of-freedom parallel actuated triangular platform. Each of its links has two rotational degree-of-freedom and one planar degree-of-freedom, totaling nine possible degree-of-freedom of actuation. Hence, the Delta Robot's motion is fully-actuated with the use of six motors at the base of the mechanism.

While the full actuation ensures control and feedback linearization within the totally of the defined workspace, it requires additional design considerations and cost due to the large number of actuators required to fulfill the motion. On the other hand, the under-actuated mechanisms are not required actuators as much as fully-actuated mechanisms. Thus, the cost, design and control complexity can substantially be reduced through under-actuation, given the proper linearization. Hence, underactuated mechanisms have obvious advantages in terms of control complexity, design and cost of the mechanism.

There are a number of studies about the underactuated mechanisms. The following articles are some of these studies: A simple and flexible static model of a three degree-of-freedom underactuated finger is introduced by Guay and Gosselin [4]. This model focus on the internal torques acting on the individual joint but it also allows complete static simulations and numerical optimization. Another kinetostatic analysis of underactuated fingers are investigated by Birglen and Gosselin [5]. In this study, force capabilities of the underactuated fingers are analyzed. A position control of nonholonomic 3R underactuated robots is investigated by Liu et. al. [6]. In this method, a new method is introduced for the control of motion of the underactuated robots.

The purpose of this study is to show behavior of a planar 3 degree-of-freedom parallel mechanism subject to under-actuation. The particularity of this study resides in the configuration of mechanism, which uses non-zero length springs instead of prismatic joints at each of its three links. This particular design allows for three equilibrium positions determined by solving static Newton's equation of resulting forces. The objective of designed control will therefore consist of determining accuracy of under actuation in reaching a desired equilibrium goal position given a different equilibrium starting position.

II. DESIGN

A. Design and Assumptions

The mechanism is a parallel actuated triangular platform. The end-effector platform of the mechanism is an equilateral triangle, each vertex connected to the base though springs of stiffness, *k*, acting as prismatic joint. In addition to that, the springs are connected by rotational joints on the vertices and the fixed base. The entire mechanism can be viewed as a 3xRPR closed-chain robot. For the purpose of this study, following assumptions are determined:

- The motion is entirely planar
- There is not any gravity acting on the mechanism
- The joints are frictionless
- The springs are massless
- Springs have infinite torsional stiffness (springs are constrained in a linear motion)
- Springs have equal stiffness constant (k)

The entire mechanism forms a closed-chain with degrees of freedom determined by Grubler equation [2].

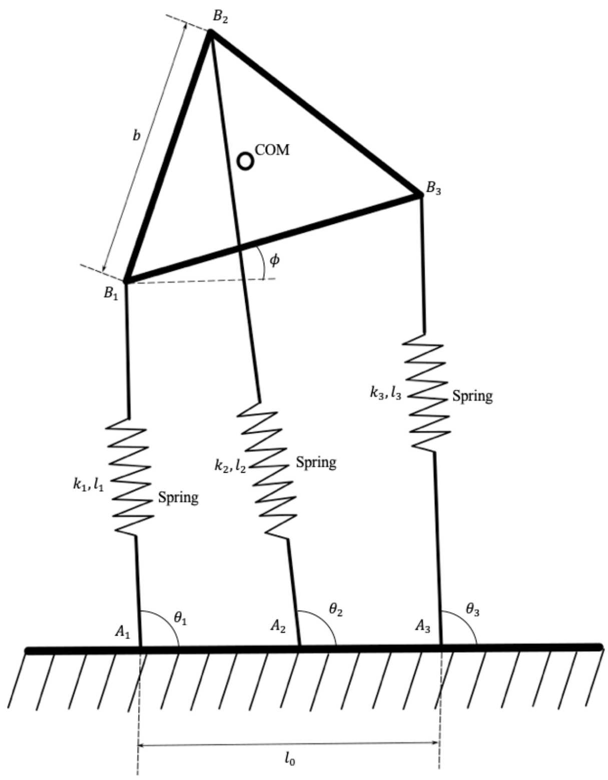

Fig. 1. The planar 3 degree-of-freedom parallel mechanism

B. Equations of Motion

In order to drive the equations of motion, the variables should be determined. The mechanism is shown in Fig. 1. The analysis is begun by considering the motion of point B_1 (lower left vertex of the triangle). The planar 3 degree-of-freedom motion is defined as a function of the coordinates *x*, *y*, and angle ϕ where angle ϕ is angle of platform with horizontal *x*-axis and it is shown in Fig. 1. In the analysis, to actuate the mechanism through the Joint A_1 is investigated at the fixed base. Therefore, the variables *x* and *y* are expressed as functions of parameters θ_1 and l_1 where parameter θ_1 refers angle of link $[A_1B_1]$ and parameter l_1 is the length of the spring (between A_1 and B_1). From Fig. 1, the following equations can be written:

$$x = l_1 \cos\theta_1 \quad (1)$$

$$y = l_1 \sin\theta_1 \quad (2)$$

$$l_2 = \sqrt{2Il_1 \cos\theta_1 + 2Jl_1 \sin\theta_1 + I^2 + J^2 l_1^2} \quad (3)$$

$$l_3 = \sqrt{2Hl_1 \cos\theta_1 + 2Gl_1 \sin\theta_1 + H^2 + J^2 l_1^2} \quad (4)$$

where

$$I = b\cos\left(\frac{\pi}{3} + \phi\right) - \frac{l_0}{2} \quad (5)$$

$$J = b\cos\left(\frac{\pi}{3} + \phi\right) \quad (6)$$

$$H = b\cos\phi - l_0 \quad (7)$$

$$G = b\sin\phi \quad (8)$$

With the newly established parameterization, the equations of motion are begun to drive by driving the potential and kinetic energy, then applying the Laplace Method:

$$K = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}I\dot{\phi}^2 \quad (9)$$

$$V = \frac{1}{2}k\left[\left(l_{0_1} - l_1\right)^2 + \left(l_{0_2} - l_2\right)^2 + \left(l_{0_3} - l_3\right)^2\right] \quad (10)$$

where parameter K refers kinetic energy of the all mechanism and parameter V refers potential energy of the all mechanism. The Lagrange relationship is applied for the system and obtain:

$$\begin{bmatrix}\tau_\phi \\ F_{l_1} \\ \tau_{\theta_1}\end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & ml_1^2 \end{bmatrix}\begin{bmatrix}\ddot{\phi} \\ \ddot{l}_1 \\ \ddot{\theta}_1\end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2ml_1\dot{\theta}_1 \\ 0 & 2ml_1\dot{\theta}_1 & 0 \end{bmatrix}\begin{bmatrix}\dot{\phi} \\ \dot{l}_1 \\ \dot{\theta}_1\end{bmatrix} + \begin{bmatrix}A \\ B \\ C\end{bmatrix} \quad (11)$$

where

$$A = k\left[\frac{\left(l_{0_2} - l_2\right)}{l_2}\left(I'l_1C_1 + J'l_1S_1 + I'I + J'J\right) + \frac{\left(l_{0_3} - l_3\right)}{l_3}\left(H'l_1C_1 + G'l_1S_1 + H'H + G'G\right)\right] \quad (12)$$

$$B = k\left[\left(l_{0_1} - l_1\right) + \frac{\left(l_{0_2} - l_2\right)}{l_2}\left(IC_1 + JS_1 + l_1\right) + \frac{\left(l_{0_3} - l_3\right)}{l_3}\left(HC_1 + GS_1 + l_1\right)\right] \quad (13)$$

$$C = k\left[\frac{\left(l_{0_2} - l_2\right)}{l_2}\left(Jl_1C_1 - Jl_1S_1\right) + \frac{\left(l_{0_3} - l_3\right)}{l_3}\left(Gl_1C_1 - Hl_1S_1\right)\right] \quad (14)$$

and

$$C_1 = \cos\theta_1 \quad (15)$$

$$S_1 = \sin\theta_1 \quad (16)$$

$$I' = \frac{\delta I}{\delta\phi} = -b\sin\left(\frac{\pi}{3} + \phi\right) \quad (17)$$

$$J' = \frac{\delta J}{\delta\phi} = b\cos\left(\frac{\pi}{3} + \phi\right) \quad (18)$$

$$H' = \frac{\delta H}{\delta\phi} = -b\sin(\phi) \quad (19)$$

$$G' = \frac{\delta G}{\delta\phi} = b\cos(\phi) \quad (20)$$

where parameter ϕ refers angle of triangle platform, parameter θ_i refers angle of springs, parameter l_i refers stretched length of springs, parameter l_{0_i} refers zero-spring length, (x, y) refers x and y coordinates of B_1 point, parameter m refers mass of triangle platform, parameter l_0 refers distance between points A_1 and A_3, parameter b refers lenth of triangle side, parameter k refers spring constant, $\dot{x}$ and $\dot{y}$ refer x and y components of translational velocity of the triangle platform, parameter $\dot{\phi}$ refers rotational velocity of the triangle platform and parameter I refers moment of inertia of the triangle platform.

C. *Equilibrium Positions*

The three degree-of-freedom planar mechanism with all parameters is shown in Fig. 1. The mechanism has three comprehensive springs with the same spring constant.

In the Fig. 1, B_1, B_2 and B_3 are vertex points of the perfect triangle and the parameter b is the side length. COM refers center of mass point of this perfect triangle. The x and y coordinates of the COM can be written as:

$$COM(x, y) = \left(\frac{x_{B_1} + x_{B_2} + x_{B_3}}{3}, \frac{y_{B_1} + y_{B_2} + y_{B_3}}{3}\right) \quad (21)$$

where parameters x_{B_1}, x_{B_2} and x_{B_3} are x-axis coordinates of the B_1, B_2 and B_3 respectfully and parameters y_{B_1}, y_{B_2} and y_{B_3} are y-axis coordinates of the B_1, B_2 and B_3 respectfully. This mechanism is considered as a problem of four-bar mechanism of chain with following assumptions:

$$A_1B_1(crank), B_1B_3(coupler), A_3B_3(rocker)$$

$$A_1B_1(crank), B_1B_2(coupler), A_2B_2(rocker)$$

First, the spring lengths and angles are found with given position coordinates (x, y) and the angle ϕ. Hence, the kinematic constraints equations are shown below:

$$\overrightarrow{A_1B_1} + \overrightarrow{B_1B_3} = \overrightarrow{A_1A_3} + \overrightarrow{A_3B_3} \quad (22)$$

$$\overrightarrow{A_1B_1} + \overrightarrow{B_1B_2} = \overrightarrow{A_1A_2} + \overrightarrow{A_2B_2} \quad (23)$$

The force of compression springs can be written as:

$$F_{s_i} = k_i(l_{0_i} - l_i) \quad (24)$$

To find equilibrium position, the forward static analysis is developed and statically balanced conditions are found. Four this mechanism, the equilibrium constraint equations are;

$$\sum F_x = 0 \ \rightarrow \ f_1(x, y, \phi) = 0 \quad (25)$$

$$\sum F_y = 0 \ \rightarrow \ f_2(x, y, \phi) = 0 \quad (26)$$

$$\sum M_z = 0 \ \rightarrow \ f_3(x, y, \phi) = 0 \quad (27)$$

The Eqs. (25), (26) and (27) are three non-linear equations and have three unknown parameters. To solve this problem, SQP (Sequential Quadratic Programming) optimization method is used in MATLAB Optimization Toolbox because it is faster. To find all solutions for (x, y, ϕ), the following objective function is defined:

$$min \quad J = \sum_{i=1}^{3} f_i^2 \quad (28)$$

subject to $\quad x_{min} \leq x \leq x_{max} \quad (29)$

$$y_{min} \leq y \leq y_{max} \quad (30)$$

$$\theta_{min} \leq \theta \leq \theta_{max} \quad (31)$$

D. *Controller Design*

The equations of motion are not linearizable but computed torque control law is applied for small range of motion of mechanism which is between first and third equilibrium positions. The computed torque control equation is shown below [3]:

$$M(q)\ddot{q} + C(q, \dot{q}) = H(q)u \quad (32)$$

where $M(q) \in \mathbb{R}^{n \times n}$ is the positive definite generalized mass matrix of n degree-of-freedom system, vector $C(q, \dot{q}) \in \mathbb{R}^n$ contains the inertial (centrifugal and Coriolis) terms and all external/active forces, including gravity, spring and damping forces if present in the system. $u \in \mathbb{R}^l$ and $H(q) \in \mathbb{R}^{n \times l}$ are the generalized control input matrix. According to this main definition of computed control law, calculation results are shown on simulation and results part.

III. Simulation and Results

To verify the position of equilibrium, we check out our analysis and design by simulation. The equilibrium positions simulation and controller results of planar 3 degree-of-freedom parallel mechanism has three equilibrium positions in defined workspace. The numerical criteria for equilibrium positions is *min* (J) < 10^{-3}.

In the simulation, $b = 1$ m, $l_0 = l_{1_0} = l_{2_0} = l_{3_0} = 4$ m, $k = 10$ N/m, $m = 1$ kg, $g = 9.8$ m/s^2 boundary conditions of our mechanism to find equilibrium positions are $x_{min} = 2$, $y_{min} = 0, z_{min} = -1$ and $x_{max} = 6$, $y_{max} = 6, z_{maz} = 1$. While parameters *x* and *y* refer *x* and *y* coordinates of vertex B_1, parameter *z* refers θ_1 angle boundary conditions.

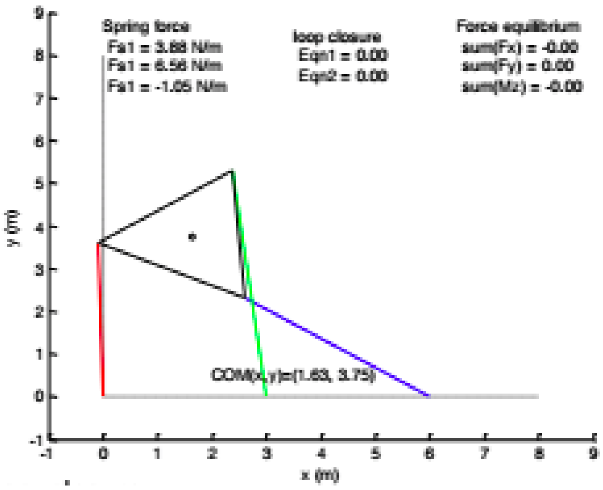

Fig. 2. First equilibrium position of 3-DOF mechanism

The Fig. 2 shows the first equilibrium position of the position of the mechanism. The spring forces are 3.88 Nm, 6.56 Nm and -1.05 Nm in order to the first link spring force (between A_1 and B_1), the second link spring force (between A_2 and B_2) and the third link spring force (between A_3 and B_3).

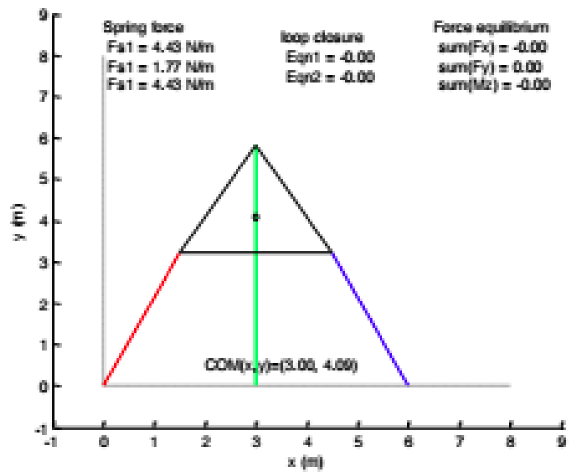

Fig. 3. Second equilibrium position of 3-DOF mechanism

The Fig. 3 shows the second equilibrium position of the mechanism. According to the results in this position, the spring forces are 4.43 Nm, 1.77 Nm and 4.43 Nm in order to the first, second and third link springs respectfully.

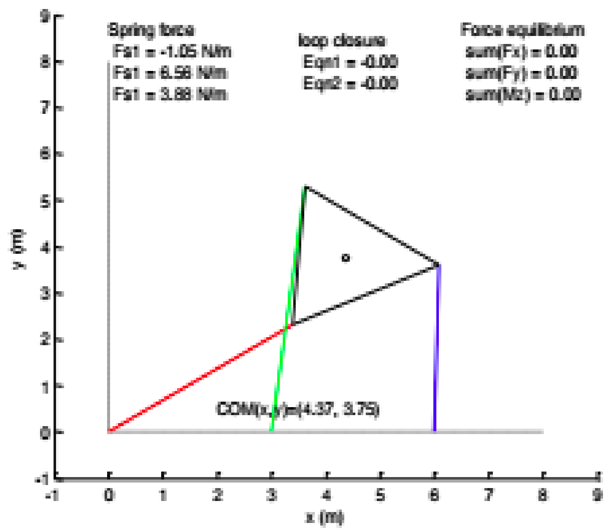

Fig. 4. Third equilibrium position of 3-DOF mechanism

The Fig. 4 shows the third equilibrium position of the mechanism. According to the Fig. 4, the spring forces are -1.05 Nm, 5.56 Nm and 3.88 Nm in order to the first, second and third link springs respectfully.

Table 1. Initial, desire and real values of motion

	Initial	Desired	Real
COM_x	1.63	4.37	3.02
COM_y	3.75	3.75	4.01
x_{B_1}	9x10^{-8}	3.64	3.38
y_{B_1}	3.38	2.41	2.31
θ_1	90.05	43.72	30.67
ϕ	-26.03	25.54	71.8

Controller design simulation results are shown in Table 1 and Fig. 5. Table 1 shows initial, desired and real values of the parameters θ_1, ϕ, x_{B_1}, y_{B_1}, COM_x and COM_y. The Fig. 5 shows the angular displacement of joint graph. According to the Table 1, there are differences between desired and real values of motion. The Fig. 5 shows that the differences between real and desired angular displacement is increasing time by time.

During the simulation, the mechanism started first equilibrium position which is shown in Fig. 2 and mechanism did not reach the desired final position which is third equilibrium position and shown in Fig. 4.

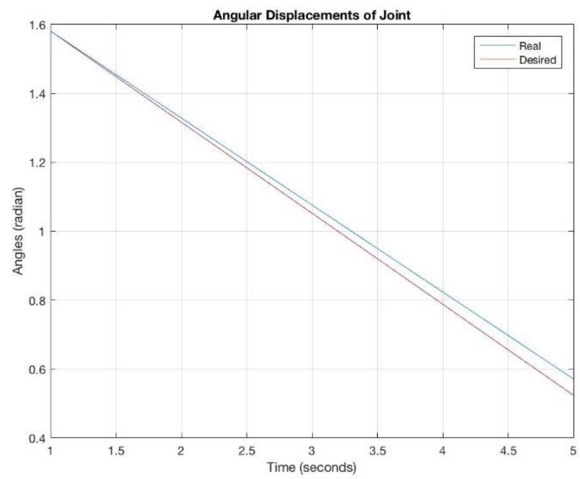

Fig. 5. Angular displacement of joint graph

IV. DISCUSSION

The analysis shows that under actuation for this particular mechanism given a specific goal is not feasible. This conclusion however leads the path for several potential improvements and future steps to be taken. Under-actuation using an additional actuator can be devised to ensure total control over 2-DOF. For pick-and-place applications in particular where the orientation of the end-effector is irrelevant, such under-actuation would most probably lead successful results.

After determining static equilibrium positions for this mechanism, driving the equations of motion, and applying feedback linearization and numerical control for under-actuation on a base rotational joint, it is observed that while coming close to the goal position, a steady-state error remains present.

Despite failure to reach to exact goal position, the mechanism was controllable with a known margin of error.

Once the accurate control is determined, it would be desirable to perform a velocity study, to investigate how such a mechanism would react to a control input. Indeed, the correct actuation is predicted in this study, the mechanism could "jump" between stable equilibrium positions, similarly to a mechanical switch.

Potential applications of such a mechanism can involve simple planar motion for pick-and-place in industrial assembly lines. Indeed, pick-and-place are usually carried out in a horizontal plane given an initial and final position. Such an under-actuated mechanism would prove to be more economical than a fully actuated pick-and-place robot, while potentially providing faster and more accurate motion.

The study of a similar mechanism in a 3-D environment, where gravity acts on the center of mass would be desirable for a more realistic problem statement.

V. CONCLUSION

In this study, a planar three degree-of-freedom parallel underactuated mechanism is presented. In design, non-zero length compression springs are used instead of prismatic joints at each of its three links. Three equilibrium positions are determined and designed control determines accuracy of the reaching desired position. According to the results, one actuator is not enough to reach the desired position.

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