

A Comparative Study for COVID-19 Forecasting Models

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Abstract – The COVID-19 was declared as an international health emergency concern by World Health Organization (WHO) in 2020. It caused about 7 million deaths and has taken interest in various disciplines. On the other hand, modeling infectious diseases can provide critical planning to control the outbreak and public health research. In this work, we consider three classical epidemic models, namely, the SI (Susceptible, Infectious) model, SIS (Susceptible, Infectious, Susceptible) model and SIR (Susceptible, Infectious, Recovered) model to simulate the spread of COVID-19 in Türkiye. We compare their performances by applying recent data of COVID-19 outbreak. We present numerical experiments to indicate which models can reproduce the epidemic dynamics qualitatively and quantitatively for forecasting.

Keywords – Infectious Diseases, COVID-19, Epidemic Model, SI Model, SIS Model, SIR Model

I. INTRODUCTION

It is well known that the COVID-19 has many characteristic features which differs from other diseases, such as high infectivity, time delay between actual dynamics and number of confirmed cases [16]. The rapid spread of the virus and the numbers related to deaths makes the study of the various strategies more significant than before. A large amount of research has been studied to understand the behavior of the coronavirus. These models are generally defined by ordinary/partial differential equations and some of them can be summarized as follows:

The SI (Susceptible, Infectious) model considers only the susceptible population and the infectious people. Here, the model assumes that there is no loss from infectious population to the recovered people [10]. We also note that, if there are no sick people, the infectious population and the spread of disease will remain zero. In [7], the authors applied the SI model to predict the future evolution of COVID-19 spread in China. They also compared the SI model with the reported COVID-19 data. A SI model with

fractional derivative studied in [1]. The SI model used to fit COVID-19 case data throughout the world in [13].

The SIS (Susceptible, Infectious, Susceptible) model can be seen as one of the simplest epidemics models to understand the spread of disease in a realistic way [14]. It is more complicated model compared to SI model. We remark that the SIS model can be used for diseases in which the population is not immune when the infection is cured [12]. The SIS model considers the fact that recovered population are resistant to reinfection which can be seen as an advantage of the model. Otunuga studied a time-dependent probability distribution for number of COVID-19 infections in SIS model in [17]. In [3], a fractional SIS model for COVID-19 was suggested.

The SIR (Susceptible, Infectious, Recovered) model [11] can be used to forecast the diseases in which individuals have permanent immunity after infection. It should be noted that the mentioned system is used to estimate the behavior of the COVID-19 in various countries [6,15]. The SIR

model is not as simple as the SI and SIS models, but it reflects a reassessment of predicted number of infections to compensate for undetected infections.

In the SIRD (Susceptible-Infectious-Recovered-Dead) model a susceptible person becomes infected in case of contacting with an infected person. An infected person may recover from the disease or die because of the infection. We note that, in the SIRD model each person who has exposed to the virus becomes infected and it does not consider the effect of quarantine process. In [5], a modified SIRD model was developed to capture features of the COVID-19 characteristics. The COVID-19 outbreaks for China and Italy were described by a modified SIRD model in [4]. A simulation of a SIRD model for COVID-19 including many countries studied in [8].

The SEIR (Susceptible-Exposed-Infectious-Recovered) model can be seen as an extension of the SIR model by including the exposed class as a variable which denotes the fraction of individuals that have been infected but are asymptomatic [2]. The SEIR model for the COVID-19 and its features can be found in [9].

In this work, we compare the performances of the SI, SIS and SIR models by applying recent data on COVID-19 outbreak for Türkiye. We present numerical experiments to indicate which models can reproduce the dynamics of the COVID-19 qualitatively and quantitatively. The outline of the research: In Section II, we briefly summarize the SI, SIS and SIR models and the numerical algorithm. In Section III, we present some experiments to demonstrate the performance of mentioned models.

II. MATERIALS AND METHOD

In this part, we introduce the SI, SIS and SIR models briefly and describe the numerical algorithm. The classical models include the classes of the individuals who can become infective (notated as S), the individuals who can spread the disease (notated as I), the individuals who can no longer spread the disease (notated as R). The number of classes depends on the model, but each model includes the class S and I . In the introduced models,

β denotes the contact frequency, $1/\gamma$ denotes the average infectious period, N stands for the fixed population size and t denotes the time variable.

We start with the simplest model: The SI model. The SI model is given by,

$$\begin{cases} \frac{dS}{dt} = -\frac{\beta}{N} I S, & S(t_0) = S_0, \\ \frac{dI}{dt} = \frac{\beta}{N} I S, & I(t_0) = I_0. \end{cases}$$

Next, the SIS model is given by,

$$\begin{cases} \frac{dS}{dt} = -\frac{\beta}{N} I S + \gamma I, & S(t_0) = S_0, \\ \frac{dI}{dt} = \frac{\beta}{N} I S - \gamma I, & I(t_0) = I_0. \end{cases}$$

Finally, the SIR model is given by,

$$\begin{cases} \frac{dS}{dt} = -\frac{\beta}{N} I S, & S(t_0) = S_0, \\ \frac{dI}{dt} = \frac{\beta}{N} I S - \gamma I, & I(t_0) = I_0, \\ \frac{dR}{dt} = \gamma I, & R(t_0) = R_0. \end{cases}$$

with $S(t) = S_0 > 0$, $I(t) = I_0 \geq 0$, $R(t) = R_0 \geq 0$ at $t = t_0$.

We use a robust implementation of differential equations, namely the *ode45* tool which applies a variable step size method and the fourth / fifth order Runge-Kutta method in MATLAB to approximate the solution of the system of differential equation. In order to determine unknown parameters, we apply theta-method (notated also as θ -methods) with real data. We remark that consistency and the other details of the method can be found in any numerical analysis books.

III. RESULTS

In this section, we report and compare some numerical results obtained by the SI, SIS and SIR models for the COVID 19 outbreak in Türkiye. We note that the actual data is borrowed from Worldometer and Our World in Data websites [18,19].

The SI, SIR and SIS models are compared for the 14-day period between 29.01.2021 and 12.02.2021,

when the spread of COVID-19 in Türkiye is more stable. For this purpose, the solution behaviors of the SI, SIR and SIS models are given in Figure 1- Figure 3, respectively.

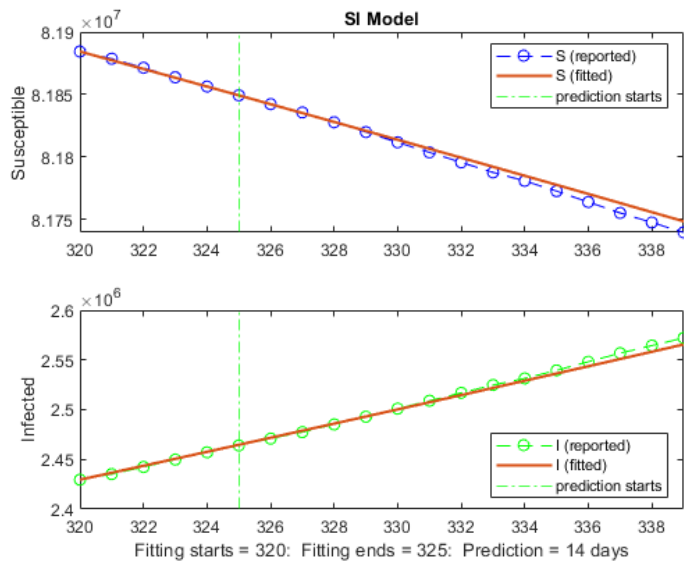


Fig. 1. 14-days forecast with SI model

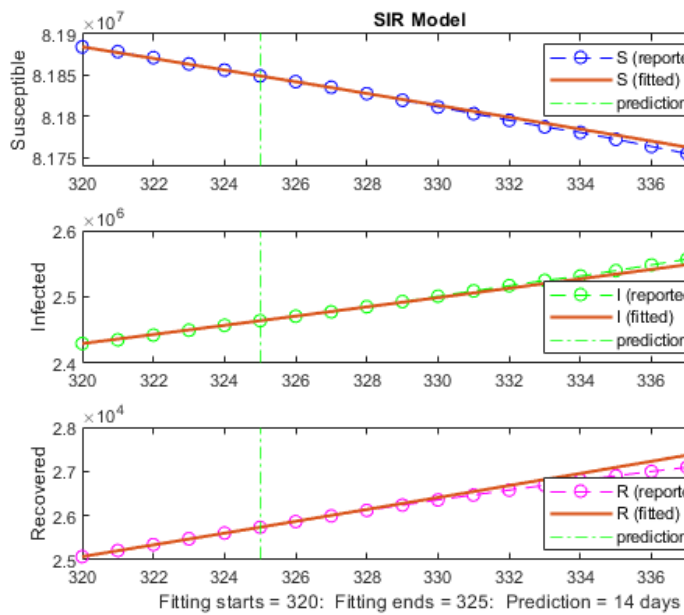


Fig. 2. 14-day forecast with SIR model

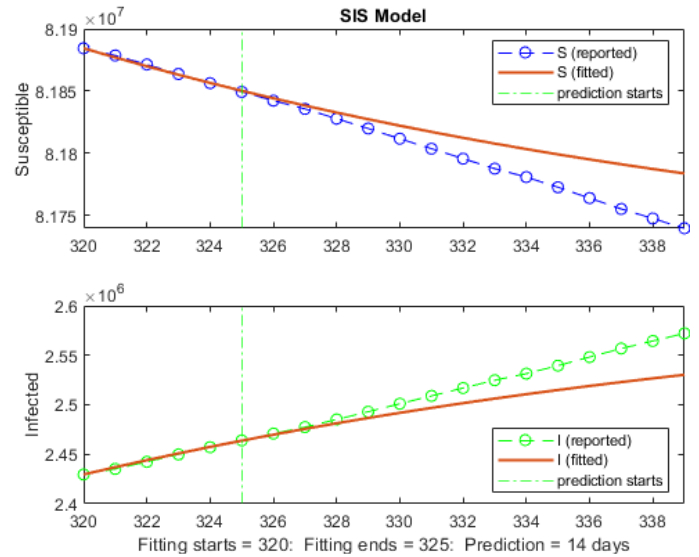


Fig. 3. 14-days forecast with SIS model

In Figure 1- Figure 3, we observe that the SI and SIR models are more compatible with the real data, while the SIS model is slightly far from the actual values.

Next, we study the case when the spread of the epidemic is rapid, and we consider the 14 and 21-day periods between 25.03.2021 and 15.04.2021. Here, the solution behaviors of the SI, SIR and SIS models are compared with real data. The Figure 4 and Figure 5 demonstrates the behavior and comparison of the solutions obtained by the SI, SIR and SIS models.

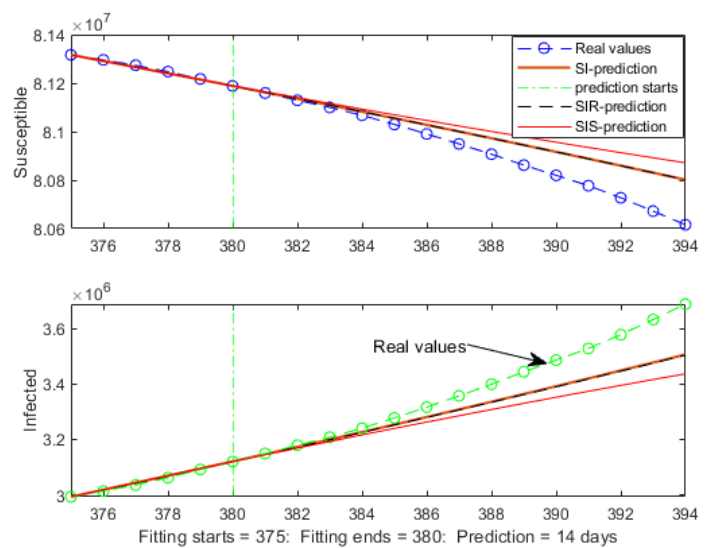


Fig. 4. Comparison of models for 14-days

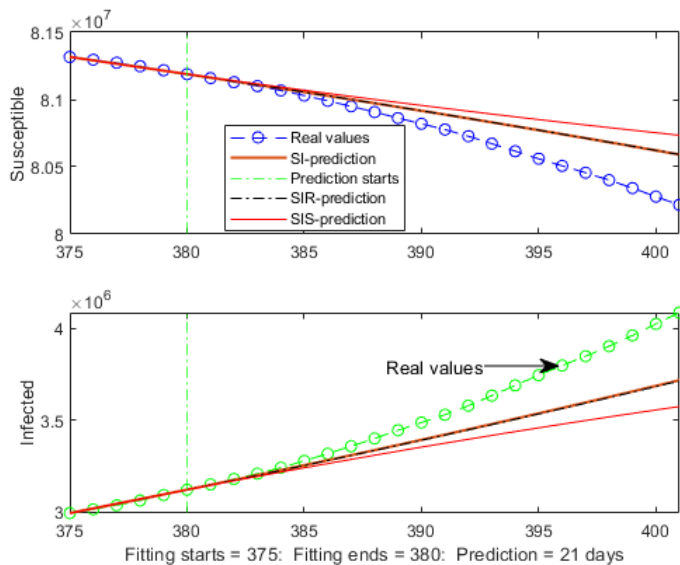


Fig. 5. Comparison of models for 21-days

In Figure 4 and Figure 5, we observe that the models for long-term predictions deviate more from the reported data. Moreover, the Figure 5 shows that the SIS model can not capture the spread of the disease well, especially during the time periods of rapid change.

We also observe that the simulations for short time period (7-days) are much better than the long time periods and we do not include the related figures here.

IV. CONCLUSION

In this paper, performances of the several models are tested for the COVID-19 outbreak in Türkiye. The detailed analysis of various suitable models for the COVID-19, the error analysis for various countries and the significance of their strengths and limits for different scenarios can be a subject of a future study.

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