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Research Article

Design and Computational Analysis of an Axisymmetric Contoured Supersonic Propulsion Nozzle

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***Abstract* –** The design of supersonic axisymmetric nozzles to a required exit Mach number necessitates applying the Method of Characteristics (MoC) that reduces the partial differential equations system of hyperbolic form governing such internal flow-fields to two ordinary differential equations known as the characteristic and the compatibility equations. These relationships are then transformed into finite difference equations and solved using an appropriate predictor-corrector algorithm.

The present study applies the MoC to the design of a contoured nozzle that accelerates the combustion gases to supersonic velocities. The selected configuration has a throat constituted of two circular arcs with the one downstream attached to a 2nd-order polynomial profile at the attachment point. The solution, in terms of the static pressure along the centerline and the wall is obtained. It was found that the flow expands gradually without any instability or possibility of shock waves. A gradual expansion represented by a decrease in pressure is noticed immediately downstream of the throat, the flow tending to expand gradually from the throat to the exit. Along the wall, the expansion is important immediately downstream of the throat while it tends to stabilize downstream to reach the ambient pressure at the outlet. In terms of performance, the thrust coefficient developed shows the great expansion process within the divergent supersonic section. The configuration is also found to develop a great specific thrust.

Keywords – Design, MoC, Supersonic Flow, Contoured Nozzle, Propulsion Nozzle

I. Introduction

The thrust generated by a propulsion nozzle is essentially dependent on the velocity of the combustion gas products discharged through its exit section. During their passage through a De-Laval nozzle, the gases are accelerated from low subsonic velocities to supersonic speeds. Several methods are used to design axisymmetric convergent-divergent propulsion nozzle contours, the simplest profile used being represented by a conical shape. The conical nozzle constituted the common shape in early rocket engines because of its ease and flexibility to be manufactured in diverse area ratios. It is however much restrained in terms of performances and weight that are mainly the result of the flow divergence at the nozzle exit that leads to a significant thrust reduction. The need to achieve a maximum thrust within a minimum length and therefore a minimum weight led to the development of several nozzle configurations including ideal, conical, contour, plug, dual bell, expansion-deflection and multi grid profiles [1].

A contoured shape allows the correction of the flow at the exit, directing it towards the axis of symmetry. Its design has been initially developed by Rao [2]. The variational computations approach used did not impose any restriction on the shape of the configuration except that it has to be a streamline. This constituted a major disadvantage because if the nozzle contour or the flow pattern is altered, then the whole calculations have to be reconsidered.

Specifying the contour of the divergent as a second-degree polynomial and employing nonlinear programming methods [3], Allman and Hoffman [4] succeeded in producing nozzles whose performances compared well to those produced by Rao [2]. Under zero pressure gradients, both methods were found to predict basically the same maximum thrust (agreement, in this case, was within 0.2%) sustaining the application of the direct optimization approaches. The contoured or Bell-type nozzles possess an expansion angle situated immediately downstream of the throat that varies between 20°-50°. It is followed by a gradual reversal of the contour slope to achieve around 5°-10° half angle at the exit lip. This configuration has been found to be approximately 20% shorter in length and consequently lighter [5], and shown to improve engine performance by up to 5% to 7% [6].

The present work focuses on the application of the combination of the Method of Characteristics and Rao method to achieve a performant design of a convergent-divergent nozzle. The former procedure was applied for the design of the divergent section [7] while the latter one led to the convergent profile. Computational Fluid Dynamics (CFD) was applied to analyze the flow field within the configuration generated. The results obtained show a smooth expansion of the combustion gases right from the inlet to the exit section where the Mach number achieved a value of 2.68.

II. Nozzle Design

A. *Supersonic Section Design*

The method of characteristics, a powerful and reliable technique, reduces the partial differential equations related to a supersonic flow into total differential equations valid along Mach lines [8]. An initial-value line is generated at the throat using a transonic computation procedure [9]. The MoC procedure led to the design of the supersonic axisymmetric contoured nozzle section represented in Figure 1.

It is an ideal profile consisting of two arcs-of-circles of different radii of curvature. The one located downstream of the throat, equaling half the throat (i.e. R_{td}=0.5y_t), is tangentially joined at the attachment point 'A' to a contour simulated by a 2nd-order quadratic polynomial wall. The arc-of-circle upstream of the throat has been taken equal to two throat radii (R_{tu}=2y_t).

The required initial geometrical and thermodynamic data needed for the computations of such an axisymmetric configuration are summarized in Table 1.

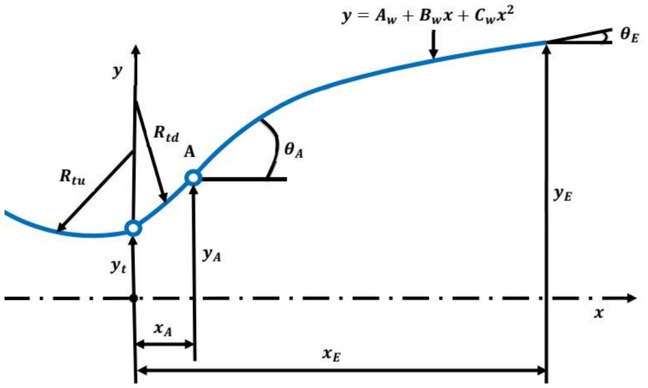

Fig. 1 Contoured nozzle configuration

Table 1. Nozzle Thermodynamic and geometrical data

Thermodynamic data	**Geometrical data**
P_a=1.013 bars	y_t=0.069 m
Pt=69 bars	R_{tu}=0.138m ; R_{td}=0.0345m
Tt=2800 K	θ_A=20°
R_G=320 J/kg. K	θ_E=5°
γ=1.2	y_E=0.223m

One of the advantages of the method of characteristics is that as the calculations of the profile proceed so the mesh of the inner domain that represent the junction of the Mach lines. The computations are also carried out solving the gas dynamic equation, the irrotationality condition and the relationship of the flow velocity to that of the sound simultaneously [10]. Figure 2 depicts the intersection points of the right-hand and left-hand characteristics along with the supersonic divergent section of the nozzle.

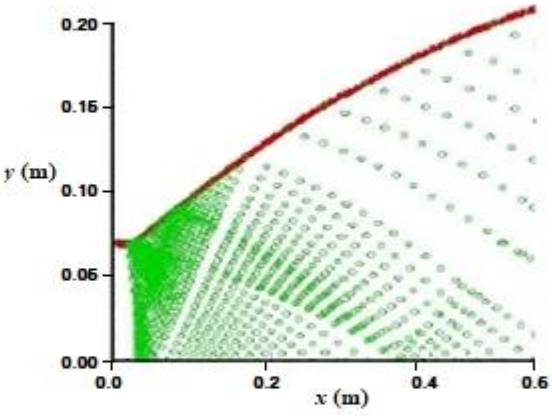

Fig. 2 Profile with its mesh (Characteristics network)

The application of the method of characteristics resulted in the profile constituted by two an arc of circle linked to a 2nd-order polynomial at the attachment point whose coordinates are shown in Table 2. The 2nd-order polynomial constants noted A_W, B_W and C_W along with the divergent section length are also displayed in Table 2, and the final profile represented in Figure 3.

Table 2. Divergent section profile parameters

Attachment point 'A'	x_A= 0.01180 m y_A= 0.07108 m
Nozzle length	x_E= 0.54 m
2nd-order polynomial coefficients	A_W= 0.06676 B_W=0.36882 C_W=−0.20535

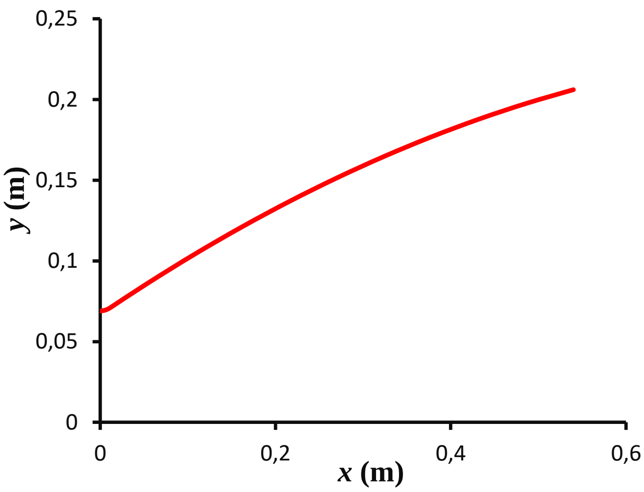

Fig. 3 Divergent section profile

B. *Subsonic Section Design*

The converging section positioned between the combustion chamber and the throat expands the flow subsonically to achieve a transonic velocity at the throat. This would lead to a supersonic expansion within the divergent section designed previously. The convergent section was produced through the application of Rao method [11]. This approach gained great success as it is the result of numerous experimental investigations.

The Rao's approach results in a converging profile determined mainly on the basis of the throat radius through:

$$\begin{cases} x = 1.5 \cdot y_t \cdot \cos(\theta) \\ y = 1.5 \cdot y_t \cdot \sin(\theta) + 2.5 \cdot y_t \end{cases} \qquad (1)$$

Where: $-130° \le \theta \le -90°$

The computations carried out led to the layout of the converging section configuration whose constituting parameters are shown in Table 3. And whose profile is represented in Figure 4. Linked to the divergent supersonic section designed earlier, this would result in the complete convergent-divergent de-Laval nozzle.

Table 3. Convergent section profile parameters

θ (°)	***x*** **(m)**	***y*** **(m)**
-90	6.3401e-18	0.069
-94	-0.0072198	0.06925212
-98	-0.01440442	0.07000725
-102	-0.02151886	0.07126172
-106	-0.02852847	0.07300941
-110	-0.03539908	0.07524181
-114	-0.04209724	0.07794805
-118	-0.04859031	0.08111492
-122	-0.05484664	0.08472702
-126	-0.06083577	0.08876674
-130	-0.06652852	0.0932144

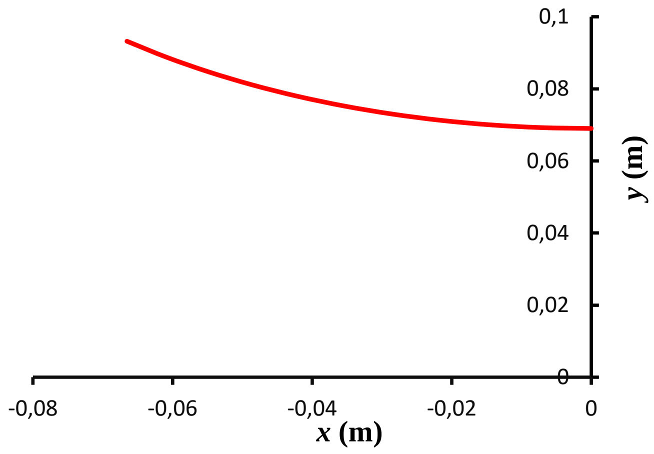

Fig. 4 Convergent section profile

III. CFD Computations

The compressible flow-field within the entire nozzle was simulated using the Ansys-Fluent finite-volume based platform.

The geometry comprised a total of 5000 elements with a number of 200 elements in the axial (x) direction and 50 elements in the radial direction (y). The property gradients of the velocity and the effects of the viscosity being expected to be high neat the throat and along the wall respectively, the mesh was refined at both positions. It is represented in Figure 5.

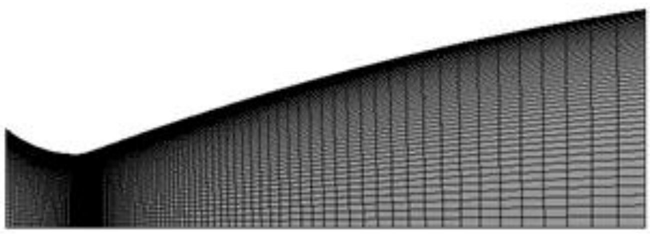

Fig. 5 Nozzle grid topology

The equations governing such a flow are represented by the universal conservation equations representing the conservation of mass, momentum and energy. The flow being turbulent, a closure of the system was performed through integrating a two-equation model represented the k-ω SST.

Table 4 displays the most representative computational parameters applied to carrying out the simulation.

Table 4. Criteria and data used for the CFD simulation

Criteria and data	
Solver	Density based
Property	Ideal gas
Viscosity model	k-ω SST
Operational pressure	0
Inlet boundary conditions	69.10^5Pa ; $2800K$
Solution method	Second order upwind
Residuals	10^{-6}

IV. Results and Discussions

A. *Pressure Distribution along Divergent*

The pressure distribution along the centerline and wall of the divergent section of the nozzle is shown in Figure 6. The sharp decrease in the first portion situated immediately downstream of the throat, particularly along the wall, is essentially due to the high value of the attachment angle (θ_A=20°) that leads to a severe expansion within this region.

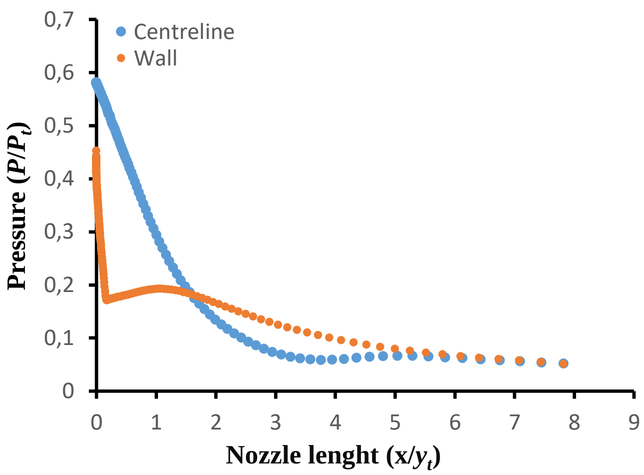

Fig. 6 Pressure distribution along centerline and wall

B *Pressure and Mach contours along nozzle*

The same trend may be noticed when we consider the pressure contours represented in Figure 7. It may be seen that pressure decreases abruptly near the throat from its stagnation value of 69 bars to stagnate further downstream.

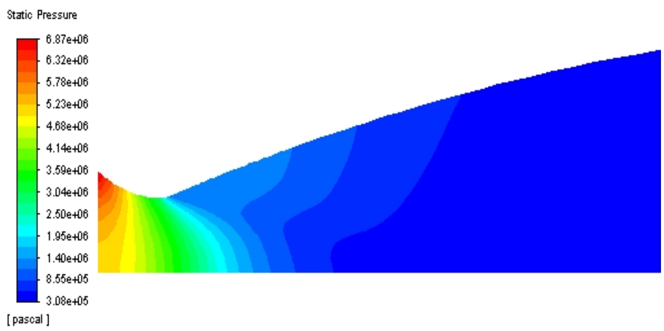

Fig. 7 Pressure contour within the entire nozzle

The supersonic expansion leads to an acceleration of the flow from its stagnation velocity at the entry section of the divergent where the Mach number is approximately equal to 0.15 (Figure 8). In the transonic region, the Mach reaches a value of unity that leads to a supersonic expansion within the divergent section. It finally reaches a Mach of 2.68 at the exit.

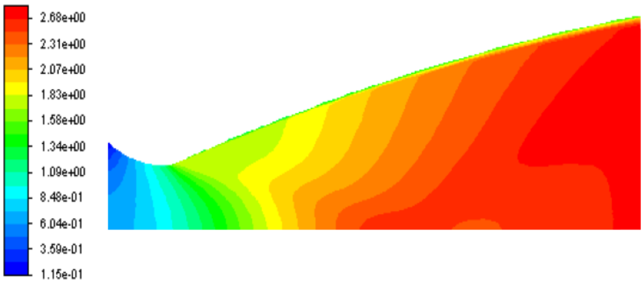

Fig. 8 Mach contour within the entire nozzle

The smooth acceleration of the flow due to the subsonic, transonic and supersonic expansions is moreover shown in Figure 9. Most of the acceleration may be seen to be occurring immediately downstream of the throat which is typical of the bell-shaped nozzle configurations. The remaining portion of the divergent section is used to turn down the flow in the axial direction in order to produce the maximum thrust in that direction.

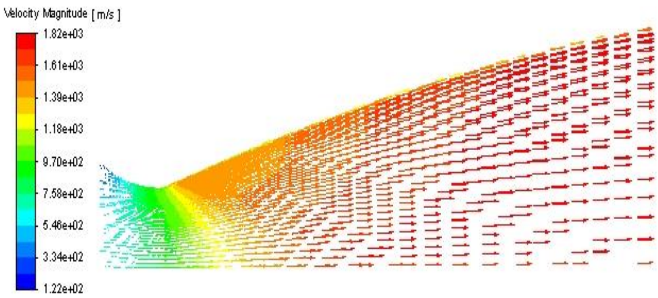

Fig. 9 Velocity vectors contour within the entire nozzle

C Nozzle Performances

The performance parameters inherent to the nozzle configuration considered have been computed. The results are shown in Table 5.

Table 5. Nozzle performance characteristics of the contoured nozzle

Performance parameters	
Developed thrust; [$\boldsymbol{T}$ (N)]	167,852.6
Masse flow rate; [$\dot{\boldsymbol{m}}$ (kg/s)]	70
Nozzle discharge coefficient [$\boldsymbol{C_d}$]	0.993
Thrust coefficient [$\boldsymbol{C_T}$]	1.62
Effective velocity [$\boldsymbol{V_{eff}}$ (m/s)]	2397
Exit Mach number [$\boldsymbol{M_e}$]	2.68
Specific impulse [$\boldsymbol{I_s}$]	244.4

The thrust delivered, a function not only of the ejection speed of the combustion gases but also of the difference between the outlet and ambient pressures, is found to be important. The thrust coefficient is one of the most important parameters characterizing a nozzle performance. It is expressed as the thrust per unit chamber pressure and throat area. It is therefore a dimensionless multiplication factor that represents the degree to which the thrust is amplified by the nozzle supersonic expansion. It is found equaling 1.62, thus showing the effectiveness of the divergent section and its contribution to the accelerations of the flow. The results show that the configuration displays a higher mass flow rate specified by the conservation of mass to be constant through a nozzle.

The actual performance of a nozzle typically deviates slightly from that predicted and therefore based on the assumption of isentropic flow mainly because of friction issues. The ratio of the actual current flow to that estimated from the isentropic relationships is known as the discharge coefficient. It is found nearing unity, thus showing the effectiveness of the approach followed in the simulation. The effective velocity is the exit velocity of the adapted nozzle. It takes the atmospheric correction into account, and is defined as the ratio of the thrust to the mass flow rate. The value delivered is representative of the accelerations the flow gained within its passage through the nozzle. Finally, the specific impulse that is defined as the ratio of thrust to the product of mass flow rate and acceleration due to gravity is a parameter generally used to compare the performance of propellants used for propulsion.

V. CONCLUSIONS

A convergent-divergent nozzle design has been carried out using method of characteristics. The configuration chosen was bell-shaped because of its numerous advantages represented essentially by its high attachment angle that allows that allows much of the expansion to take place immediately downstream of the throat. A simulation of the turbulent compressible flow-field that takes place within the convergent, throat and divergent sections constituting the nozzle has also been undertaken. The 'Ansys-Fluent' platform has been used.

The analysis of the results obtained in terms of pressure, Mach and velocity distributions exhibited the smooth expansion of the flow that led to an acceleration of the flow from its exit from the combustion chamber to the exit cross-section of the divergent section.

In terms of performances represented essentially by the thrust coefficient as it characterizes the efficiency of the expansion process control by the configuration generated, the axisymmetric contoured configuration has been found to perform adequately resulting in an appreciable value of the propulsive thrust. Finally, the nozzle discharge coefficient that neared unity comforted the authors in the approach followed particularly in the turbulence model applied.

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REFERENCES

[1] S. Khare and U.K. Saha, "Rocket nozzles: 75 years of research and development", *Sådhanå*, 46:76, pp. 1-22, 2021.

[2] G.V.R. Rao, "Exhaust nozzle contour for optimum thrust", *Journal of Jet Propulsion*, 28, pp. 377-382, 1958.

[3] D.M. Himmelbau, *Applied Nonlinear Programming*, McGraw Hill Book Co., New York, USA, 1972.

[4] J.G. Allman and J.D. Hoffman, "Design of Maximum Thrust Nozzle Contours by Direct Optimization Methods", *AIAA journal*, vol. 19, N4, pp.750-751, 1981.

[5] G.P. Sutton and O. Biblarz, *Rocket propulsion elements*, Wiley India Pvt. Ltd., 2013.

[6] T. Elnady and W. Shyy, "Recent advances in contoured supersonic nozzle design and optimization", *Progress in Aerospace Sciences*, 119, 100668, 2021.

[7] A. E. Puckett, "Supersonic Nozzle Design", Journal of Applied mechanics, Transaction of ASME, vol. 13, n° 4, pp. 265-270, 1946.

[8] M.J. Zucrow and J.D. Hoffman, *Gas Dynamics-Multidimensional Flow,* Krieger, Malabar, FL, vol. 2, 1986.

[9] A. Haddad and H. Kbab, "Application of De-Laval Nozzle Transonic Flow Field Computation Approaches", International Journal of Mechanical, *Industrial and Aerospace Sciences*, vol 1, n2, pp. 328-323, 2013.

[10] N. Boughazi, a. Haddad and H. Kbab, "Design approach and performance analysis of a truncated ideal contour propulsion nozzle", U.P.B. Sci. Bull., Series D, Vol. 84, Iss.2, pp. 167-184, 2022.

[11] B.D. Baloni, S.P. Kumar and S.A. Channiwala, "Computational Analysis of Bell Nozzles", Proc. of the 4th Int. Conf. of Fluid Flow, Heat and Mass Transfer (FFHMT'17), Toronto, Canada, 2017. https://doi.org/10.11159/ffhmt17.110.