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Image edge detection and fractional calculation

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Abstract – Fractional calculus has begun to play a significant role in various research domains, including image and signal processing. Within image processing, fractional calculus offers intriguing possibilities for filtering and edge detection, presenting a novel approach to enhance image quality. Fractional calculus involves the generalization of differentiation and integration to non-integer orders. As debated by numerous researchers, the term "generalized calculus" might be more suitable than "fractional calculus," which is the prevalent terminology.

In this study, we aim to elucidate and delve into two pivotal articles: "A Novel Edge Detection Operator Based On Fractional Gaussian Differential" [1] and "Fractional Differentiation Based Image Processing" [2]. These articles elucidate how fractional calculus can confer advantages to image processing. Specifically, we will explore its application in image edge detection and image quality enhancement. The detection of image edges is of paramount importance within image processing, meriting thorough investigation.

The objective of edge detection is to identify points within a digital image that correspond to abrupt changes in luminance intensity. These alterations in image properties often reflect significant events or changes in the world's characteristics. They encompass discontinuities in depth, surface orientation, material properties, and scene illumination. Edge detection constitutes a research field situated within image processing and computer vision, particularly within the realm of feature extraction.

Consequently, we shall meticulously examine the utilization of fractional calculus for image edge detection, offering comprehensive insights into its application and presenting the outcomes of this endeavor. Additionally, we shall provide MATLAB programs developed during this research.

Keywords – Image; Edge Detection; Fractional Calculus; Fractional Differentiation; Luminance Intensity.

I. INTRODUCTION

There exist significantly superior methods for edge detection, such as the fractional calculus-based edge detector. It plays a pivotal role in image enhancement and delineation of image boundaries. We will scrutinize this enhancement within the context of two distinct sections, focusing on the derivative of order v (where v is a real number) in the ranges 0 < v < 1 and 1 < v < 2 .

II. FRACTIONAL CALCULUS OF NON-INTEGER ORDER 0 < V < 1 :

The same principle is employed for fractional calculus between 0 and 2, i.e., a convolution between the image and a filter is used to compute the fractional derivative between 0 and 2. Implementation of the Fractional Differentiation Filter (FIR) The fractional differentiation filter can be derived from the integer-order differentiation filter.The transfer function of the Fractional Impulse Response Differentiation Filter (FIR) is as follows:

$$D^V(z) = \left(\frac{1-z^{-1}}{T}\right)^V$$

Referencing the Extension of the Binomial Series

$$(1+x)^v = 1 + vx + \sum \frac{v(v-1)\ldots(v-k+1)}{k!}x^k$$

By substituting z^{-1} instead of x, the equation above can be written as follows:

$$D^V(z) = \frac{1}{T^v}(1 - vz^{-1} + \sum \frac{v(v-1)\ldots(v-i+1)}{i!}(-z^{-1})^{-i}) = \frac{1}{T^v}\sum(-1)^i \frac{T(v+1)}{T(i+1)T(v-i+1)}z^{-1}$$

Where T represents the sampling period, z is the shift operator, and T(*) denotes the Gamma function. For a more detailed step-by-step explanation, refer to [3].

Fractional Gradient Operators (Computed through Convolution):

Considering the constrained influence of the fractional differentiation filter's transfer function (FIR), the suitable selection of N, and the derived approximation of the first-order finite difference formula

$$D^V(z) = \frac{(1-z^{-1})^v}{T} = \frac{1}{T^v}\sum(-1)^i \frac{T(v+1)}{T(i+1)T(v-i+1)}z^{-1}$$

This can lead to a differential equation of the signal

$$\frac{d^v f(t)}{dt^v} = f(t) + (-v)f(t-1) + \frac{v(v-1)}{2}f(t-2) + \cdots + (-1)^n \frac{T(v+1)}{n!\,T(v-n+1)}f(t-n)$$

For digital images, based on the discrete signal difference equation, the fractional differential gradient formula can be derived in various directions.

Horizontal direction ($D^V_{XL\leftrightarrow XR} = D^V_{XL} - D^V_{XR}$)

$$\frac{d^v f(x,y)}{dx^v} = a_1 f(x-1,y) - a_2 f(x+1,y) + \cdots + a_n f(x-n,y) - a_n f(x-n,y)$$

Such as

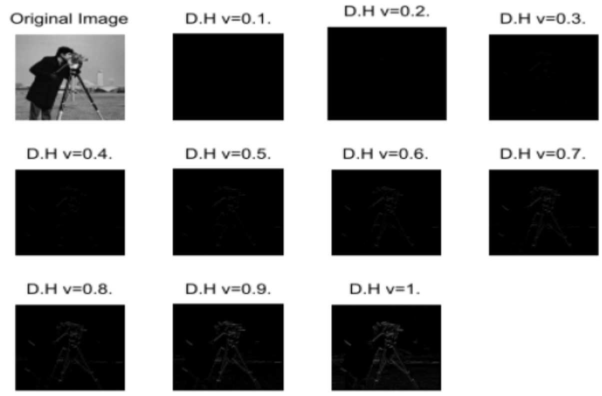
$$a_1 = -v, a_2 = \frac{v(v-1)}{2}, \ldots., a_n = (-1)^n \frac{T(v+1)}{n!\,T(v-n+1)}$$

By selecting the preceding n elements, the four directions of the fractional differential gradient mask can be obtained through truncation

To mitigate excessive filtering errors, we opt for the three preceding elements from the fractional order difference definition to construct the fractional differential gradient mask in different 5x5 directions.

$$w_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{v^2-v}{2} & -v & 0 & v & \frac{v^2-v}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The following figure illustrates the application of the mask w1:



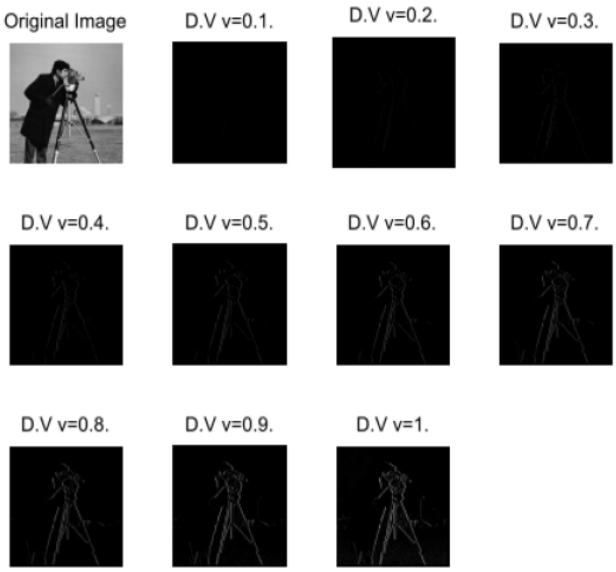
Horizontal direction ($D^{V}_{YU\leftrightarrow YD} = D^{V}_{YU} - D^{V}_{YD}$)

$$\frac{d^v f(x,y)}{dy^v} = a_1 f(x,y-1) - a_2 f(x,y+1) + \cdots + a_n f(x,y-n) - a_n f(x,y+n)$$

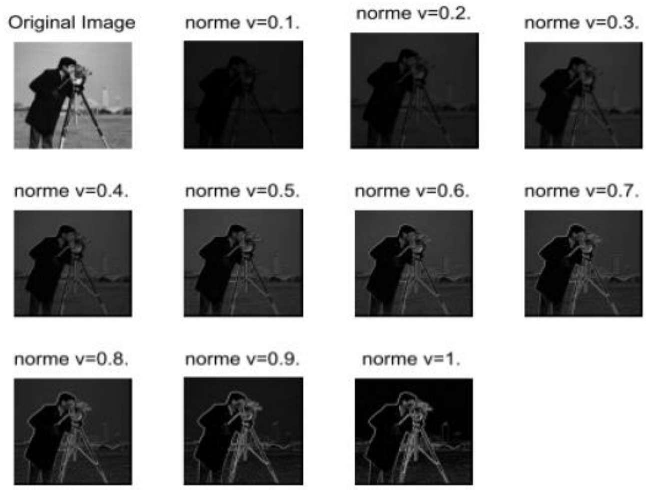
The fractional differential gradient mask in various 5x5 directions:

$$w_1 = \begin{bmatrix} 0 & 0 & \frac{v^2-v}{2} & 0 & 0 \\ 0 & 0 & -v & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & 0 \\ 0 & 0 & \frac{v^2-v}{2} & 0 & 0 \end{bmatrix}$$

The following figure illustrates the application of the mask w2:



The following figure depicts the application of the norms of masks w1 and w2:

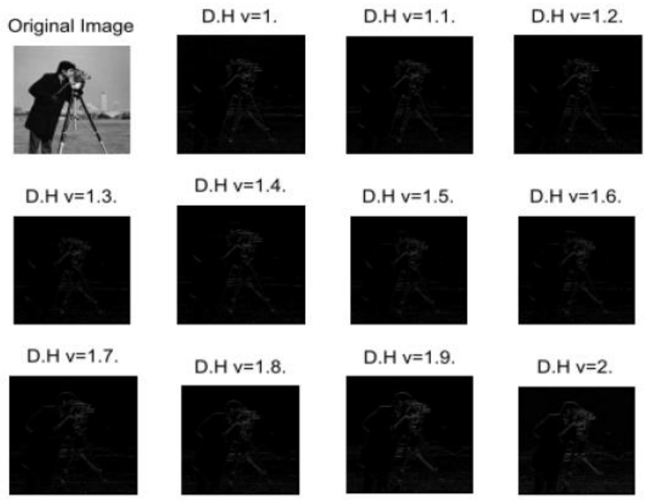


III. FRACTIONAL CALCULUS OF NON-INTEGER ORDER $1 < v < 2$:

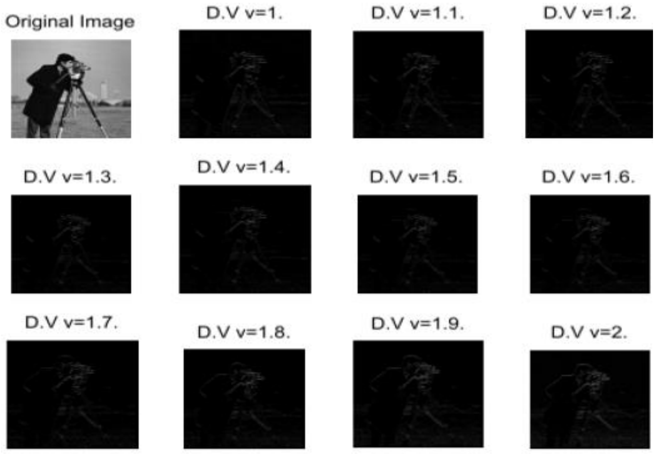
The performance of edge detection algorithms will inevitably be influenced by noise. Therefore, a certain level of smoothing must be applied prior to the detection operation on images contaminated by noise.

The Gaussian mean operator performs image smoothing excellently. As the surface of the two-dimensional Gaussian function has a bell-shaped form, the smoothing effect can be adjusted by controlling the variance of the Gaussian function and the size of the kernel. This allows for the generation of various kernels tailored to different images contaminated by noise.

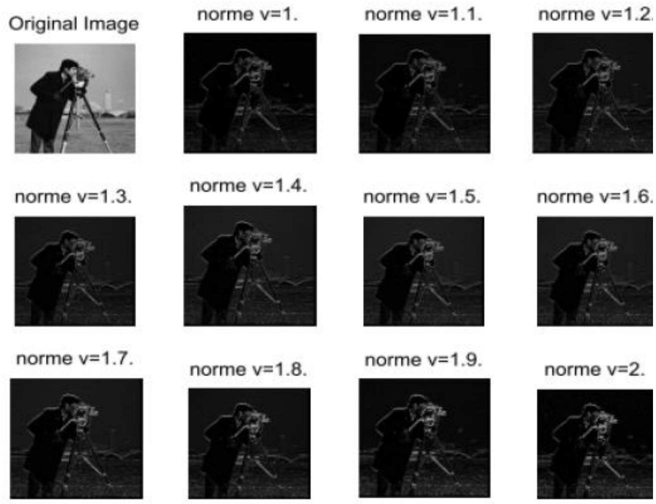
The following figure depicts the application of the mask w1:



The following figure illustrates the application of the mask w2:



The following figure demonstrates the application of the norms of masks w1 and w2.



IV. RESULTS

If v is between 0 and 1:

Whenever the variable v changes from 0 to 1, it enhances the clarity of the image edges.

If v is between 1 and 2:

Regardless of the change in the variable v from 1 to 2, image edges are present, with higher precision approaching 2, and there is an increase in image illumination.

V. DISCUSSION

Through our study, it becomes clear the importance of studying partial derivatives in determining the edges of images with high accuracy and opening a field for scientific research in the future for researchers interested in processing images and videos, and this contributes to the medical, military and other fields.

VI. CONCLUSION

In this study, we have introduced methods for image edge detection. Utilizing the gradient method and the Laplacian zero-crossing method, which are the most widely employed techniques in this field, we have particularly focused on enhancing image contours through the employment of fractional gradient, fractional Laplacian, and fractional Gaussian operator. Finally, a comparative analysis among these diverse methods has been presented. Our deduction demonstrates that fractional methods prove to be the most effective for image enhancement and edge detection.

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