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A Hybrid Quadratic Programming and Evolutionary Single-objective Optimization Algorithm: Empirical Study on CEC 2022 Benchmark Problems

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Abstract **–** Optimization methods are used in many fields of study to find solutions that maximize or minimize some operating parameters. Optimization can be considered constrained or unconstrained, as well as computational and traditional optimization algorithms. Both has advantages and disadvantages among them. Therefore, to improve the performance of the algorithm it is possible to use both in a hybrid manner. In this research, hybrid computational and traditional optimization method is proposed. For this purpose, two algorithms are selected as the examples of both categories, which are as a mathematical algorithm Sequential Quadratic Programming (SQP) and as a metaheuristic algorithm Genetic Algorithm (GA). As hybrid algorithm whose are named as SQP-GA and GA-SQP, are used. In addition to GA-SQP hybrid algorithm which is composed of two different forms named as V1 and V2 with respect to the collaboration of these algorithms. In this paper, this proposed hybrid algorithms were applied to the CEC 2022 benchmark problems are used to solve with boundary constrained optimization.

Keywords – Optimization, SQP, GA, Objective Function, Single-Objective Constrained Optimization

I. Introduction

Mathematical optimization or mathematical programming is the process of selecting the best element from a set of feasible alternatives based on some criterion. There are many fields in which optimization is used, such as engineering [1], computer science, operational research, economics, etc.

While there are many optimization methods, there is no single method that performs well for all functions. To compare the performance of these methods and each newly proposed optimization method, benchmark functions, some of which have been used in this study, are used. In this way, we can have an idea about which type of optimization algorithms are more useful for which function types. We can classify these optimization algorithms in various ways; however, we can basically examine them under two headings as conventional and meta-heuristic methods in Figure 1 [2].

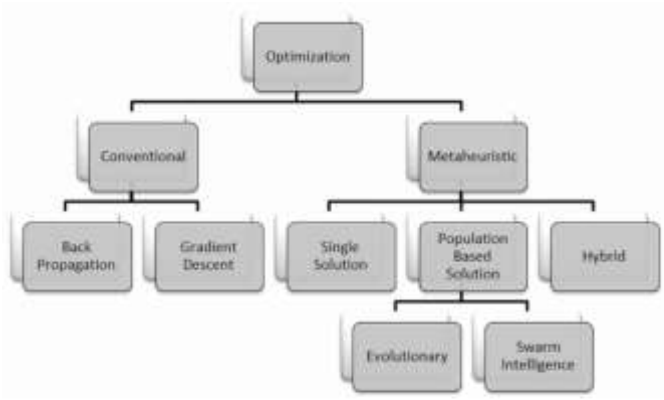

Figure 1: Classification of Optimization Methods [2]

We examine two algorithms from under different headings; the genetic algorithm (GA) [3], which is a meta-heuristic method and the gradient-based sequential quadratic programming (SQP) algorithm [4], which is a conventional method. The genetic algorithm is a stochastic method which means it produces different results each run. The algorithm has the capability of global search and there is no need to specify gradient.

The sequential quadratic programming (SQP) method is one of the efficient methods for addressing smooth constrained nonlinear optimization problems [8]. The method is deterministic, when it is given same initial point, it produces same result. The SQP needs gradient information to find search direction in the design space.

The paper organization is as follows: the details about the algorithms SQP, GA and hybrid algorithms presented after the introduction section. In addition, optimization problems will be mentioned in Section II: Materials and Method. The results which we have compare mentioned algorithms on different optimization problems are in Section III. The last part, which is Section IV, we conclude our research.

II. Materials and Method

In this study, we use 12 benchmark problems. These benchmark functions have various properties which means they have complexities from their nature, e.g. the function can be convex/nonconvex, uni-modal/multi-modal or highly multi-modal, hard to find its global optimum or having many local optima, etc. [5]. In addition to these properties' dimensionality is another issue that affects performance of the optimization algorithm.

A. *Benchmark Problems*

For the optimization process, we use 12 different problems which are grouped as basic functions, hybrid functions and composition functions, are found in *Table 2*. In the set of basic functions, we use shifted and rotated version of some benchmark functions. In the set of hybrid functions, the variables are randomly divided into some subcomponents and different benchmark functions are used for different subcomponents. The formulation is stated as follows:

$$F(\boldsymbol{x}) = g_1(z_1^*) + g_2(z_2^*) + \cdots g_N(z_N^*) + F^*(\boldsymbol{x}) \qquad (1)$$

$F(\boldsymbol{x})$: hybrid function
$g_i(\boldsymbol{x})$: i^{th} benchmark function
N: number of benchmark functions
z_i^*: randomly divided subcomponents

The set of composition functions is formed by combining some biased benchmark functions multiplied by some weight values. In this way, a more complex problem set has been obtained. The formulation is shown below:

$$F(\boldsymbol{x}) = \sum_{i=1}^{N} \{ w_i^* [\lambda_i g_i(\boldsymbol{x}) + bias_i]\} + F^* \qquad (2)$$

$F(\boldsymbol{x})$: composition function
$g_i(\boldsymbol{x})$: i^{th} benchmark function
N: number of benchmark functions
λ_i: used to control each $g_i(\boldsymbol{x})$'s height.
w_i: weight value for each $g_i(\boldsymbol{x})$

The detailed description of these functions is in the study [6].

B. *Optimization Algorithms*

Two different algorithms were chosen for optimization. First one is Sequential Quadratic Programming (SQP), which is an exact gradient-based method, and the other is Genetic Algorithm (GA), which is a metaheuristic approach.

The SQP algorithm is one of the powerful methods for solving constrained nonlinear problems [7]. We begin with defining nonlinear, constrained optimization problem:

$$\begin{gathered} \min f(x) \\ x \in \mathbb{R}^n : h(x) \geq 0 \end{gathered} \qquad (3)$$

Where x is an n-dimensional parameter vector and $h(x)$ contains one or more nonlinear inequality constraints. The fundamental concept of SQP method involves formulation and solution of a quadratic programming sub-problem during each iteration. This sub-problem is derived by linearizing the given constraints and making a quadratic approximation of the Lagrangian function $L(x, \lambda)$:

$$L(x, \lambda) = f(x) - \lambda^T h(x) \qquad (4)$$

Here, $\lambda \in \mathbb{R}^m$ and the vector contains Langrangian multipliers of the nonlinear programming problem. With the help of Lagrangian function, a quadratic program (QP) has the form:

$$\begin{gathered} minimize\ \frac{1}{2} d^T H_k d + \nabla f(x_k)^T d \\ d \in \mathbb{R}^n : \nabla h(x_k)^T d + h(x_k) \geq 0 \end{gathered} \qquad (5)$$

Table 1: Benchmark Problems

No		Functions	F^* (bias)
Unimodal Function	1	Shifted and full Rotated Zakharov Function	300
Basic Functions	2	Shifted and full Rotated Rosenbrock's Function	400
	3	Shifted and full Rotated Expanded Schafer's f7 Function	600
	4	Shifted and Rotated Non-Continuous Rastrigin's Function	800
	5	Shifted and Rotated Levy Function	900
Hybrid Functions	6	Hybrid Function 1 *(used benchmark funcs: F06, F07 and F04)*	1800
	7	Hybrid Function 2 *(used benchmark funcs: F07, F09, F13, F4, F12 and F16)*	2000
	8	Hybrid Function 3 *(used benchmark funcs: F09, F10, F11, F12 and F13)*	2200
Composition Functions	9	Composition Function 1 *(used benchmark funcs: F02, F08, F06 and F14)*	2300
	10	Composition Function 2 *(used benchmark funcs: F12, F04 and F07)*	2400
	11	Composition Function 3 *(used benchmark funcs: F03, F12, F15, F02, and F04)*	2600
	12	Composition Function 4 *(used benchmark funcs: F07, F04, F12, F06, F08 and F03)*	2700
Search range: $[-100{,}100]^D$			

where $H_k \epsilon \mathbb{R}^{n \times n}$ approximates the Hessian of the Langragian function, d is the optimal search direction of related subproblem. An estimation of Langrangian multipliers for the next iteration is obtained by using Newton's method. The Hessian of Lagrangian function can be updated by a quasi-Newton method (e.g. the Broyden–Fletcher–Goldfarb–Shanno (BFGS formula).

Genetic Algorithm (GA) is an evolutionary single objective optimization algorithm where the members of the population is named as chromosomes. The chromosomes are applied to three operators named as crossover, mutation, and selection operator. In crossover the new offspring is generated from the parent population by selecting and crossover the chromosomes. The roulette wheel method is selected because of its efficiency. Then the offspring applied to the polynomial mutation operators. Finally, the best members from these two populations are selected/survived to the new generation.

In this research two novel hybrid algorithms named as SQP-GA and GA-SQP are proposed so that the exploration property of the Genetic Algorithm and the exploitation property of the SQP algorithm is merged for a better performance with respect to the solution quality. However, this collaboration is not straightforward.

In SQP-GA approach, as the name suggests this hybrid algorithm begins with SQP algorithm. We employ more than one SQP run, the number of these runs depend on the number of population of GA algorithm. Each SQP runs terminates for the specified number of iterations. In the next step, we form a matrix whose rows contains each SQP solutions with the dimension of *D*. GA uses this matrix as an initial population. After specified number of iterations, GA choose the solution with the best fitness value and calculates the function value.

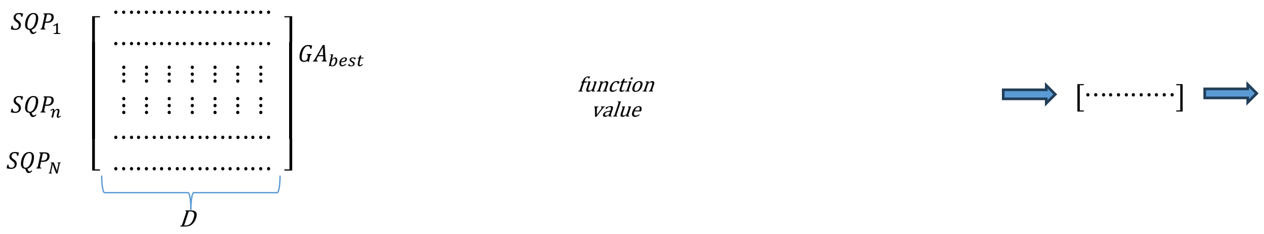

Figure 2: SQP-GA approach

The second hybrid algorithm is GA-SQP, this time we begin with GA then SQP algorithm is applied. In this approach, two different forms are investigated. The first form is called GA-SQP_V1, we take all GA solutions and run SQP each of the solutions. At the end, we calculate function values for each SQP solutions and choose the minimum value of the functions.

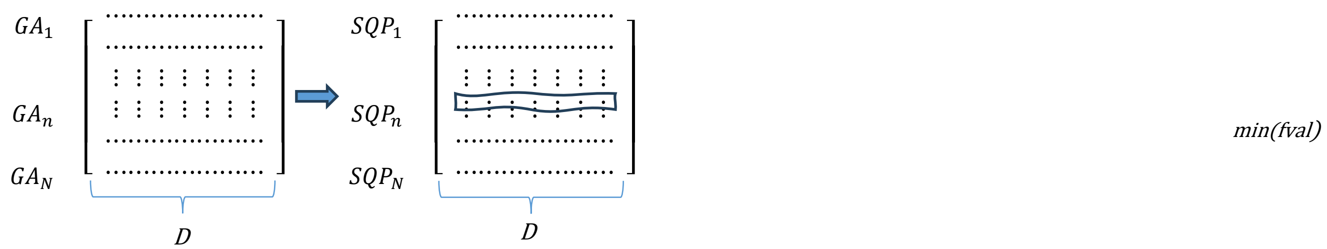

Figure 3: GA-SQP approach; the first form V1

Table 2: Optimization results (over 10 runs), for 5 optimization approaches, for D = 10, for all 12 optimization problems.

	Problem No:	SQP		GA		SQP_GA		GA_SQP_V1		GA_SQP_V2	
		mean	std	mean	std	mean	std	mean	std	mean	std
Unimodal Function	1	300,00	6,12e-12	300,88	2,79e+00	300,00	1,15e-11	300,00	1,15e-11	300,00	8,73e-12
Basic Functions	2	407,53	3,07e+00	417,74	2,77e+01	400,00	8,73e-03	403,47	4,08e+00	403,47	4,08e+00
	3	683,23	2,33e+01	614,72	5,78e+00	609,42	5,79e+00	609,78	2,91e+00	610,20	2,75e+00
	4	910,54	3,20e+01	820,89	6,36e+00	839,70	8,90e+00	822,78	7,70e+00	822,78	7,70e+00
	5	3369,24	1,10e+03	1068,17	1,32e+02	1290,45	1,96e+02	1026,00	1,20e+02	1110,28	1,56e+02
Hybrid Functions	6	1825,53	2,57e+01	2447,66	7,69e+02	1800,73	6,93e-01	1809,78	1,01e+01	1813,43	9,37e+00
	7	2302,74	1,38e+02	2038,46	7,27e+00	2029,97	1,08e+01	2033,17	1,09e+01	2039,98	1,04e+01
	8	2404,72	1,76e+02	2220,80	1,86e+00	2221,14	7,59e-01	2220,67	4,97e-01	2221,35	1,07e+00
Composition Functions	9	2529,28	2,85e-11	2529,82	8,20e-01	2503,43	7,21e+01	2529,28	6,06e-13	2529,28	2,54e-11
	10	3728,81	6,39e+02	2525,44	5,07e+01	2529,22	5,95e+01	2513,79	3,96e+01	2513,95	3,95e+01
	11	3146,31	7,90e+02	2685,30	1,32e+02	2600,00	1,15e-04	2735,82	4,77e+01	2735,83	4,77e+01
	12	3025,11	2,01e+02	2873,04	7,21e+00	2866,12	2,21e+00	2874,44	8,33e+00	2875,22	8,11e+00

The second form is called GA-SQP_V2, in this approach, we take the GA solution which has the best fitness value. Then run the SQP algorithm and obtain the function value.

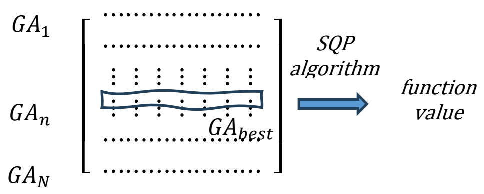

Figure 4: GA-SQP approach; the second form V2

III. Results

A meta-heuristic algorithm GA (Genetic Algorithm) and a gradient-based optimization algorithm SQP (Sequential Quadratic Programming) with the proposals of GA-SQP, SQP-GA_V1 and SQP-GA_V2 were employed to obtain optimization results on benchmark problems in Table 2 when *D=10* for both algorithms. The statistical properties of the solutions are collected from the 10 independent run of the algorithms. These properties are the mean and standard deviation of the solutions. When we compare the algorithms with the help of Table 2, the Unimodal Function *P01* has convex characteristics and produces same and better optimization results with the approaches which contains SQP algorithm. From the basic functions *P04* is the only function that produce better function value with GA (Even that result is obtained, the solutions for the hybrid algorithm is almost same). As for the results produced by first hybrid algorithm, it is clear from the Table 2 most of the functions give their minimum function value with SQP-GA approach. *P02, P03, P06, P07, P09, P11* and *P12* are from basic, hybrid and composition functions which have different level of complexities. From this perspective, it can be inferred that the function type is not a determining factor in the results. The reason could be the order of the algorithms, because SQP algorithm finds the better result for itself, and GA improves the result with population-based approach. The second hybrid algorithm with its two forms produce better results for the functions *P05, P08* and *P10* which are again from different function types. The functions *P05* and *P08* generate better results with SQP-GA_V1, the other function gives its best result with SQP-GA_V2.

IV. CONCLUSION

In this study, 12 different problems which have different level of complexities were optimized. While performing the optimization, we use two well-known algorithms GA and SQP in a hybrid manner to improve the performance of these algorithms. In total, five different approaches were investigated. Considering that the optimization results for different approaches and problems are calculated, many optimization problems have been studied. As a general inference, all five approaches produced good results for some functions individually, however the hybrid SQP-GA was the approach that produced the best results for many functions. As future study, the hybrid methodology between heuristic and metaheuristics will be discussed on more challenging problems with respect to the real-word applications.

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