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Research Article

Different Epileptiform Regimes in the Neural Population Modelled by the Generalized Telegraph Equation

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Abstract **–** The field-type approach to the neural cortical activities serves as a good alternative to the ANN-type models. It represents different states of the spiking and bursting neurons as a continuous field with a certain initial spatial distribution. The evolution of the neural populations is described with the generalized telegraph partial differential equations. In this paper, we use the Cattaneo generalization of Fick's model to describe the evolution of the epileptiform behavior in the small- and middle-scale neural clusters. We study the factorization procedure for the generalized telegraph equation and investigate the exact particular solutions to different dynamical regimes, which depend on the separation constant playing the role of a control parameter in our model. Additionally, we derive the traveling wave solutions and discuss briefly their properties.

Keywords – Neural Population, Epileptiform Behavior, Cattaneo Equation, Factorization Procedure, Traveling Waves.

I. INTRODUCTION

The mathematical models for appearing epileptiform patterns at the limited-scale neural clusters and their spreading to the greater scales of the neural populations cover a wide spectrum, including difference and differential approaches [1-3]. Correspondingly, the models based on artificial neural networks (ANNs) with the complex neural elements demand more powerful computational tools and are much more time-consuming [4].

The epileptiform dynamics study in this case can be focused as on *ex vivo*, *in vitro* modeling [5], as on the fittinf the experimental data [6].

A good alternative to ANN-type models are field-type models of the cortical activities, which represent different states of the spiking and bursting neurons as a continuous field with a certain initial spatial distribution. The evolution of the neural populations is described with partial differential equations [8].

This paper deals with the Cattaneo generalized formulation of the telegraph-type partial differential equations [7], which describes the distribution function of the hyper-synchronized cells as a smooth differentiable function of the space and time coordinates, together with the diffusing current, which corresponds to the spreading of the epileptiform dynamics to the greater scales of the neural population [9]. For simplicity, we discuss here one-dimensional case, but the multi-dimensional model will be based on the similar approach, and will lead us to the same type of regimes.

We analyze the factorization procedure for the generalized telegraph equation and investigate the exact particular solutions to different dynamical regimes, which depend on the separation constant playing the role of a control parameter in our model.

Additionally, we derive the traveling wave solutions and discuss briefly their properties.

II. METHOD: FACTORIZATION OF THE GENERALIZED TELEGRAPH EQUATION FOR EPILEPTIFORM DYNAMICS

The family of the Fick diffusion-type models made a long way from their classical formulation [10] to their modern generalized form [11]

For 1D case, the distribution of the hyper-synchronized neural elements (i.e. the epileptiform phase) $E(x,t)$ can be represented as Fick's second law in the form of the following diffusion equation [9]:

$$\frac{\partial E(x,t)}{\partial t} = D\frac{\partial^2 E(x,t)}{\partial x^2} \ , \tag{1}$$

with the positive 'diffusion constant' D. The flux of the epileptiform phase $j(x,t)$ is represented with the Fick's first law:

$$j(x,t) = -D\frac{\partial E(x,t)}{\partial x} \ . \tag{2}$$

Usually the neural networks do not demonstrate the *immediate* reaction to the gradient of the field $E(x,t)$, for that reason we should replace Eq.(2) with its time shifted analog proposed by Cattaneo in 1948 [7]:

$$j(x,t) - \tau\frac{\partial j(x,t)}{\partial t} = -D\frac{\partial E(x,t)}{\partial x} \ , \tag{3}$$

with the time delay constant τ. LHS(3) is the result of the expansion in the form $g(t-\tau) \cong g(t) - \tau \cdot dg/dt$ for a small τ.

The modified Cattaneo model can be unified in the form of the generalized telegraph equation [8]:

$$\tau\frac{\partial^2 E(x,t)}{\partial t^2} + \left(1 - \tau\frac{\partial f}{\partial E}\right)\frac{\partial E(x,t)}{\partial t} = -D\frac{\partial E(x,t)}{\partial x} + f(E) \ , \tag{4}$$

where usually $f(E)$ is chosen as a polynomial function of the epileptiform phase distribution E.
We investigate here the ***factorized solutions*** to the equations (1), (3) in the form:

$$E(x,t) = A(x)B(t)\ ;\ j(x,t) = F(x)G(t).\ , \tag{5}$$

Then after the substitution of the first Eq.(4) to (1), one can easily obtain by the separation of variables:

$$\frac{1}{A(x)}\frac{d^2A(x)}{dx^2} = \frac{1}{DB(t)}\frac{dB(t)}{dt} = k\ , \tag{6}$$

with the separation constant k. As a rule, following the decay concept, this k is chosen as a negative number producing the space-harmonic diffusion with the exponential time decay:

$$E(x,t) = E_0 \exp\{-D\,|\,k\,|\,t\}\sin(|\,k\,|\,x)\ ;\ k<0,\ E_0 = \text{const.} \tag{7}$$

Nevertheless, the solution (7) is more appropriate for a certain boundary condition, while for a local epileptiform process there is no well-defined boundary. Thus, in the open sub-set of the neural network, one should focus on the case of the positive k, then:

$$E(x,t) = E_0 \exp\{Dkt\}\sinh(kx)\ ;\ k>0,\ E_0 = \text{const.} \tag{8}$$

The solution (8) demonstrates an exponential growth both in time and space for the epileptiform phase distribution $E(x,t)$.
The substitution of (8) to (3) gives us after the variable separation:

$$-\frac{1}{DB(t)}\left[G(t) - \tau\frac{dG(t)}{dt}\right] = \frac{1}{F(x)}\frac{dA(x)}{dx} = c\ , \tag{9}$$

with the separation constant c. Then the flux function becomes:

$$j(t) = j_0\frac{Dk}{\tau Dk - 1}\exp\{Dkt\}\cosh(kx)\ , \tag{10}$$

where j_0 is a constant. Eq.(10) stands for the exponentially growth solution in space and time.
Finally, let's analyze the traveling wave solution to (1), (3). One should take the distribution function E in the form:

$$E(x,t) = E(x+vt)\ ;\ j(x,t) = j(x+vt)\ , \tag{11}$$

with the epileptiform phase spreading speed constant v. Then by (1) we get: $v = D$, and by (3):

$$j(x+vt) = \frac{D}{\tau D - 1}E(x+vt)\ . \tag{12}$$

By (12), the epileptiform flux is traveling in the neural environment with the constant speed $v = D$ without decay preserving its shape.

III. RESULTS

The choice of different signs for the separation constant produces different dynamical regimes in the epileptiform dynamics of a small-scale neural cluster with the open boundaries: alternatively to the exponentially time-decaying spatially-periodic solution, which is typical for the physical applications of diffusion-type equations, one can reproduce another solution exponentially increasing both in spatial and temporal coordinates. This type of solution corresponds to the appearing of the macro-scale in the epileptiform dynamics, when the initially small-scaled hyper-synchronization spreads itself to the bigger scales.

IV. DISCUSSION

According to our observation, in the Cattaneo model applied to the epileptiform phase, the standard standing wave spatial sine/cosine-shape and traveling wave solutions which are typical to the diffusion modified partial differential equation are not a matter of great interest, because they describe a relatively artificial behavior of the hyper-synchronize neural phase at the very small scale of the whole population. From other hand, the exponentially-increasing solution looks to be much more realistic for the explanation of the seizures gradually extending in space and time.

In the majority of physical application to the physical diffusion processes, such types of solutions are not considered due to the boundary conditions and the constraints coming from the energy conservation. In our case, the small neural cluster is a sub-set of a bigger neural network, which feeds it with the energy. The boundaries of such clusters are not fixed, thus, we get an extra argument not to mimic a spatial periodic solutions.

Playing with the sign of the separation constant, one can trigger the regime of the hyper-synchronization spreading in space. At the same time, switching to the inverse direction, this control may play just an opposite role as a tool for suppression of the macroscopic epileptiform behavior in the ANN. Such driving response of our model can be much more powerful to compare with various open-loop stimulations to identify the effects of external control on the cortical output [12].

All the properties that we observe for the 1D spatial model will be also valid for the multidimensional case.

V. CONCLUSION

In practical applications, the choice of the certain sign (plus or minus) in the separation constant is a form of control which is capable to re-shape the epileptiform phase from the local dynamics to its macroscopic realization. This mechanism is extremely interesting, and it may serve as a very useful tool for studying the special development of epilepsy in human cortical networks.

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