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Research Article

Comparison and Analysis of Different Approaches in Fractional Order Systems

Ibrahim TUTAL*[1], Turgay KAYA[2]

[1]*Fırat University, Department of Electrical and Electronics Engineering, Turkey, ibrahimtutal01@gmail.com*
[2] *Fırat University, Department of Electrical and Electronics Engineering, Turkey, tkaya@firat.edu.tr*

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Abstract – Fractional order calculations, which have been around since the 1700s, have become an effective method used for better modeling and control of systems in many fields of science and engineering in recent years. This system, which gives very successful results in modeling, recently has been frequently used in engineering applications such as filter modelling. Filter design, which is an example of these applications, is a rich research field with a complete design theory, starting with design conditions and ending with circuit implementation. In this context, the differential equations used in modeling the system mostly include fractional derivative and integral operators. Physical interpretation of fractional operators is not as easy as integer operators. Since the fractional operator is not local and depends on the past values of the function as required by the derivative operation, it creates a long memory effect in the system. In this study, two different approaches are presented as solutions to the difficulties encountered in modeling fractional order systems. The outputs of these approaches used in modeling and analysis of the fractional order system are compared and their advantages and disadvantages are stated.

Keywords – Fractional-Order Systems, Filters, Modeling of System, Method of Oustaloup Fractional-Order Derivative.

I. INTRODUCTION

Differential equations in which the integral or derivative degrees have any real number are called fractional order systems. This system is a sub-branch of mathematics and has been used for many years [1]. The development of this system has increased in parallel with computer technology.

The foundations of fractional mathematics were laid in the conversations between L'Hospital and Leibniz in the late 17th century [2]. Later, theoretical studies were continued by Euler and Lagrange, and systematic studies were put forward by Holmgren, Liouville and Riemann [1-3]. In the 1800s, Boole used fractional calculations in the symbolic solution of the differential equation of constant degree, and Heaveside used fractional calculations in the solution of electromagnetic field theory problems [4].

Fractional calculations have been the subject of many studies in the field of mathematics and have affected other fields as well, and many studies on engineering and physics have emerged [5]. Since fractional order modeling gives better results than integer modeling in these areas, interest in these areas is increasing.

This system, which provides very successful results in modeling, has recently been frequently used in engineering applications such as filter design. Filter design is a rich field of research in which there is a complete and error-free design theory, starting with design conditions and ending with circuit implementation. Conventional filter design includes primary, secondary, etc. degrees are limited. Fractional order filters were first proposed by Radwan and his team, a system in which one element (resistor, capacitor or inductance) is fractional and can be used for all filter designs [6]. General expressions for maximum frequency, quality factor, correct phase, half power frequencies have been obtained.

M. V. Bhat created the fractional order all-pass filter design with a degree between 0<a< 1, using an electronic element operational transconductance amplifier (OTA). Unlike the literature, a fractional order capacitor (OTA-C) was used in the filter [7]. G. Singh and his colleagues designed a voltage-mode fractional-order filter using two fractional-order capacitors and three single-output OTAs. With this designed structure, all second-order filter types can be realized [8].

X. Qunwei and his colleagues carried out a frequency-based study by designing a fractional and recursive active power filter. They preferred recursive control because it is the superior steady state. They applied the Lagrange interpolation algorithm to obtain overlapping fractional models [9].

D. Kubanek and his colleagues conducted a study on the transfer functions of four bandpass filters with fraction orders between 1<α<2. They found the coefficients using numerical least squares optimization, which aims to minimize magnitude errors between these transfer functions and the sample functions corresponding to the 2nd order Butterworth filter responses [10].

Anil K. Shukla and his team have developed a fractional-order filter-based algorithm for retinal blood vessel cell division. The filter in the study was created with the help of fractional derivative and an exponential weight factor. Fractional filter of a local covariance matrix and eigenvalue maps were used to develop the retinal vessel segmentation algorithm. The local covariance matrix consists of a quadratic image moment [11].

J. Nako and his team designed fractional order low-pass and high-pass Butterworth and Chebyshev filters using an active circuit element. Their work is carried out using only a single active element, which seems to minimize the use of active circuit elements [12].

D. Song and colleagues designed an adaptive fractional order Kalman filter to calculate the charge states of lithium-ion batteries. First, a model with a fractional-order constant phase element module was created to mathematically express the charge state in the lithium battery. Then, state equations were created with the help of the augmented vector, and the fractional order Kalman filter was used to calculate the coefficients of the equation [13].

II. MATERIAL AND METHOD

The format of the derivative and integral part of fractional calculations involving fractional derivatives or integrals is the commonly known $\frac{df}{dt}, \frac{d^2f}{dt^2}$ or $\int_0^t f(u)du$. Likewise, functions have first-order derivatives, second-order derivatives, first integrals, and double integrals. With the developing computer technology, it is now possible to calculate the 0.5th degree, π, degree derivative and integral of a function. Therefore, derivatives and integral fractional calculations with arbitrary degrees of real or complex numbers are described.

Fractional calculation is a branch of mathematics concerned with non-integer derivatives and integrals. When it comes down to it, fractional order calculations (non-integer order calculations) also include integer order calculations. The fundamental operator of fractional order calculations is ${}_aD_t{}^r$ (a and t are the lower and upper limits, r ∊ R), can be considered as its generalization to non-integer expression [14].

$$ {}_aD_t{}^r = \begin{cases} \frac{d^r}{dt^r} & : r > 0 \\ 1 & : r = 0 \\ \int_a^t (dt)^{-r} & : r < 0 \end{cases} \quad (1) $$

Here r denotes the degree of the derivative, t and a denotes the integral limits..

Filter design with the help of fractional order calculations is different from normal design. Filters are generally classified as first, second or n'th order systems and their degrees are integers. Transfer functions, which are mathematical expressions of filters, are generally in the form $T(s) = \frac{N(s)}{D(s)}$. Here D(s) and N(s) are polynomials raised to the integer power of the Laplace operator, such as s^2 or s^n. However, the Laplace equation of s^a, whose degree is not an integer between 0<α<1, is a representative representation of a fractional degree system. Conventional continuous time filters are of integer order. However, using fractional calculus, filters can also be represented by more general fractional order differential equations. Integer filters are just a subset of fractional order filters anyway.

Conventional filter design is limited to first, second or third order. However, fractional filter design allows using rational numbers as degrees. The design of such filters is achieved by generalizing the degree domain of conventional filters. According to the generalized theory, three critical points are emphasized:

1. The half power frequency, X, is the frequency at which the power drops to half the passband power.

$$|\mathrm{F}(j\omega_h)| = \frac{|\mathrm{F}(j\omega_{geçirme\ bandı})|}{\sqrt{2}} \tag{2}$$

The bandwidth of any filter is related to the half power frequency. Here $|\mathrm{F}(j\omega_h)|$ is the transfer function of the filter.

2. The maximum or minimum frequency ω_m is the frequency at which the response magnitude is maximum or minimum and is obtained by solving the equation $\left(\frac{d|\mathrm{F}(j\omega)|}{d\omega}\right)_{\omega=\omega_m} = 0$.
3. The true phase frequency ω_{rp} is an imaginary parameter with the frequency of the phase $\angle\mathrm{F}(\omega_{rp}) = \pm\frac{\pi}{2}$ and the transfer function F(s).

In fractional order calculations, the solution of derivative and integral operations is not as easy as in integer order derivative and integral operations. In other words, it does not have a local operator like integer degree derivative and integral operators, but also depends on the past values of the function. It is difficult to make applications or model them in real terms with these infinite-dimensional functions. For this reason, integer approximation models have been used to analyze and model fractional order calculations more easily.

One of the methods used as the integer approximation model in this study is the Oustaloup method. Assuming that the function $F(s) = s^a$ is a function with fractional degree α, M zeros and M pole values, the integer degree Oustaloup approximation function,

$$F(s) = C_0 \prod_{n=1}^{M} \frac{1+\frac{s}{\omega_{zn}}}{1+\frac{s}{\omega_{pn}}} \tag{3}$$

is obtained as. With this method, the given frequency width is divided into small intervals and the function is converged for each value. These converged functions are connected in series to form the fractional order filter.

Another approximation method is the Regulated Oustaloup method. This method is designed to ensure that the poles and zeros of the filter are in the stable region compared to the previous method. Thus, an improvement in the convergence performance of the discrete filter in the desired frequency range was achieved. Arranged approximation model for fractional order functions,

$$F(s) = \lim_{N\to\infty} L_N(s) = \lim_{N\to\infty} \prod_{k=-N}^{N} \frac{1+s/\omega' k}{1+\omega k} \quad (4)$$

is obtained as.

III. RESULTS

In this study a 4th-order discrete-time fractional order filter is designed in both proposed approaches Oustaloup and Regulated Oustaloup. In each approach, the phase and amplitude responses of the fractional order filter were compared. Additionally, the convergence errors of both methods are given in Table 1.

Table 1. Convengerce Errors of Recommended Approaches

Method	Convengerce Errors
Oustaloup	0,0251
Regulated Oustaloup	0,0205

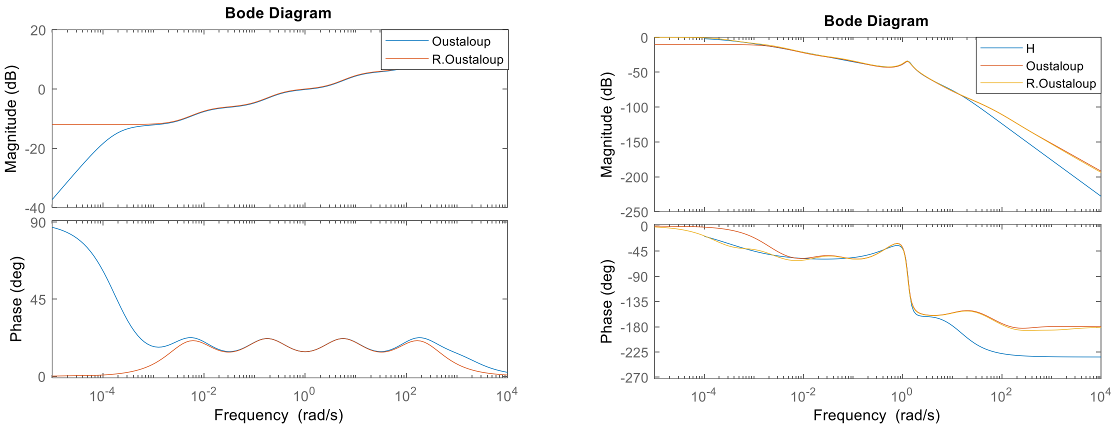

Figure 1. Phase and Amplitude Responses of Recommended Approaches

IV. DISCUSSION AND CONCLUSION

According to the information obtained as a result of the study, it was seen that the filter designed with the Oustaloup method provided good convergence to its function in the low frequency region, but its convergence performance decreased as it moved to the high frequency region. It has been observed that the fractional order discrete filter designed with the Regulated Oustaloup method has better convergence performance at high frequencies.

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