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Research Article

Koopman Operator-Based Identification of Twin Rotor Aerodynamic System

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Abstract- This research paper provides an unusual approach for identifying the Twin Rotor Aerodynamic System (TRAS) through the utilization of Koopman operator theory. The TRAS, known for its inherent nonlinear dynamics, poses significant challenges in modeling and control due to its complex aerodynamic interactions. Traditional modeling techniques often struggle to capture the intricate dynamics accurately. In this work, we propose an effective method to identify the TRAS model: the Koopman operator, a potent mathematical tool for investigating nonlinear dynamical systems. System identification can be obtained by the Koopman operator, which gives an infinite-dimensional linear system by transforming the nonlinear dynamics. Through rigorous analysis and simulation studies, we demonstrate the effectiveness and accuracy of our proposed approach in capturing the nonlinear behavior of the TRAS. This research contributes to advancing our understanding of complex aerodynamic systems and lays the groundwork for developing robust control strategies for applications such as unmanned aerial vehicles (UAVs) and rotorcraft.

Keywords: Koopman Operator, TRAS, Lifting Data, Observables, Gauss RBF.

I. INTRODUCTION

The Twin Rotor Aerodynamic System (TRAS) serves as a prominent testbed for studying complex nonlinear dynamics in the realm of aerospace engineering. Its unique configuration, comprising two counter-rotating propellers mounted on a single platform, presents a challenging yet insightful platform for investigating aerodynamic interactions and control methodologies. To create efficient control schemes and improve the functionality of different rotorcraft applications, such as unmanned aerial vehicles (UAVs), helicopters, and quadcopters, precise modeling and identification of the TRAS dynamics are essential. However, the inherent nonlinearities and aerodynamic complexities of the TRAS pose significant hurdles in traditional modeling and identification techniques.

In recent years, the Koopman operator theory has emerged as a promising tool for analyzing nonlinear dynamical systems [1]. Named after the mathematician Bernard Koopman, this theory offers a powerful framework for representing and understanding the evolution of nonlinear systems in an infinite-

dimensional space [2]. By transforming the nonlinear dynamics into an equivalent linear framework, the Koopman operator enables the analysis and prediction of system behavior with unprecedented accuracy and efficiency. This approach has found applications across various domains, including fluid dynamics, robotics, and control systems.

In the context of aerospace engineering, the Koopman operator approach has garnered significant interest for its potential in modeling and identifying complex aerodynamic systems. Several researchers have explored its utility in characterizing the behavior of aircraft dynamics [3-4]. However, its application to the TRAS identification problem remains relatively unexplored. This paper aims to bridge this gap by proposing a novel approach for identifying the TRAS model using the Koopman operator framework.

Through a comprehensive literature review, we highlight the existing methodologies and challenges associated with TRAS modeling and identification. Traditional techniques often rely on simplified linear or quasi-linear models, which may fail to capture the intricate dynamics accurately. Moreover, the nonlinearity and coupling effects inherent in the TRAS further complicate the identification process. Recent advancements in system identification techniques, including machine learning and optimization algorithms, have shown promise in addressing these challenges [5]. However, their applicability to the TRAS identification problem remains to be explored.

In this paper, we present a systematic approach for identifying the TRAS model based on the theory of Koopman operators. We demonstrate the effectiveness of our proposed methodology through rigorous analysis and simulation studies. By leveraging the inherent structure of the Koopman operator, we aim to capture the nonlinear dynamics of the TRAS accurately and efficiently. This research contributes to advancing our understanding of complex aerodynamic systems and lays the groundwork for developing robust control strategies for rotorcraft applications.

The rest of the paper is structured as follows: An overview of the Twin Rotor Aerodynamic System is given in Section II. The Koopman operator's theoretical foundation is introduced in Section III. The methods used to model TRAS using the Koopman operator is described in depth in Section IV. The experimental setup and findings are presented in Section V, and Section VI contains the discussions that follow. The paper is finally concluded in Section VII, which also suggests options for future research.

Twin Rotor Aerodynamic System

The Twin Rotor Aerodynamic System (TRAS) embodies a sophisticated MIMO laboratory apparatus, meticulously designed to emulate the intricate dynamics of helicopter models [6]. Its versatile functionality enables motion along two principal axes: lateral and longitudinal [7]. Structurally, the TRAS consists of a sturdy beam. This beam has two pivotal components essential for its operation: a primary propeller and a tail propeller [8].

The primary propeller assumes a pivotal role in controlling pitch motion within the system [9]. Through precise manipulation, it governs the angular orientation of the TRAS, facilitating changes in its pitch angle to achieve desired flight dynamics [9]. The tail propeller serves a crucial role in regulating azimuthal motion [10]. By exerting controlled forces, it enables the TRAS to execute smooth rotational movements around its vertical axis, thereby enhancing its maneuverability and agility [6].

The visual representation of the TRAS in Figure 1 provides a comprehensive depiction of its structural layout, offering valuable insights into the spatial arrangement of its components [6].

Figure 1. Twin rotor aerodynamic system [11].

In summary, the TRAS represents a sophisticated laboratory system meticulously crafted to replicate the complex dynamics of helicopter flight [7]. Its integration of multiple inputs and outputs, coupled with its ability to execute motion in lateral and longitudinal directions, renders it an invaluable tool for aerospace research and development endeavors.

In the realm of control studies concerning the Twin Rotor Aerodynamic System (TRAS), paramount attention is directed towards the precise regulation of azimuth and pitch angles through the computation of controlled inputs for both the main and tail propellers [12]. TRAS, characterized by its highly coupled, nonlinear, and inherently unstable nature, presents formidable challenges in control design and implementation [13]. The nonlinear model of TRAS, as delineated in Equation [11], provides a comprehensive representation of its complex dynamics:

$$\ddot{\psi} = (u_\phi - J\frac{\psi^2}{2}\sin(2\phi) - C_\phi\dot{\phi} - K_g g) / J_\phi$$

$$\ddot{\phi} = (u_\psi + J\,\dot{\psi}\,\dot{\phi}\,Sin\,(2\phi) - K_\psi\psi - C_\psi\dot{\psi}) /\ J_\psi$$

Where

$$J_\psi = (m_m l_m^2 + m_t l_t^2)\,Cos^2(\phi) + 2m_{c\omega}l_{c\omega}\,sin^2(\phi) + J_z$$

$$J_\phi = m_m l_m^2 + m_t l_t^2\ + 2m_{c\omega}l_{c\omega}\ + J_x$$

$$J = m_m l_m^2 + m_t l_t^2\ - 2m_{c\omega}l_{c\omega}$$

$$K_g = (m_m l_m - m_t l_t)\cos(\phi) + 2m_{c\omega}l_{c\omega}\sin\,(\phi)$$

Here, ϕ and ψ symbolize the azimuth and pitch angles, correspondingly, while $\dot{\phi}$ and $\dot{\psi}$ depict their respective velocities. The system parameters encompass distances of the main rotor and tail rotor from the origin $l_m\ and\ l_t$ respectively), alongside the masses of counter weights at the main rotor and tail rotor ($m_m\ and\ m_t$). Additionally, $m_{c\omega}$ and $l_{c\omega}$ denote the mass of the ends levers and the lengths of end levers, respectively. For comprehensive parameter values, reference [11] provides detailed information. The state vector for TRAS integrates four pivotal variables. According to TRAS, that state vector is as

$$x = \left[\phi\ \dot{\phi}\ \psi\ \dot{\psi}\ \right]^T$$

In our study, we incorporate two controlled inputs: yaw input (u_1) and pitch input (u_2), both measured in radians. Additionally, we focus on two outputs: yaw angle (ϕ) and pitch angle (ψ), measured in radians. Upon applying these control inputs, the model generates data for four states. These states are simulated using MATLAB Simulink. However, for our analysis, we selectively extract the yaw and pitch angles as the outputs of interest. We then proceed to identify these outputs using the Koopman operator approach.

Koopman Operator

This study uses the notion that every finite-dimensional nonlinear system has an equivalent limitless-dimensional linear representation in the space of actual-valued functions of the system's input and state. This property is employed in the system identification technique. The flow of trajectories along the paths in this space of actual-valued functions is characterized by the (linear) Koopman operator. There is a

bijective and well-defined link between the system's finite and infinite dimensions representations [15]. This allows us to derive the comparable nonlinear system representation and then use linear regression on observed data to approximate the Koopman operator. The model estimation technique described in [16] and [17] is summed up in the remaining portion of this section and applied to nonlinear model with known control input before being used and verified on an actual, highly complex, coupled, and nonlinear TRAS system.

1. Koopman spectral theory

A basic introduction to Koopman spectral theory and a numerical approach for approximating the $Koopman\ operator$ are given in this subsection.

The set of linked ordinary differential equations is taken into consideration in the classical geometric theory of dynamical systems.

$$\dot{x} = F(x, u) \qquad (1)$$

The input of the system is represented by "u" and u ϵ R^m, and F is differentiable continuously in "x". The variables(states) of the system is denoted by " x " and $x \epsilon R^n$. At time t, the solution to (1) is indicated by φ(t,x_0,u) where u is the applied controlled input at all times from 0 to t(sec), and x_0 is the initial. To keep things simple, we refer to this map as the flow map using φ(x_0,u) rather than φ(t,x_0,u). The nonlinear model can be converted into an limitless-dimensional function space $\mathcal{F}$=$\mathbb{L}^2$(X × U) , where $\mathcal{F}$=$\mathbb{L}^2$(X × U), is the space of square integrable actual valued functions with domain X × U, and X⊂ R^n and U⊂ R^m are compacted subsets. We refer to $\mathcal{F}$ constituents as observables. The system's flow in $\mathcal{F}$ is defined by the set of $Koopman\ operator \mathcal{K}^t: \mathcal{F} \longrightarrow \mathcal{F}$, for any t $\geq$ 0. This set, by definition, explains how the "Observables" $\oint \in \mathcal{F}$ evolve along the system's path:

Based on definition:

$$\mathcal{K}^t \oint = \oint \circ \varphi \qquad (2)$$

In spite of the nonlinearity of the system (1), $\mathcal{K}^t$ is a linear operator of nonlinear model. Since for $\oint_1, \oint_2 \epsilon \mathcal{F}^n$ and $\lambda_1, \lambda_2 \epsilon \mathbb{R}$

$$\mathcal{K}_t\ (\lambda_1 . \oint_1, \lambda_2 . \oint_2) = \lambda_1 (\oint_1 \circ \varphi_t) + \lambda_2 (\oint_2 \circ \varphi_t)$$

$$\mathcal{K}_t\ (\lambda_1 . \oint_1, \lambda_2 . \oint_2) = \lambda_1 \mathcal{K}_t \oint_1 + \lambda_2 \mathcal{K}_t \oint_2 \qquad (3)$$

So the $Koopman\ operator$ gives a linear representation of the system's flow in the lmitless-dimensional space of observables [18].

We introduce the $Koopman\ operator$'s infinitesimal generator $\mathcal{A} : \mathcal{F} \longrightarrow \mathcal{F}$[15, Equation 7.6.5], which is described in terms of the vector field "F" as follows because this equation applies for all observables.

$$\mathcal{A} = F\ .\ \nabla_x \qquad (4)$$

Thus, $\dot{\oint} = \mathcal{A} . \oint$ infinitesimal generator characterizes the dynamics of the observables along the trajectories of the system. Keeping in mind that the $Koopman\ operator$ describes the flow of observables, the relationship between the $Koopman\ operator$ and its infinitesimal generator can be seen by making the following connection:

$$\mathcal{K}^t = e^{\mathcal{A}t} = \sum_{k=0}^{\infty} \frac{t^k}{k!} \mathcal{A}^k \qquad (5)$$

Knowing the Koopman operator " $\mathcal{K}^t$," the vector field F may be found using equations (4) and (5). Algorithm 1 summarizes the three phases that make up this technique.

Algorithm 1: $Koopman$ Base Model Identification
Input: $f(x_K; u_K)$; $(x_K+1; u_K)$ g for $k = 1; :::; K$
Step 1: Lifting data using (9)
Step 2: $Koopman\ operator$ approximation, $\mathcal{K}^{T_S}$ using (13)
Step 3: Evaluate Vector Field, F^{-} through (17) and (20)
Output: F

II. METHODOLOGY

This section involves a comprehensive review of lifting techniques employed in system identification and nonlinear dynamics. Lifting techniques aim to transform nonlinear systems into higher-dimensional linear systems, thereby facilitating their analysis and modeling.

1 Lifting the Date

The first step in identifying a linear system in observables space is to convert empirical data into a format that is compatible with the Koopman-based system identification process. This would theoretically require "lifting" state measurements into the limitless-dimensional "$\mathcal{F}$" observables space. However, to be implemented, measurements can only be hoisted into a limited-dimensional subspace. $\bar{\mathcal{F}} \in \mathcal{F}$ is the subspace of "$\mathcal{F}$" that is covered by "N" linearly independent basis functions (e.g., exponentials, radials, and monomials) $\{\psi_k\}_{k=1}^{N}$. Every observable $\bar{\mathcal{F}} \subset \mathcal{F}$ can be expressed as a linear combination of the components of the basis as

$$\bar{f} = \alpha_1\psi_1 + \cdots + \alpha_N\psi_N \tag{6}$$

It should be noted that $\bar{f} \in \bar{\mathcal{F}}$ has a *vector representation* provided by the vector coefficient, $\alpha = [\alpha_1 \cdots \alpha_N]^T$. We introduce the lifting function, $\psi: \mathbb{R}^n x \mathbb{R}^m \longrightarrow \mathbb{R}^N$, to express $\bar{f}$ at a given state "x" and constant control input "u" :

$$\psi(x,u) = [\psi_1(x,u) \cdots \psi_N(x,u)]^T \tag{7}$$

So, $\bar{f}(x,u)$ may be written in a vector form as.

$$\bar{f}(x,u) = \alpha^T\psi(x,u) \tag{8}$$

We consider $\psi(x,u)$ as N dimensional "lifted" version of (x,u), So multiplying $\psi(x,u)$ by the observable's vector representation produces the observable's value at (x,u),

2. Approximating the Koopman Operator

In this step in the Koopman-based system identification approach is to choose the Koopman operator that best represents the flow of the lifted versions of the measured data points. Theoretically, the Koopman- operator has limitless dimensions, but we are able to locate a limited-dimensional approximation of it in $\bar{\mathcal{F}}$. Keep in mind that $\mathcal{K}^T$may be expressed as an "N × N" matrix that uses matrix multiplication to work with observables:

$$\mathcal{K}^T\alpha = \beta \tag{9}$$

Where observables in $\bar{\mathcal{F}}$ have vector representations denoted by α, β.Our objective is to determine a $\mathcal{K}^T$, that in the $\mathbb{L}^2$-norm sense, as precisely as possible characterizes the activity of the infinite dimensional Koopman- operator $\mathcal{K}^T$ on the finite dimensions subspace $\bar{\mathcal{F}}$ of all observables. Consequently, the following must be true in order to precisely replicate the action of $\mathcal{K}^T$ operating on an observable in $\bar{\mathcal{F}} \subset \mathcal{F}$.

$$(\mathcal{K}^T\alpha)^T\psi(x,u) = \alpha^T\psi(\varphi^t(x,u),u) \tag{10}$$

Since this is a linear equation, the best approximation of $\mathcal{K}^T$ on $\bar{\mathcal{F}}$ in the sense of the $\mathbb{L}^2$-norm may be obtained by solving (12) for $\mathcal{K}^T$ for a given $x \in \mathbb{R}^n$ *and* $u \in \mathbb{R}^m$.

$$\mathcal{K}^T = (\psi(x,u)^T)^{\dagger}\psi(\varphi^t(x,u),u)^T \qquad (11)$$

Where the least-squares pseudoinverse is indicated by the superscript "†". We use K+1 discrete state measurements with sampling time Ts to construct an approximate Koopman operator from a set of experimental data. We split the data into K "snapshot pairs"$\{(x_K, u_k); (y_k, u_k)\} \in \mathbb{R}^{(n\times m)\times 2}$ of the following form.

$$y_k = \phi^{T_s}(x_k, u_k) + \sigma_k \qquad (12)$$

σ_k represents the measurement noise. For our base of x and u monomials, we select those whose total degrees are equal to or less than w, implying $N = (n+m+w)!/(n+m)!\,w!$ [16, III, Section]. After that, we lift every pair of snapshots in accordance with (10) and combine them to create the ensuing K×N matrices:

$$\Psi_x = \begin{bmatrix} \psi(x_1,u_1)^T \\ \vdots \\ \psi(x_k,u_k)^T \end{bmatrix} \qquad \Psi_y = \begin{bmatrix} \psi(y_1,u_1)^T \\ \vdots \\ \psi(y_k,u_k)^T \end{bmatrix} \qquad (13)$$

Based on (14), $\mathcal{K}^{\mathrm{Ts}}$ is chosen to offer the least squares optimal match to all of the observed data, as supplied by

$$\bar{\mathcal{K}}^{T_s} = \Psi_x{}^{\dagger}\Psi_y \qquad (14)$$

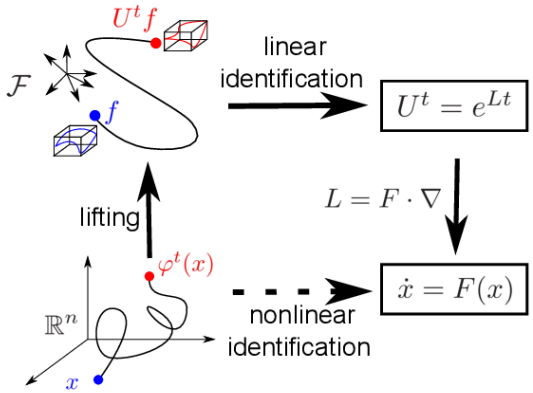

Figure 2. The identification of a classical nonlinear system takes place directly in the state space. On the other hand, the suggested method for identifying the Koopman operator involves lifting the data and linearly identifying the Koopman operator.

3. Obtaining Vector Field

The nonlinear vector field is identified in the final stage of the Koopman-based system identification approach by using the one-to-one correspondence between the infinite and finite dimensions system representations. As said earlier, our goal is to find a $\bar{\mathcal{F}}$ that as accurately as possible characterises the behaviour of the vector field F in the sense of the $\mathcal{L}_2$ -norm on the finite dimensional subspace $\bar{\mathcal{F}}$.

Equation (4) uses the Koopman operator's infinitesimal generator to connect the vector field to it. Using the approximation of the Koopman operator, we may invert (6) and get the infinitesimal generator $\bar{\mathcal{A}}$ of the set of Koopman operators $\mathcal{K}^{\mathrm{Ts}}$obtained in Section IV:

$$\bar{\mathcal{A}} = \frac{1}{T_s} log\mathcal{K}^{\mathrm{Ts}} \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}} \qquad (15)$$

Where log represents the logarithm of the primary matrix [22, Chapter 11]. Remember that when the number of data points is too small, $\mathcal{K}^{\mathrm{T}}$may have zero or negative eigenvalue and that the principal matrix logarithm is defined only for matrices whose eigenvalues all have non-negative real components

[12]. Therefore, if there are not enough data points, this strategy may not work. To fix the problem in this case, more system measurements might be made.

Once $\bar{\mathcal{A}}$ is known, $\bar{\mathcal{F}}$ can be found using (5). Regarding an observable $\not{f} \in \mathcal{F}$, consider $\mathcal{A}$ applied. According to equation (4), this is equivalent to the inner product of the vector field F and the gradient of $\not{f}$ with respect to x:

$$\mathcal{A}\not{f}(x,u) = \frac{\partial \not{f}(x,u)}{\partial x} F(x,u) \qquad (16)$$

As the projection of $\not{f}$ onto $\bar{F}$, Let $\alpha \in \mathbb{R}^N$be the vector representation of $\bar{\not{f}}$. The finite dimensional equivalent of (17) is therefore obtained from (11) by

$$(\bar{\mathcal{A}}\alpha)^T \psi(x,u) = \alpha^T \frac{\partial \psi(x,u)}{\partial x} \bar{F} \qquad (17)$$

We search for the vector field $\bar{F}$ for each batch of observed data such that (18) holds as much as is practical given the $\mathcal{L}_2$-norm. We choose the least-square solution to (18) across the set of all observed data points, which is $\{(x_k, u_k)|k = 1, \dots, K\}$ This solution is given by

$$\bar{F} = \begin{bmatrix} \frac{\partial \psi(x_1,u_1)}{\partial x} \\ \vdots \\ \frac{\partial \psi(x_k,u_k)}{\partial x} \end{bmatrix}^{\dagger} \begin{bmatrix} \bar{\mathcal{A}}^T & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \bar{\mathcal{A}}^T \end{bmatrix} \begin{bmatrix} \psi(x_1,u_1) \\ \vdots \\ \psi(x_k,u_k) \end{bmatrix} \qquad (18)$$

A more detailed discussion of this procedure can be found in [14], [15].

Koopman identification of TRAS

We used the approach described in Section 4 to a continuously nonlinear TRAS model in order to illustrate and assess its effectiveness, and we contrasted the outcome with that of many other nonlinear identification methods. This section includes a detailed description of the TRAS model, Experimental setup, and data Collection used in the system estimation process, and the method used to assess and compare performance between models.

1. Controlled Input

To produce a sample of the system's behaviour that is typical of the whole operating range, two multi sine signal was applied as input. To make the performance best, we reduced the correlation up to 95%. The one signal applied as a yaw input u_1 and the other applied as a pitch input u_2 in radian. The range of control-input into the model *u* was a set of -3.1415 - 3.1415 rad (-180 to 180 in degree) for yaw input and the control-input into the model *u* was a set of -1.22173 - 1.22173 rad (-70 to 70 in degree) for pitch input.

$u(t) = [u_1\ u_2]$ rad

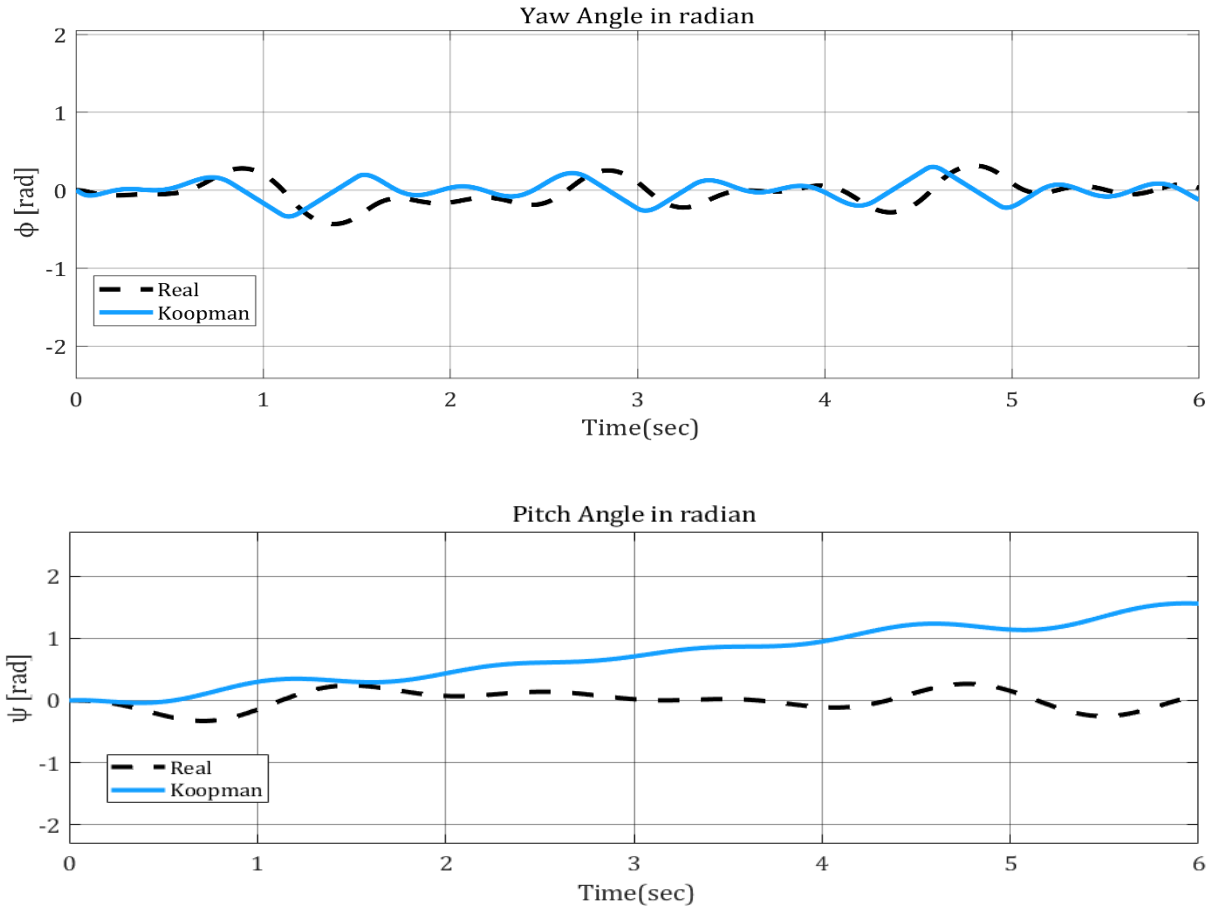

Figure 3. Under identical initial conditions and control inputs, the Koopman-based model's projected position (blue) is superimposed on the measured ϕ and ψ angles of the TRAS during a 6-second time window (black, dotted).

1. Data Collection

For data collection of TRAS, we consider the SIMULINK model of TRAS. The TRAS is highly nonlinear model, so I designed PID controller for TRAS to obtain the appropriate data for TRAS. We used two multi sine excited signal as input for data collection. The maximum frequency of u_1 and u_2 are 0.8 and 0.2 rad/cycle respectively while minimum frequency is 0 rad/cycle. The time period for data collection is 500 seconds, and the sampling time is 0.01 sec. As we have four states, so we collect "4 x 50001" data point for state, and used "2 x 50001" data point as a control input. We convert this data into "501" sample step and "100" trajectories.

2. Model Comparison

We derived a state-space model for the Twin Rotor Aerodynamics System (TRAS) utilizing the methodology outlined in section 4, which focuses on the identification of systems using the Koopman operator. This involved employing a "Gauss radial basis function (RBF)" of maximum degree w = 2 in conjunction with the training data gathered. Subsequently, we assessed the model's accuracy by conducting simulations and comparing them against each of the validation datasets, as illustrated in Figure 3.

III. RESULT AND DISCUSSION

Here, we present a comprehensive analysis of the results obtained from the identification of TRAS using the Koopman Operator, focusing on the error-to-norm ratio, best-fit ratio, and normalized square error metrics for evaluation.

We first examined the error-to-norm ratio, which provides a measure of the discrepancy between model predictions and observed data normalized by the norm of the observed data [15].

The following formula defines the error to norm ratio.

$$Enr = \frac{||x-\hat{x}||_2}{||x||_2} \quad (19)$$

The $\mathcal{L}_2$- norm is represented by $||x||_2$, and the states of the system's full-order and approximation Koopman models are indicated by x and $\hat{x}$. The performance of the estimated Koopman model is better the smaller the value of Enr. Our findings indicate that the error-to-norm ratio remains low across various operating conditions, suggesting that the Koopman Operator framework effectively captures the system's dynamics.

Furthermore, we assessed the best-fit ratio, which quantifies the similarity between model simulations and validation datasets. According to [20], the optimal best fit ratio is as follows:

$$\%\boldsymbol{BFR} = 100\% \times \boldsymbol{max}\{\frac{||x-\hat{x}||_2}{||x-\bar{x}||_2}, \mathbf{0}\} \quad (20)$$

The states of the actual model are denoted by x, the state of approximated model are represented by $\hat{\boldsymbol{x}}$, and the real model's mean is represented by $\bar{\boldsymbol{x}}$. If the percent best fit ratio is largest and the error to norm ratio is smallest, the model approximation is good; if the opposite is true, the approximation is bad and the approximation error is big. Our analysis demonstrates that the models derived from the Koopman Operator exhibit a high best-fit ratio, indicating a close match between simulated and observed data.

Additionally, we evaluated the normalized square error to provide a comprehensive assessment of model performance. The normalized square error metric accounts for both the magnitude and direction of errors, offering a nuanced understanding of the discrepancies between model predictions and actual observations. NRMSE values close to zero indicate a good fit between the model and the observed data, while higher values suggest larger discrepancies between them.

Table 1. Comparison of estimated TRAS

Model	Error to norm ratio (Enr)		% BFR Best fit Ratio (%)
	ϕ	ψ	
Koopman	0.933	1.554	93.052
Polynomial KF	0.971	0.0721	96.94
Laplacian KF	7.225	8.1435	91.09
Cauchy KF	7.745	8.362	89.72
Affine KF	31.23	13.514	87.19

The formula for Normalized Root Mean Square Error (NRMSE) is as follows:

$$\%\, NRMSE = \frac{RMSE}{y_{max}-y_{min}} \times 100\% \quad (21)$$

Where

$$RMSE = \sqrt{\frac{\sum_{k=1}^{N_{total}}(x_k - \hat{x}_k)^2}{N_{total}}}$$

The yaw angle NRMSE is determined to be 4.0589%, while the pitch angle NRMSE is slightly higher at 5.5889%. These metrics serve as crucial indicators of the system's ability to accurately predict and control its orientation. Our results indicate that the normalized square error remains consistently low, highlighting the robustness of the Koopman Operator-based models in representing the TRAS dynamics accurately.

In conclusion, our study demonstrates the efficacy of the Koopman Operator framework for the identification of nonlinear Twin Rotor Aerodynamic Systems. Through comprehensive analysis using error-to-norm ratio, best-fit ratio, and normalized square error metrics, we have shown that the models derived from the Koopman Operator offer accurate representations of TRAS dynamics across various operating conditions.

IV. CONCLUSION

We have effectively utilized Koopman operator theory to identify the Twin Rotor Aerodynamic System (TRAS) nonlinearly, exhibiting superior performance compared to conventional linear models. Unlike other approaches, the Koopman model was excellent at capturing the nonlinear dynamics of TRAS and could start simulations from real system conditions without requiring iterative adjustment. Computational difficulties arise when using the Koopman method to higher-dimensional systems. However, this problem might be mitigated by using past system information, for example, by choosing an appropriate foundation for observables. Subsequent research endeavors will center on surmounting these obstacles and broadening the Koopman methodology to encompass higher-dimensional TRAS models, non-polynomial models, and models including external factors. This demonstrates how Koopman operator theory can be used to create precise nonlinear dynamical models for TRAS, which will help with customized control plans for TRAS.

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