Uluslararası İleri Doğa Bilimleri ve Mühendislik Araştırmaları Dergisi
Sayı 8, S. 279-286, 4, 2024

Araştırma Makalesi

https://as-proceeding.com/index.php/ijanser
ISSN: 2980-0811

International Journal of Advanced Natural Sciences and Engineering Researches
Volume 8, pp. 279-286, 4, 2024

Research Article

The missing mathematics in high school. Theorems, proofs and geometric constructions.

Zenel Sina[1], Robert Kosova[2*], Arben Reka[2], Aurela Biçaku[2], Arben Starova[3]

[1]*Department of Pedagogy. University "A. Moisiu" Durres. Albania.*
[2]*Department of Mathematics. University "A. Moisiu" Durres. Albania.*
[3]*High Schol "Kristaq Rama" Tirana. Albania.*

[*]robertkosova@uamd.edu.al

(Received:15 May 2024, Accepted: 25 May 2024)

(3rd International Conference on Engineering, Natural and Social Sciences ICENSOS 2024, May 16-17, 2024)

ATIF/REFERENCE: Sina, Z., Kosova, R, Reka, A., Biçaku, A & Starova, A. (2024). The missing mathematics in high school. Theorems, proofs and geometric constructions. *International Journal of Advanced Natural Sciences and Engineering Researches*, 8(4), 279-286.

Abstract **–** In the last 30 years, several reforms have been implemented in the Albanian education system. They certainly aimed to improve the education system, especially pre-university education, by improving the school curriculum, teaching and learning methods, infrastructure, teacher training, and the student assessment system. Many curricula and textbooks have been revised, changed, and added to meet the challenges of today's society. The advancement of technology has made it possible to create new materials and interactive teaching methods for a more interesting and attractive school. However, at the end of the day, beyond the good intentions, discussions, and promises, what matters is what the students have learned and what they know. What happens to high school students when they reach their first year of university? What we are witnessing today is that, for quite some time, there has been a growing gap between the knowledge obtained in high school and the knowledge that students should have in their first year at Albanian or European universities. In this article, we will discuss some of the most important elements of mathematics, theorems, and proofs that do not exist anymore in our high school curricula, and in particular the geometry constructions, as well as the impact on students' achievement and results in university.

Keywords –Curricula, High School, Theorems, Proofs, Geometry Construction

I. Introduction

The fall of communist regimes across Eastern Europe in the late twentieth century started a period of transition and reforms, and Albania was no exception. The Albanian education system, once characterized by ideological indoctrination and centralization, underwent significant transformations following the fall of communism in 1990. The education system, which had long been controlled by the state and used as a tool for ideological indoctrination, faced urgent calls for reform to adapt to the new democratic realities. Initial reforms focused on decentralization, curriculum revision, and democratization of educational institutions [1].

The post-communist period saw efforts to expand access to education and promote equity, particularly in rural and underserved areas. Initiatives such as school construction projects, the provision of scholarships, and targeted interventions aim to reduce disparities in access and improve educational outcomes for marginalized groups, including girls, children from low-income families, and ethnic minorities [2].

Curriculum Revisions: Curriculum revisions in Albania typically involve a comprehensive review and update of the national curriculum across all levels of education, from primary to secondary [3]. Changes in the school curriculum in Albania. These revisions are aimed at aligning educational objectives with contemporary needs, improving the quality of education, and ensuring that students acquire the knowledge, skills, and competencies necessary for success in the 21st century [4].

Review of Learning Objectives: Curriculum revisions often begin with a review of existing learning objectives to ensure they are relevant, achievable, and aligned with national educational priorities and international standards. This process may involve input from various stakeholders, including educators, policymakers, and experts in the respective fields [5].

Update of Content and Pedagogy: Curriculum revisions involve updating content and pedagogical approaches to reflect advances in knowledge, technology, and teaching methodologies. This may include introducing new subjects or topics, revising the sequence of subjects, and incorporating innovative teaching strategies that promote active learning, inquiry-based instruction, and problem-solving skill development [6].

Emphasis on Competency-Based Learning: There is often a shift towards competency-based learning, where the focus is on students' ability to apply knowledge and skills in real-world contexts rather than memorization of facts. Curriculum revisions may emphasize the development of key competencies such as communication, collaboration, critical thinking, creativity, and adaptability [7].

Teacher Training and Professional Development: To support the implementation of revised curricula, there is a need for teacher training and professional development programs. Teachers may receive training on new subject content, teaching methodologies, assessment practices, and the use of instructional materials and resources, and together with students, they are encouraged to search for information outside the curricula, such as wikis, tutorials, etc. [8–9].

Assessment and Evaluation: Curriculum revisions also entail changes in assessment and evaluation practices to ensure alignment with revised learning objectives and instructional approaches. Assessment methods may include a mix of formative and summative assessments, performance tasks, project-based assessments, and standardized tests to measure students' progress and attainment of learning outcomes [10].

Pedagogical Approaches: Reforms might focus on shifting pedagogical approaches in math education. There could be a move towards more inquiry-based learning, problem-solving methodologies, and collaborative activities. Textbooks may be revised to include more real-world applications and interactive exercises to engage students actively in the learning process [11–12].

Integration of Technology: With the increasing role of technology in education, reforms may emphasize the integration of digital tools and resources in math instruction. Textbooks could be adapted to include online resources, interactive simulations, and multimedia elements to enhance learning experiences and cater to diverse learning styles [13–14].

Teacher Training: Education reforms often include provisions for teacher training and professional development. In the context of math education, teachers may receive training on new teaching methodologies, instructional strategies, and the effective use of updated textbooks and resources [14–15].

Assessment and Evaluation: Reforms may also entail changes in assessment practices to ensure alignment with revised curricula and instructional goals. Textbooks might include assessment tasks and exercises designed to measure students' mastery of key concepts and skills in line with updated standards and objectives; they should also include the problems that mathematicians are struggling to solve today to encourage the students to be part of the marvelous world of discovery [16].

II. LITERATURE REVIEW

Math textbooks have changed several times following the implementation of reforms in Albania. Until the 1990s, mathematics texts were translated from Russian and were similar to the eastern bloc. After the 1990s, there was great pressure to change everything that was inherited from communism. This made sense for history or philosophy books, but not for mathematics or natural science books. Anyway, all the books were changed, and the math books, too. Subsequently, implementing another reform, two kinds of high schools were created—social and natural, and two different programs and correspondent math textbooks were used [17]. This reform didn't seem to be very successful, maybe because high school students and parents were looking for easier mathematics. Later on, in the years 2009–2011, math-alternative texts were used in both programs: core math and advanced math.

Anyway, until 2008, the Albanian math textbooks were similar to the European textbooks in terms of curriculum, materials, and topics, such as in Germany, Italy, France, Greece, etc. [18].

The last reform replaced the previous textbooks with translated ones from the British education system. Compared with the previous math text books, they lack many important topics, materials, and, especially, theoretical math, definitions, theorems, and proofs.

The previous core mathematics text book for high school students has been considered among the best math books. The text book contained all the necessary topics, concepts, definitions, theories, and proofs with applications. The core math text book of the year 2017–2018 contained 39 definitions, 27 theorems, 182 solved exercises, and 18 hours dedicated to the Matura exam. On the other side, the textbook for 2020–2021, which is still in use today, has only 8 definitions, 2 theorems, 119 solved exercises, and only 8 hours dedicated to the Matura exam. The important topics such as continuous functions, derivates, applications of derivates, second-grade curves, integrals, probability, and statistics have been reduced significantly.

Geometry, which is an important subject in mathematics, does not exist anymore as it used to be 20–30 years ago. Geometry topics are presented in several chapters in math textbooks for grades 10–12. All parts of geometry, plane geometry, analytic geometry, and geometry in space are simplified, and it is all about formulas and calculations [19].

Concepts such as axioms and theorems are no longer present in high school textbooks, let alone proofs. Concepts of geometric figures, such as circles and triangles, as well as their properties, are given without definitions, theorems, or proofs, or too little of them. For example, Euclid's theorems are just presented with symbols or formulas, never with words, and when university students are asked to present or describe easy theorems in words, they never do it correctly because they have never been asked to do it before [20].

As far as space geometry is concerned, most of it has vanished, creating big problems for the students at universities, especially for those who study engineering programs, natural sciences, etc. [21]. What is left is just calculations of the areas or volumes of simple space pyramids, cones, and spheres.

III. MATERIALS AND METHOD

Constructive geometry is an important part of teaching mathematics in high school. It focuses on the construction and analysis of geometric figures using only simple construction tools such as a pencil and ruler (Fig. 1). Through this geometry, students develop skills to understand and apply concepts such as lines, triangles, squares, circles, and other figures. Constructive geometry is important because it can only be used when you have good theoretical foundations. Every construction is based on knowing its properties, such as the congruence of triangles, perpendicular segments, the properties of bisectors, etc.

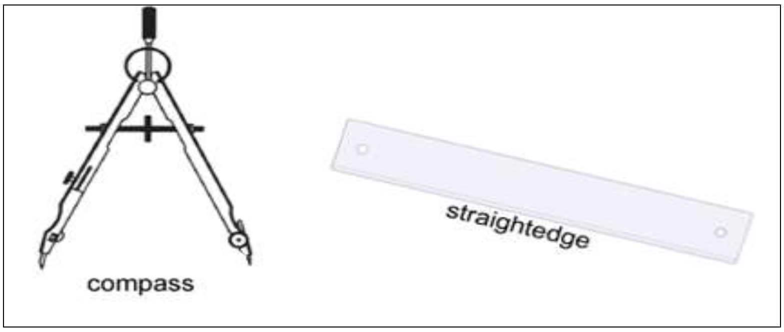

Figure 1. Compass and straightedge for geometric constructions.

Basic geometric constructions

Below are a few common examples that use the three rules of congruent triangles:

Rule 1: *SAS* (Side-Angle-Side)

If any two sides and the angle included between the sides of one triangle are equivalent to the corresponding two sides and the angle between the sides of the second triangle, then the two triangles are congruent by the SAS rule.

Rule 2. *ASA* (Angle-Side-Angle)

If any two angles and the side included between the angles of one triangle are equivalent to the corresponding two angles and the side included between the angles of the second triangle, then the two triangles are congruent by the ASA rule.

Rule 3. *SSS* (Side-Side-Side)

If all three sides of one triangle are equivalent to the corresponding three sides of the second triangle, then the two triangles are congruent by the SSS rule.

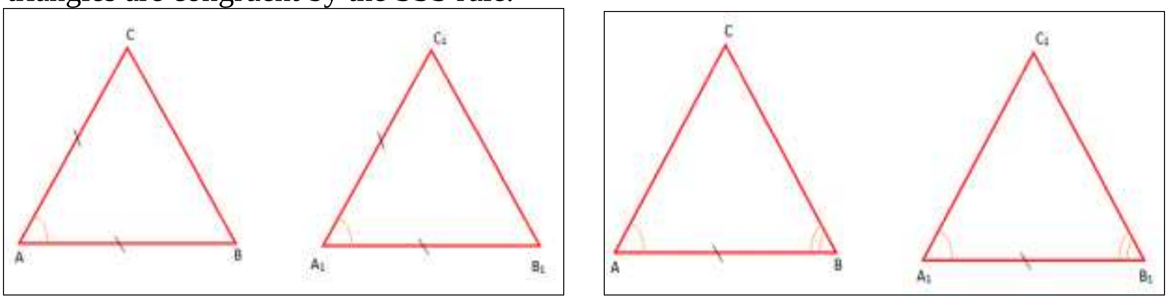

Figure 2. SAS and ASA rule of congruent triangles.

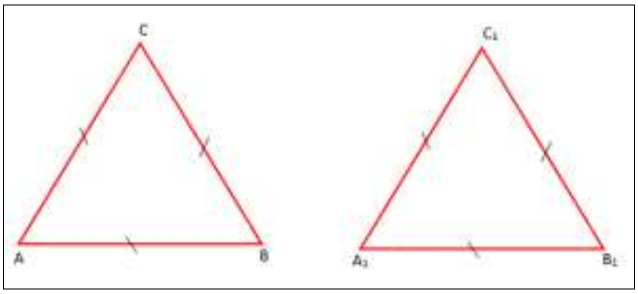

Figure. 3. SSS rule of congruent triangles.

IV. RESULTS

The geometric construction will work only if the students have complete knowledge of necessary math concepts, definitions and theorems.

Definition. Perpendicular bisector of the segment AB is the line which is perpendicular to AB at its middle point.

Definition. The angle bisector is the line, or segment which divides a given angle into two equal parts.

Properties of an Angle Bisector:
An angle bisector divides an angle into two equal parts.
Any point on the bisector of an angle is equidistant from the sides or arms of the angle.
In a triangle, it divides the opposite side into the ratio of the measure of the other two sides.

1. Midpoints and perpendicular bisectors

Construct a midpoint and a perpendicular bisector for line segment AB:

Place the compass point on point A and open the compass such that its width is greater than half the length of the line segment. Maintain the width of the compass and draw an arc above and below the line segment. The arc will need to intersect the arc drawn in the next step, so draw arc that are relatively long.
Maintain the compass width, place the compass point on B, and draw arc above and below the line segment such that they intersect the first arc at two points.
Draw a line through the intersections of the two arcs. Call the point of intersection of the line and line segment AB point C. Point C is the midpoint and the line is the perpendicular bisector of line segment AB.

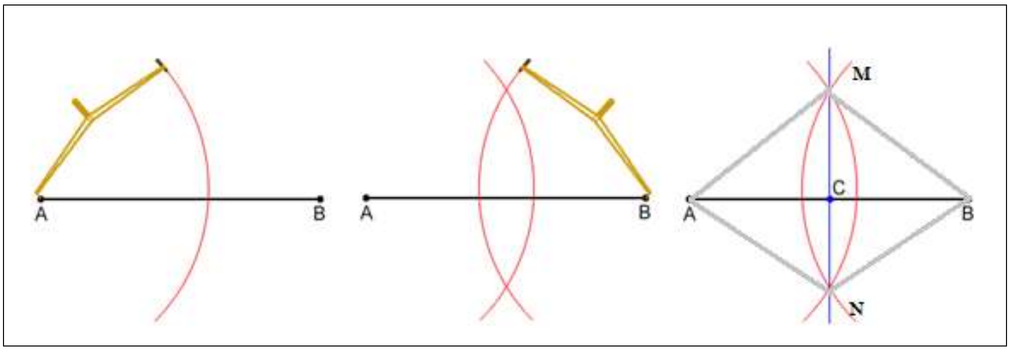

Figure 4. Construction of perpendicular bisector.

Theorem. Point C is the middle point of line segment AB and MC is perpendicular with AB (MC is perpendicular bisector of line segment AB).
Proof. Let's consider the tringles AMN and BMN.
AM=AN=BM=MB, the same setting of compass,
MN=MN, the same side segment
Then, by the third rule of congruent tringles, we have triangle AMN is congruent to triangle BMN.
As a result, all the angles are equals, too.
$\sphericalangle MAC = \measuredangle MBC, \sphericalangle AMC = \sphericalangle BMC.$
Comparing the triangles $\Delta\ ACM\ \equiv \Delta\ BCM$.
Sides $CM = CM,$
Sides $AM\ = CM,$
Angles $\sphericalangle AMC = \sphericalangle BMC,$
The first rule of congruence, SAS.
As a result, we have equal segments $AC\ = CB$, meaning that C is the midpoint of AB.
Also, MC is perpendicular to $AB; MC \perp AB$.

2. Bisecting an angle

Construct an angle bisector:

Place the compass point on vertex E and draw an arc through both ray ED and EF.
Place the compass point on the intersection of the arc (drawn in step 1) and ray EF, draw an arc in the interior of angle DEF. Maintain the width of the compass and repeat the process with the compass point instead on the intersection of the arc and ray ED, and making sure that two arcs drawn intersect.
Label the intersection of the two interior arcs G. Draw a ray from E through G. Ray EG should bisect angle DEF forming congruent angles DEG and FEG; $\sphericalangle DEG = \sphericalangle FEG$.

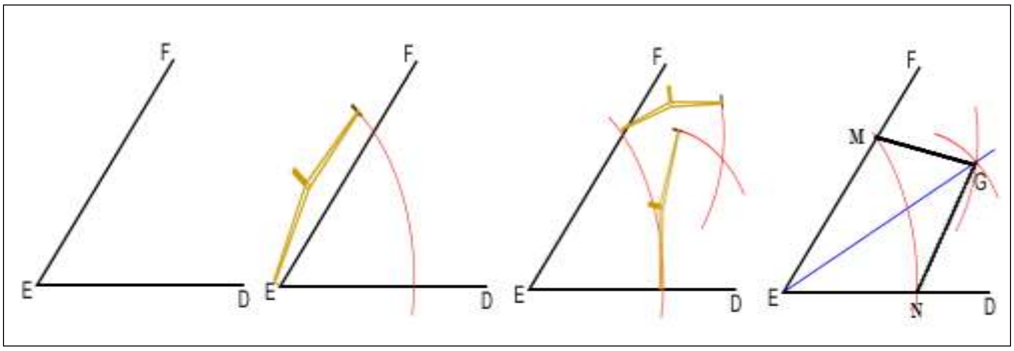

Figure 5. The construction of angle bisector.

3. Finding the center of a circle

We can find the center of a circle by constructing the intersection of the perpendicular bisectors of two chords of a circle.
For the circle given above, draw two chords with a straightedge. Make sure the two chords are not parallel to each other. Label the chords AB and CD.
Construct the perpendicular bisectors of both chords following the steps in the "midpoints and perpendicular bisectors" section above. Label the point of intersection of the two perpendicular bisectors O. Point O is the center of the circle.
Theorem. Point O is the center of circle.
Proof. From the geometric construction, we have $OA = OB = OC = OD$, meaning that point O is the center of the circle.

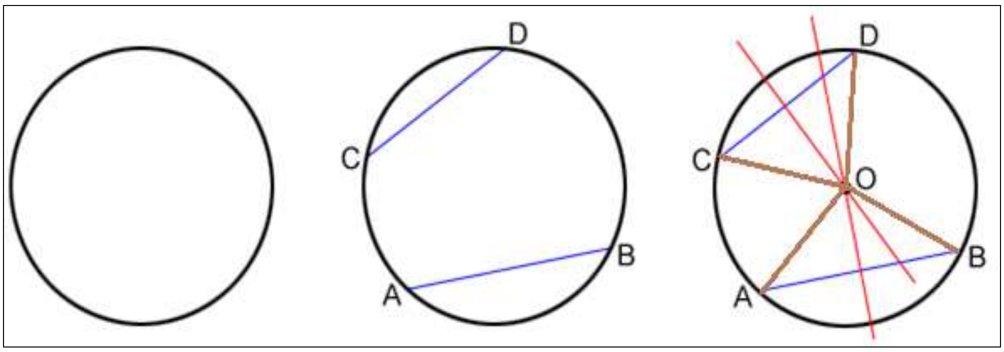

Figure 6. Finding the centre of circle.

4. Copying an Angle

Construct a copy of angle ABC using a compass and straightedge:
Draw ray FG.
Place the compass point on B and draw arc DE through rays AB and BC.
Maintaining the compass width, place the compass point on F and draw an arc through ray FG.
Label the intersections of arc DE and angle ABC (from step 2) P and Q. Place the compass point on P and draw an arc through Q. The radius of the arc is line segment PQ.

Again, keeping the compass width the same, place the compass point on the intersection of the arc, drawn in step 3, and ray FG, then draw an arc above ray FG such that it intersects the other arc.
Draw a ray from F through the intersection of the two arcs. Label the head of the ray H. The measure of angle GFH should be equal to the measure of angle ABC.

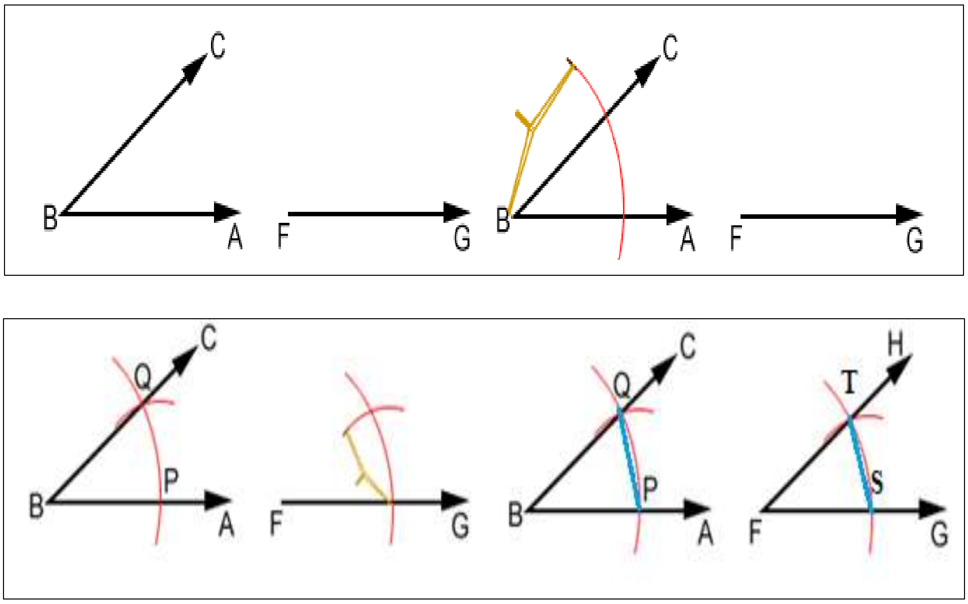

Figure 7. Copying the angle.

Theorem. The angles $\sphericalangle PBQ = \sphericalangle SFT$
Proof. Sides $BP = BQ = FT = FS$, the same setting of compass,
Sides $PQ = TS$, the same setting of compass.
Triangles are congruent, $\Delta BPQ \equiv \Delta FST$, the third rule of congruent triangles, SSS.
As a result, the angles are equal, $\sphericalangle B = \sphericalangle F$.

V. Discussion and Conclusion

Albania's education system, particularly at the preuniversity level, urgently needs to reintroduce the majority of fundamental theoretical math concepts, theorems, proofs, and definitions that were included in curricula many years ago.
Needless to say, theoretical mathematics, definitions, theorems, and proofs, among which the usage of geometric constructions, are vital and very important for the preuniversity education system.
The study of theorems and proofs helps students improve their logical reasoning skills. Understanding the logical framework of mathematical arguments allows students to evaluate problems, create hypotheses, and construct rigorous proofs.
Theorems and proofs help people understand mathematical concepts more deeply. Instead of memorizing formulas and algorithms, students learn the basic principles that govern mathematical events. This conceptual understanding not only enhances problem-solving abilities, but it also fosters mathematical intuition and fluency.
Among many valuable math tools and strategies, geometric construction allows students to explore mathematical concepts directly. Students learn about the properties, relationships, and symmetries inherent in shapes and structures by drawing geometric figures with a compass and straightedge.
The study of theorems, proofs, and geometric construction encourages creativity and imaginative problem-solving. When faced with difficult issues and conjectures, students are urged to think outside the box, devise new methodologies, and seek inventive solutions. This innovative problem-solving method is relevant not only to mathematics but also to other subjects, fostering an environment of creation and

originality.

Advanced mathematics courses, as well as related subjects such as science, engineering, and computer science, require a solid understanding of theoretical mathematics. Mastery of these fundamental ideas provides students with the analytical skills and mathematical fluency required to flourish in higher-level studies and pursue jobs that require strong numeric reasoning capabilities.

The inclusion of theorems, proofs, and geometric construction in high school math education is crucial for bridging the gap between high school and university requirements. These elements enrich the mathematical learning experience and empower students to become confident, proficient mathematicians.

REFERENCES

[1] Mernacaj, M. (2011). *The reforms in mathematics education for grades 1 through 12 in Albania from 1945 to 2000*. Columbia University.

[2] Lutaj, L., & Duka, A. (2023). Students-Teachers of the Future, Citizenship, and Inclusive Rights in Education. *Interdisciplinary Journal of Research and Development*, *10*(2), 45-45.

[3] Haxhihyseni, S. (2013). Using Literature through the Curriculum, as a Way to Increase Children's Potential. *Academic Journal of Interdisciplinary Studies*, *2*(4), 113-120.

[4] Kosova, R., Bushi, F., Kapçiu, R., Cullhaj, F., & Kosova, A. M. (2024). An analyses of Albanian pre-university education system, and the missing surveys.

[5] Tase, M., & Aliaj, V. SOME TIPS FOR DEVELOPING AN INTEGRATED CURRICULUM. CASE STUDY ALBANIA. In *International Marmara Social Sciences Congress (Spring 2020)* (p. 161).

[6] Shara, J., & Gjirokaster, A. (2021). The Automation of Mathematical Reasoning. *Journal of Research in Applied Mathematics*, *10*(2), 122-131.

[7] Papajani, A. (2021). Curriculum reform in pre-university education in Albania. *Polis*, (20), 7-19.

[8] Sula, G., Haxhihyseni, S., & Noti, K. (2021). Wikis as a tool for co-constructed learning in higher education–An exploratory study in an Albanian higher education. *International Journal of Emerging Technologies in Learning (IJET)*, *16*(24), 191-204.

[9] Sina, Z. (2013). Effects of Application of Independent Research-Based Assignments on Students' Results.

[10] Tartari, E., & Lutaj, L. (2021, June). The Impact of Pedagogical Discourse on the Performance of the Teacher's Work. In *Pearson Journal International Conference on Social Sciences & Humanities* (pp. 131-137).

[11] Kaiser, G. (2020). Mathematical modelling and applications in education. *Encyclopedia of mathematics education*, 553-561.

[12] Bartolini, M. G., & Martignone, F. (2020). Manipulatives in mathematics education. *Encyclopedia of mathematics education*, 487-494.

[13] Hajrulla, S., Demir, T., Bezati, L., & Kosova, R. (2023). The impact of constructive learning applied to the teaching of numerical methods. *CONSTRUCTIVE MATHEMATICS: FOUNDATION AND PRACTICE*, 39.

[14] Kosova, R., Bushi, F., Kapçiu, R., Cullhaj, F., & Kosova, A. M. (2024). A review of primarily tests and algorithms: Engaging students to code for mathematics.

[15] Kosova, A. G. R. The Performance of University Students and High School Factors. Statistical Analyses And ANCOVA.

[16] Kosova, R., Kapçiu, R., Hajrulla, S., & Kosova, A. M. (2023). A Revıew of Mathematical Conjectures: Exploring Engaging Topics for University Mathematics Students. *International Journal of Advanced Natural Sciences and Engineering Researches (IJANSER)*, *7*(11), 180–186. https://doi.org/10.59287/as-ijanser.581

[17] Gjoci, B. (2017). *Analyzing Mathematics High School State Examinations in Albania in the 1970s and 2006-2015: Two Decades, Two Historical Periods*. Columbia University.

[18] Islam Alikaj/ Tekstet e matematikës për gjimnazin dhe provimet e maturës shtetërore. Gazeta Telegram. Retreaved from https://telegraf.al/arsim/islam-alikaj-tekstet-e-matematikes-per-gjimnazin-dhe-provimet-e-matures-shteterore/. Accessed on July 20, 2014.

[19] Stylianides, A. J., & Stylianides, G. J. (2022). Introducing students and prospective teachers to the notion of proof in mathematics. *The Journal of Mathematical Behavior*, *66*, 100957.

[20] Seah, R., & Horne, M. (2020). The construction and validation of a geometric reasoning test item to support the development of learning progression. *Mathematics Education Research Journal*, *32*(4), 607-628.

[21] Sıla, E., & Lıftaj, S. (2023). An Overview of Geometry Curriculum and Its Understanding by Albanian Students. *The Eurasia Proceedings of Educational and Social Sciences*, *33*, 56-62.