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Research Article

Make Science Visible through Numerical Methods in Computer Graphics. Water Wave Equation through Graphical Bezier Solution

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Abstract – This article shows how computer graphics can be used to visualize science concepts and operationalize inquiry practices in engineering design to support integrated learning and teaching of science and engineering. The main results of the study demonstrate the importance of numerical methods in computer graphics, particularly in terms of their ability to generate smooth curves, detailed surfaces, and realistic transformations. This article provides real-world examples in the field of sustainable energy engineering based on open-source design and analysis Web app. Simulations were performed to showcase the capabilities of numerical methods. The graphics results obtained highlighted the strengths and limitations of each technique, providing valuable insights for practitioners and researchers. Additionally, the article discusses code simulations for B-spline curves, surface subdivision, and interpolation, providing practical examples for implementation.

The findings of this study can be useful for researchers, practitioners, and students seeking to develop their understanding and implementation of numerical methods in computer graphics. Based on these graphical capabilities, generative design driven by evolutionary computation can also be visually illustrated to give students a glimpse into how artificial intelligence is transforming engineering design.

Keywords –Concepts, numerical methods, Numerical Simulations, design tools, spline curves, Visualization

I. INTRODUCTION

Curve fitting techniques, such as Bezier curves and B-spline curves, provide flexible shape control and local modifications. Surface fitting techniques, including surface subdivision and interpolation, refine meshes and fit smooth surfaces through control points or samples. 3D transformations using numerical methods enable object manipulation, camera movement, and scene composition in computer graphics. Overall, this article provides insights into the practical applications of numerical methods in computer graphics and their impact on various industries, including video games, animation, and film. Numerical methods play a crucial role in computer graphics, allowing for the creation of realistic and complex visual images (1). This article explores the application of numerical methods in computer graphics, focusing on curve fitting, surface fitting, and 3D transformations [31]. The research methodology involves a review of relevant literature and numerical simulations using software tools such as OpenGL, MATLAB, and Python. Data is collected from various sources, including academic articles, textbooks, and online resources. Numerical methods are essential in computer graphics, enabling the creation of visually stunning digital imagery. This study focuses on curve fitting techniques, surface fitting techniques, and 3D transformations. It also presents evidence of learning from pilot tests at culturally diverse high schools. Science educators interested in incorporating engineering design into their lesson plans may find this article helpful.

Curve fitting techniques, such as Bezier curves and B-spline curves, generate smooth and flexible curves in computer graphics [17], [29]. Bezier curves use control points and mathematical principles like Bernstein polynomials and de Castelão's algorithm. B-spline curves provide local control and find applications in CAD, animation, and freeform surface modeling [22].

Surface fitting techniques, including surface subdivision and interpolation, create complex 3D surfaces [3]. Surface subdivision refines meshes for smoother surfaces, while interpolation fits surfaces through control points or samples for applications like 3D modeling and medical imaging [4]. We analyse 3D transformations like translation, rotation, and scaling manipulate objects in 3D space. These transformations utilize numerical methods and matrices for precise control and dynamic manipulation.[1]

This research explores the practical applications and impact of numerical methods on visual quality, realism, and interactivity. Simulations and graphics result comparisons will provide insights into strengths, limitations, and suitability for various applications.

By delving into theory, algorithms, and techniques, this study contributes to the understanding and advancement of numerical methods in computer graphics. One of the fundamental equations used in B-spline curve representation is the blending function equation, which calculates the influence of each control point on the curve. The equation for *k*-order B-spline speaks about with *n+1* control points.

II. Materials and Method

2.1 Literature Review

Curve fitting techniques are crucial in computer graphics for generating smooth and visually appealing curves that accurately represent desired shapes. These techniques involve finding a mathematical function or representation that best fits a set of given control points, allowing for precise control and manipulation of the curve.

2.2 Curve Fitting Techniques:

Bezier curves are widely used in computer graphics for smooth curve generation. They are defined by control points and employ mathematical principles such as Bernstein polynomials and de Casteljau's algorithm. The graph for a Bezier curve of degree 3 is given by:

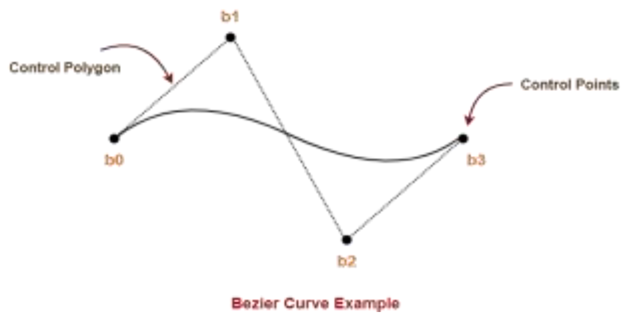


Fig. 1. Bezier curve of degree 3

- t is any parameter where $0 <= t <= 1$
- P(t) = Any point lying on the bezier curve
- B_i = i^{th} control point of the bezier curve
- n = degree of the curve
- $J_{n,i}(t)$ = Blending function

$$C(n,i)t^i(1-t)^{n-i} \text{ where } C(n,i) = n! / i!(n-i)! \quad (2.1)$$

The de Casteljau's algorithm recursively evaluates the control points to determine the position of points on the Bezier curve. This algorithm provides flexibility in shape control and is extensively used in shape design, animation, and modeling [2].

The curve is completely contained in the convex hull of its control points.

So, the points can be graphically displayed & used to manipulate the curve intuitively [7].

2.3 B-Spline Techniques for spline curves:

Given n + 1 control points P_0, P_1, ..., Pn and a knot vector U = { u0, u1, ..., um }, the B-spline curve of degree p defined by these control points and knot vector U is

$$\mathbf{C}(u) = \sum_{i=0}^{n} N_{i,p}(u)\mathbf{P}_i \quad (2.2)$$

where Ni,p(u)'s are B-spline basis functions of degree p. The form of a B-spline curve is very similar to that of a Bezier curve. Unlike a Bezier curve, a B-spline curve involves more information, namely: a set of n+1 control points, a knot vector [17, 18, 19, 20] of m+1 knots, and a degree p. Note that n, m and p must satisfy m = n + p + 1. More precisely, if we want to define a B-spline curve of degree *p* with *n + 1* control points, we have to supply *n + p + 2* knots *u0, u1, ..., un+p+1*. The degree of a B-spline basis function is an input, while the degree of a Bezier basis function depends on the number of control points [25]. To change the shape of a B-spline curve, one can modify one or more of these control parameters: the positions of control points, the positions of knots, and the degree of the curve. If the knot vector does not have any particular structure, the generated curve will not touch the first and last legs of the control polyline it is called *OPEN*.

We may want to clamp the curve so that it is tangent to the first and the last legs at the first and last control points, respectively, as a Bezier curve does. To do so, the first knot and the last knot must be of multiplicity p+1. This is called *CLAMPED*, [2] [8].

If we repeat the process we get a *CLOSED B-SPLINE*.

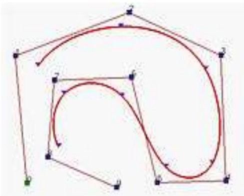
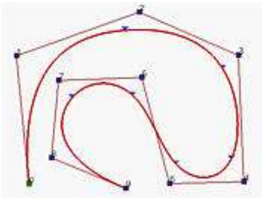
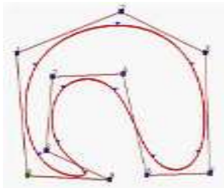
OPEN:  CLAMPED:  CLOSED: 

Fig. 2. B spline

III. Results

Surface Fitting Techniques

3.1 *Surface Subdivision*

Surface subdivision is a surface fitting technique that iteratively refines a mesh representation, resulting in smoother and more detailed surfaces. The basic idea behind surface subdivision is to divide each polygonal face into smaller sub-faces and adjust the positions of the vertices based on certain rules or algorithms. This process is repeated iteratively until the desired level of smoothness and detail is achieved [5].

Surface subdivision techniques, such as the Loop subdivision algorithm or the Catmull-Clark subdivision algorithm, are commonly used in 3D modeling, sculpting, terrain modeling, and medical imaging. These techniques provide precise control over the shape and detail of surfaces, enabling the creation of realistic and visually appealing 3D objects [3]. Quadrilateral based meshes generally use Catmull-Clark, while triangular based meshes generally use loop subdivision.

3.2 *Surface Loop subdivision:*

Loop subdivision, do not involve explicit equations but operate based on vertex and face subdivision rules, The algorithm begins by calculating new positions for each vertex in the mesh based on a weighted average of its neighboring vertices. These weights are determined by the topology of the mesh and ensure that the new positions preserve the overall shape of the surface [5]. After updating the vertex positions, the algorithm creates new edges and faces by connecting the newly generated vertices. This step increases the level of detail in the mesh and improves the smoothness of the surface. The loop subdivision algorithm is recursive, meaning that it can be applied multiple times to achieve even higher levels of refinement. Each iteration further smooths the surface and adds more detail to the mesh. Loop subdivision has several advantages. It produces visually pleasing surfaces with smooth and continuous curvature. It also maintains the overall shape of the original mesh while adding additional geometric detail. Additionally, it is relatively efficient in terms of computational complexity compared to other subdivision techniques [3].

Catmull-Clark is an algorithm that maps from a surface (described as a set of points and a set of polygons with vertices at those points) to another more refined surface. The resulting surface will always consist of a mesh of quadrilaterals.
The process for computing the new locations of the points works as follows when the surface is free of holes. For each face, a face point is created which is the average of all the points of the face. For each edge, an edge point is created which is the average between the center of the edge and the center of the segment made with the face points of the two adjacent faces. For each vertex point, its coordinates are updated [10].

3.3 Interpolation

Interpolation is another surface fitting technique used in computer graphics. It involves fitting a smooth surface through given control points or samples. The goal is to find a surface that passes through the given points while maintaining smoothness and continuity.

Various interpolation methods can be employed, such as polynomial interpolation, spline interpolation, or radial basis function interpolation. These techniques are used in applications like 3D modeling, image reconstruction, and medical imaging to create smooth and continuous surfaces based on available data points [3]. 3D transformations are fundamental operations in computer graphics that allow for the manipulation and positioning of objects in 3D space. The three main types of 3D transformations are translation, rotation, and scaling.

Translation involves shifting objects along the x, y, and z axes. Rotation changes the orientation of objects around a specified axis or point. Scaling adjusts the size of objects, either uniformly or along specific axes

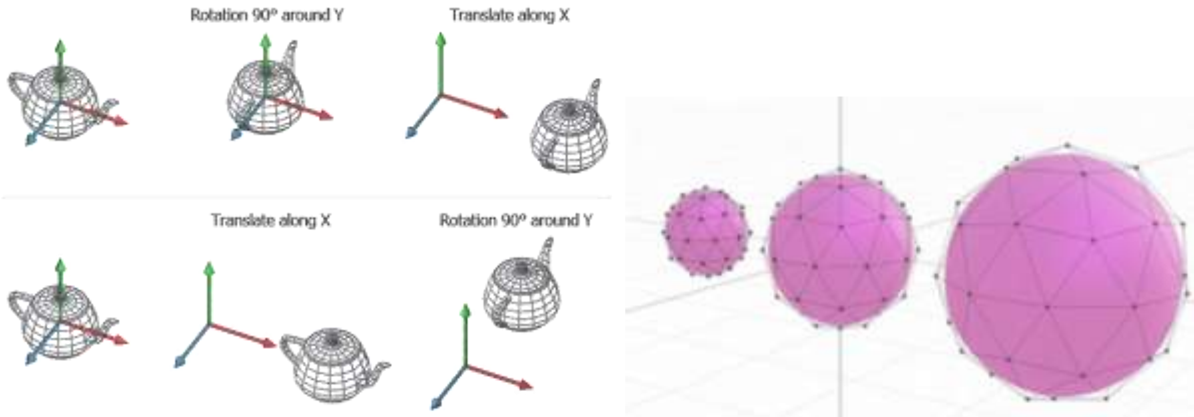


Fig. 3. Visual quality of computer-generated scenes

These transformations are represented using transformation matrices and are crucial for object manipulation, camera movement, and scene composition in computer graphics. By applying appropriate transformation matrices, practitioners can achieve dynamic and interactive 3D graphics, enhancing the realism and visual quality of computer-generated scenes [4].

3.4 Simulations

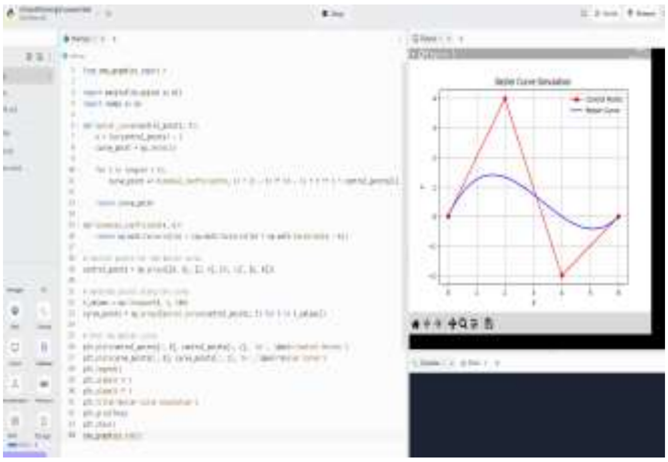


Fig. 4. Beizer simulation and graph-generated scene 1

In order to validate the effectiveness and practicality of the numerical methods discussed, simulations were conducted using software tools such Python and online compiler Replit. These simulations involved generating and manipulating various graphical elements.

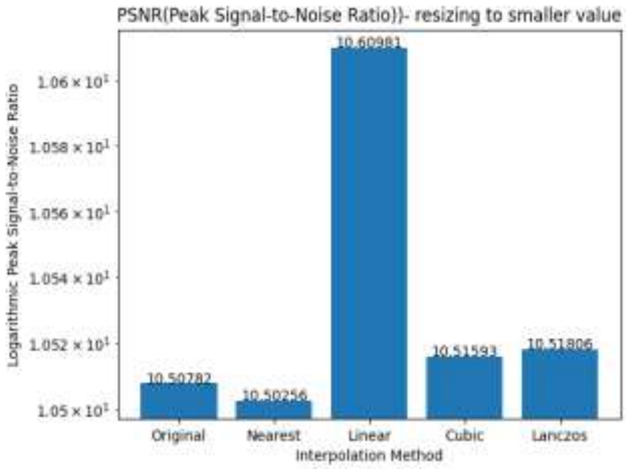


Fig. 5. Beizer interpolation scene 1

Fig. 6. Beizer simulation and graph-generated scene 2

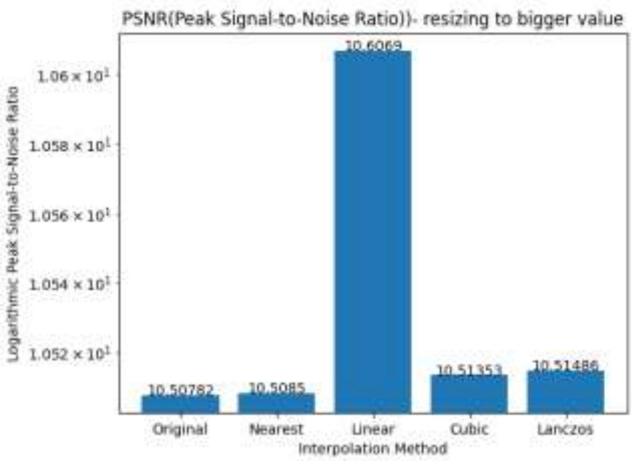


Fig. 7. Beizer interpolation scene 2

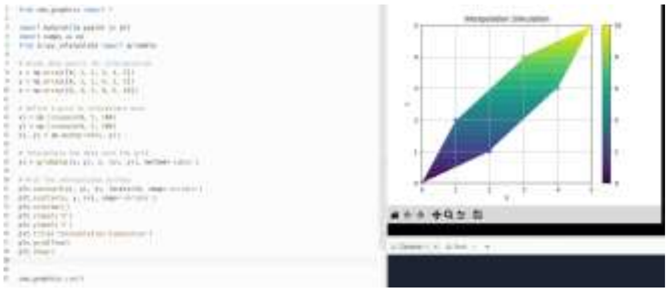


Fig. 8. Beizer simulation and graph-generated scene 3

IV. DISCUSSION

Based on the analysis of Mean Squared Error (MSE) and Peak Signal-to-Noise Ratio (PSNR) metrics for image resizing using different interpolation methods, we can draw the following conclusions:

4.1 Resizing Down: When comparing the resized images to the original ones after downsizing, the interpolation methods exhibit distinct characteristics:

Nearest Neighbor interpolation performs the poorest, as it introduces noticeable differences and reduces the similarity between the resized and original images.

Linear interpolation demonstrates the most significant change in similarity, resulting in resized images that are more similar to the original images compared to other methods. This indicates that linear interpolation tends to preserve the overall structure and visual fidelity during the downsizing process.

Cubic and Lanczos interpolation methods show similar performance, as they produce resized images that are relatively close to the original images. However, considering their computational complexity, the similarity improvement achieved by these methods may not justify the additional resources required.

4.2 Resizing Up

When comparing the resized images to the original ones after upsizing, similar trends emerge as observed during the downsizing process:

Nearest Neighbor interpolation still demonstrates a lower level of similarity compared to the original images, although the difference is not as pronounced as during downsizing. This method tends to introduce blocky artifacts and limited smoothness during the upsizing process.

Linear interpolation continues to exhibit a significant improvement in similarity, making the resized images more similar to the originals. It maintains the overall structure and visual fidelity, enhancing the upsized images' quality.

V. CONCLUSION

In conclusion, based on both MSE and PSNR metrics, linear interpolation consistently performs well in terms of maintaining similarity between the resized and original images, both when downsizing and upsizing. Nearest Neighbor interpolation shows poorer results, while Cubic and Lanczos interpolation methods offer relatively good performance but may not justify their increased computational complexity. It is important to consider the specific requirements of the image resizing task and the trade-off between similarity and computational efficiency when selecting an interpolation method for image processing.

Alternatives to affirmative action, such as socioeconomic-based admissions or hiring practices, are being touted as potentially more fair methods of achieving diversity without overtly considering race or gender. Such alternatives, however, also come with their own set of challenges and implications, necessitating in-depth research and pilot programs to assess their viability.

Although some may argue that affirmative action can lead to reverse discrimination, the evidence suggests that it is an important and effective instrument for achieving greater equality. Affirmative action policies may not always be enough to help corporations and colleges reach their diversification and inclusion objectives.

In summarizing affirmative action, we reflect on a complex landscape characterized by divergent views and constant evolution. Through this blog post, we have explored the historical roots, the myriad arguments, and the real-world applications of affirmative action. Whether these policies stand as a remedy to past discrimination or a challenge to meritocratic principles depends largely on where one stands in the ongoing debate.

Cubic and Lanczos interpolation methods show similar performance, as they produce resized images that are relatively close to the original images. However, considering their computational complexity, the similarity improvement achieved by these methods may not justify the additional resources required. The discourse extends an invitation to engage in constructive dialogue, deepen understanding, and potentially influence the course of affirmative action in the future. The pursuit of true workplace diversity is far from a finished journey, but through continued evaluation and thoughtful conversation, progress can be made.

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