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Picture Fuzzy B-spline Curve Approximation Model

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Abstract *–*In this paper, the picture fuzzy set approach was applied to introduce the approximation of B-spline curve model. Firstly, picture fuzzy control point relation is described by using basic concepts of picture fuzzy set which is involved picture fuzzy number and picture fuzzy relation. The B-spline basis function is then combined with the picture fuzzy control point relation. Afterwards, using an approximation technique, fuzzy B-spline curves are created and displayed. In conclusion, a few numerical instances for creating the desired curve are displayed.

Keywords –B-Spline Curve, Approximation Method, Picture Fuzzy Control Point Relation, Picture Fuzzy B-Spline Curve.

I. INTRODUCTION

Model selection in determining the perfect or smooth curve is very important especially in CAGD. One of the famous models in modelling is B-spline. B-spline or basis spline are powerful mathematical tool used to create smooth and flexible curves that can represent complex shapes and data patterns. Many researchers use approximation B-spline curve in their study with different fields [1][2].

However, traditional B-spline curves assume precise and deterministic data, which is often not the case in real-world applications. Data can be imprecise, uncertain, or incomplete due to various factors such as measurement errors, noise or inherent variability in the phenomena being modelled. Zadeh in 1965 [3] developed the concept of the fuzzy set to address the limitations of classical set theory in dealing with uncertainty and imprecision. Classical set theory operates under the principle of binary logic where an element either belongs to a set or does not. This binary approach is often insufficient for real-world situations where boundaries are not always clear-cut, and things can be partially true or false. The fuzzy set concept allows for membership degree, meaning an element may only partially be a part of a set. to varying degrees between 0 and 1. However, there is a weakness in this fuzzy set concept where it cannot handle more complex and nuanced situations of uncertainty and vagueness. Coung and Kreinovich [4] introduced the picture fuzzy set (PFS) in 2013 as an expansion of the fuzzy set (FS) and intuitionistic fuzzy sets [6]. PFS is designed to handle more nuanced and complex situations of uncertainty, incorporating additional parameters to represent neutrality or hesitancy, along with degree of membership and non-membership. To further their investigation, Coung in 2014 [5] has included the membership degree of positive, neutral, and negative regarding a component that extends an IFS. After PFS was developed, it was thought to be a powerful mathematical tool that worked well in scenarios where human opinions involved a greater number of yes, abstain, no, and refuse responses. A few research have been conducted on the application of the picture fuzzy set idea to a decision-making problem and its generalizations in different disciplines. A geometric explanation of picture fuzzy sets given by Singh in 2015 [7], who also suggested correlation coefficients for picture fuzzy sets that examine the membership degree of positive, negative, neutral, and refuse. On the other hand, Wei in 2017 [8] published similarity measures between picture fuzzy sets by taking into consideration the degree of positive, neutral, negative, and refusal membership in picture fuzzy sets. In 2018 [9], Wei and Gao introduced a few new dice similarity metrics for picture fuzzy sets as well as the comprehensive dice similarity measures for picture fuzzy sets. In 2018 [10], Wei introduced a new method for calculating the degree of similarity between PFS. The author used these PFS similarity metrics for construction material and mineral field recognition. Consequently, a few applications utilizing PFS have been discussed by some academics in their study [11]-[14].

In modelling, data collection is very important to determine the shape of the curve and surface which is utilized as a control point of approximation and interpolation technique [15]. Uncertainty data has a profound impact on the shape and reliability of curves in modelling. However, approximation of B-spline curve is the main emphasis of this study. The B-spline curve is indeed non-global, meaning that they exhibit local control points over their shape. The mathematical representation of B-spline curves approximation was introduced by Carl [16]. The author's work laid the foundation for the theoretical understanding and practical application of B-spline.

To handle uncertainty data, Zadeh in 1965 [3] enhanced the fuzzy set theory with membership while in 1986 [4], Atanassov improved fuzzy set theory by incorporating true, false, and uncertainty degree membership. Many researchers use the FS and IFS approaches for geometric modeling in B-splines [17]-[22]. However, Rosli and Zulkifly [23][24], presented the neutrosophic B-spline curve by employing the interpolation method and neutrosophic bicubic B-spline surface interpolation. Meanwhile, Smarandache [25], has contributed significantly to picture fuzzy sets in various fields including geometric modelling. Additional researchers in the field have utilized PF sets in various scenarios like image processing, pattern recognition, and decision making. These researchers have explored and extended the picture fuzzy sets to enhance the accuracy and flexibility of geometric modeling and other computational techniques [26][27].

The objective in this paper is to focuses on the designing a geometric model to handle gathering uncertainty-related data which is focused on picture fuzzy B-spline curve approximation (PFBSC) model. Before constructing the PFBSC, the picture fuzzy control point needs to be established using PFS theories and their characteristics. PFBSC models are created utilizing these control points and the B-spline basis function. An approximation technique is then utilized to view the models. The structured of this paper is firstly, the background information is explained in Section 1. The relation of picture fuzzy control point (PFCPR) and relation of picture fuzzy point (PFPR) introduced in Section 2 . Then section 3 covered the technique for the PFBSC approximation using PFCPR. The PFBSC approximation is shown graphically and numerically in section 4. Lastly, the research will be concluded in section 5.

II. PICTURE FUZZY CONTROL POINT RELATION

The concept of relation is one of the most important concepts in modeling. Some preliminary picture fuzzy relation (PFR) results are shown [4][5]. Defining the PFCPR based on PFS is explained according to definition 1.

Definition 1: Suppose that P and Q are non-empty set of points and $P,Q,I \subseteq \mathbb{R}^3$ then PFCPR is stated as

$$\hat{S} = \left\{ \begin{array}{l} \left\langle \left(p_i,q_j\right), \mu_{\hat{S}}\left(p_i,q_j\right), \eta_{\hat{S}}\left(p_i,q_j\right), \nu_{\hat{S}}\left(p_i,q_j\right), \rho_{\hat{S}}\left(p_i,q_j\right) \right\rangle \mid \\ \left(p_i,q_j\right), \mu_{\hat{S}}\left(p_i,q_j\right), \eta_{\hat{S}}\left(p_i,q_j\right), \nu_{\hat{S}}\left(p_i,q_j\right), \rho_{\hat{S}}\left(p_i,q_j\right) \in I \end{array} \right\} \quad (1)$$

Where $\left(p_i,q_j\right)$ are point in ordered pair and $\left(p_i,q_j\right)\in P\times Q$. As though $\mu_{\hat{S}}\left(p_i,q_j\right)$, $\eta_{\hat{S}}\left(p_i,q_j\right)$, $v_{\hat{S}}\left(p_i,q_j\right)$ and $\rho_{\hat{S}}\left(p_i,q_j\right)$ represent the ordered pair of points membership for positive, neutral, negative and refusal in $\left[0,1\right]\in I$. The refusal degree is demonstrated by

$$\rho_{\hat{S}}\left(x\right)=1-\left[\mu_{\hat{S}}\left(p_i,q_j\right)+\eta_{\hat{S}}\left(p_i,q_j\right)+v_{\hat{S}}\left(p_i,q_j\right)\right] \tag{2}$$

and $0\le\mu_{\hat{S}}\left(p_i,q_j\right)+\eta_{\hat{S}}\left(p_i,q_j\right)+v_{\hat{S}}\left(p_i,q_j\right)\le 1$ condition is permitted.

Definition 2: [4] In relation to a fixed $x\in\hat{A}$, $\left(\mu_{\hat{A}}\left(x\right),\eta_{\hat{A}}\left(x\right),v_{\hat{A}}\left(x\right),\rho_{\hat{A}}\left(x\right)\right)$ is call picture fuzzy number (PFN), where

$$\mu_{\hat{A}}\left(x\right)\in\left[0,1\right],\eta_{\hat{A}}\left(x\right)\in\left[0,1\right],v_{\hat{A}}\left(x\right)\in\left[0,1\right]\ \rho_{\hat{A}}\left(x\right)\in\left[0,1\right]$$

and

$$\mu_{\hat{A}}\left(x\right)+\eta_{\hat{A}}\left(x\right)+v_{\hat{A}}\left(x\right)+\rho_{\hat{A}}\left(x\right)=1 \tag{3}$$

PFN is simply represented by $\left(\mu_{\hat{A}}\left(x\right),\eta_{\hat{A}}\left(x\right),v_{\hat{A}}\left(x\right)\right)$.

Definition 3: [4] Let X , Y and Z the usual non-empty sets. A relation of picture fuzzy (PFR), $\hat{R}$ is a subset of picture fuzzy, $X\times Y$ given by

$$\hat{R}=\left\{\left(\left(x,y\right),\mu_{\hat{R}}\left(x,y\right),\eta_{\hat{R}}\left(x,y\right),v_{\hat{R}}\left(x,y\right)\right):x\in X,y\in Y\right\} \tag{4}$$

for $\mu_{\hat{R}}:X\times Y\to\left[0,1\right]$, $\eta_{\hat{R}}:X\times Y\to\left[0,1\right]$ and $v_{\hat{R}}:X\times Y\to\left[0,1\right]$ fulfilled the requirement

$$0\le\mu_{\hat{R}}\left(x,y\right)+\eta_{\hat{R}}\left(x,y\right)+v_{\hat{R}}\left(x,y\right)\le 1 \text{ for each } \left(x,y\right)\in\left(X,Y\right)$$

Where $\left(X\times Y\right)$ shall stand for the set of all PFR.

Control points are the grouping of all sets of points that are utilized to ascertain a spline curve's shape. The process of creating, regulating, and constructing a smooth curve involves the control point. The fuzzy control point notion from earlier research [28][29] is used to visualize PFCPR. PFCPR is defined as follows:

Definition 4: Assuming $\hat{S}$ be a PFPR, PFCPR is seen as a collection of points $n+1$ that are utilized to describe the curve and are represented by coordinates and location as

$$\hat{C}_i=\left\{\hat{C}_1,\hat{C}_1,\hat{C}_2,..,\hat{C}_{n+1}\right\} \tag{5}$$

whereas i is one smaller than the number of points, $n+1$. Consequently, the definition of PFCPR as follows.

III. APPROXIMATION OF PICTURE FUZZY B-SPLINE CURVE MODEL

The PFBSC is defined and produced by combining the PFCPR with basis function of B-spline.

Definition 5 : Let $\hat{C}_i=\left\{\hat{C}_0,\hat{C}_1,\hat{C}_2,..,\hat{C}_n\right\}$ and $i=0,1,2,..,n$ be a PFCPR and Position vector along the curve as a function of the parameter t, denoted by $\hat{S}(t)$, represents PFBSC as

$$\hat{S}(t)=\sum_{i=1}^{n+1}\hat{C}_i N_i^k(t) \tag{6}$$

With $t_{\min}\le t\le t_{\max}$ and $2\le k\le n+1$ where $\hat{C}_i$ are the position vector of $n+1$ control polygons vertices and the N_i^k are the B-spline basis function where its described as

$$N_i^1(t)=\begin{cases}1 & if \quad t_i \le t < t_{i+1}\\ 0 & otherwise\end{cases} \tag{7}$$

and

$$N_i^k(t)=\frac{(t-t_i)N_i^{k-1}(t)}{t_{i+k-1}-t_i}+\frac{(t_{i+k}-t)N_{i+1}^{k-1}(t)}{t_{i+k}-t_{i+1}} \tag{8}$$

PFBSC in Eq. (6) is parametric function consists of positive, neutral, negative and refusal membership curve described as,

$$\widehat{S}_\mu(t)=\sum_{i=1}^{n+1}\widehat{C}_i^\mu N_i^k(t) \tag{9}$$

$$\widehat{S}_\eta(t)=\sum_{i=1}^{n+1}\widehat{C}_i^\eta N_i^k(t) \tag{10}$$

$$\widehat{S}_\nu(t)=\sum_{i=1}^{n+1}\widehat{C}_i^\nu N_i^k(t) \tag{11}$$

$$\widehat{S}_\rho(t)=\sum_{i=1}^{n+1}\widehat{C}_i^\rho N_i^k(t) \tag{12}$$

IV. VISUALIZATION OF PICTURE FUZZY B-SPLINE CURVE APPROXIMATION MODEL

To illustrate PFBSC approximation, Now, let's look at PFBSC with six PFCPR as shown in Table 1.

Table 1. Picture Fuzzy Control Point Relation with its membership degree

PFCPR $\widehat{C}_i$	**Positive membership** μ	**Neutral membership** η	**Negative membership** ν	**Refusal membership** ρ
0	0.3	0.5	0.1	0.1
1	0.6	0.2	0	0.2
2	0.2	0.6	0.2	0
3	0.5	0	0.3	0.2
4	0	0.4	0.5	0.1
5	0.1	0.2	0.3	0.4

From Figure 1 to Figure 4, the approximation curve is shown separately with the corresponding picture control points(red dots) by using Eq. (6). Positive, neutral, negative, and refusal membership control polygons make up the picture control polygon, which is the line that connects the control points.

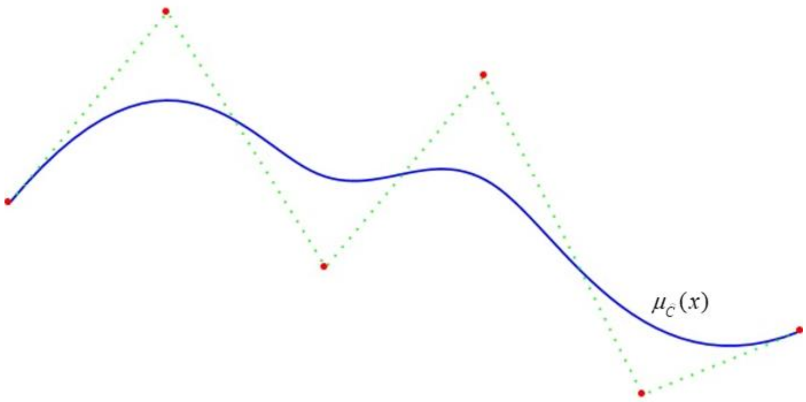

Fig. 1 Positive membership approximation curve

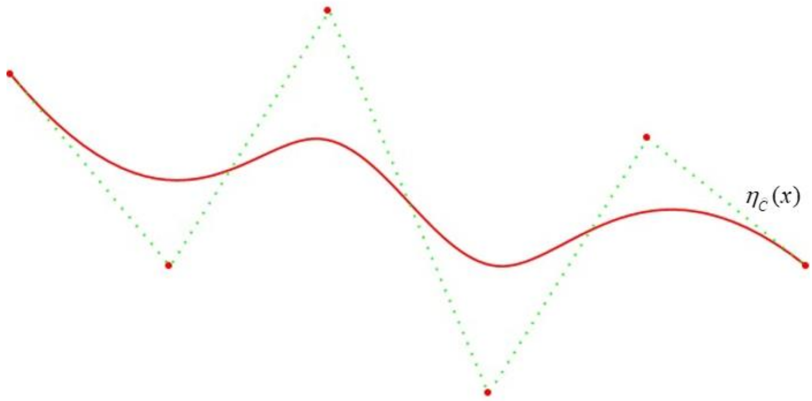

Fig. 2 Neutral membership approximation curve

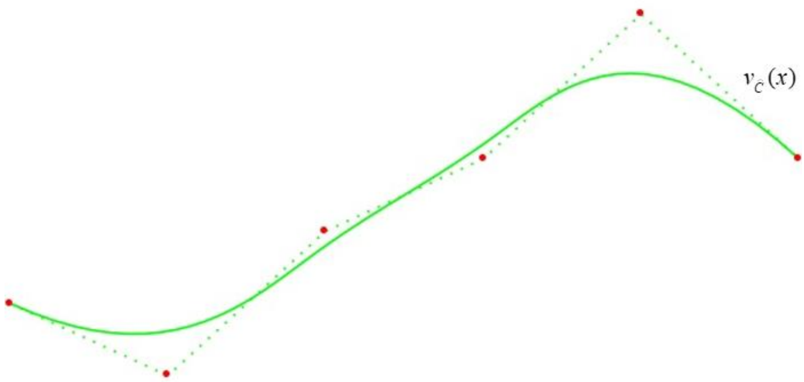

Fig. 3 Negative membership approximation curve

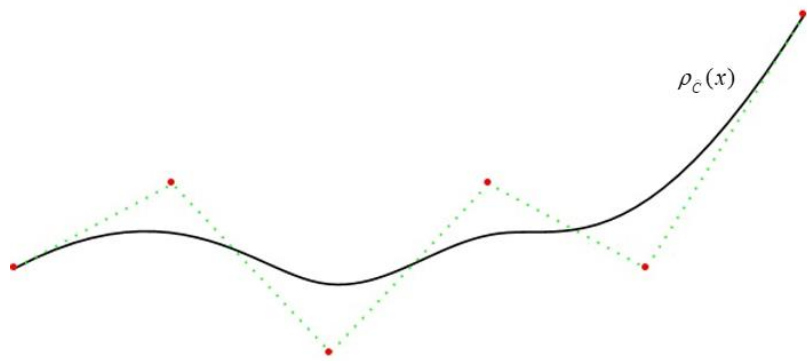

Fig. 4 Refusal membership approximation curve

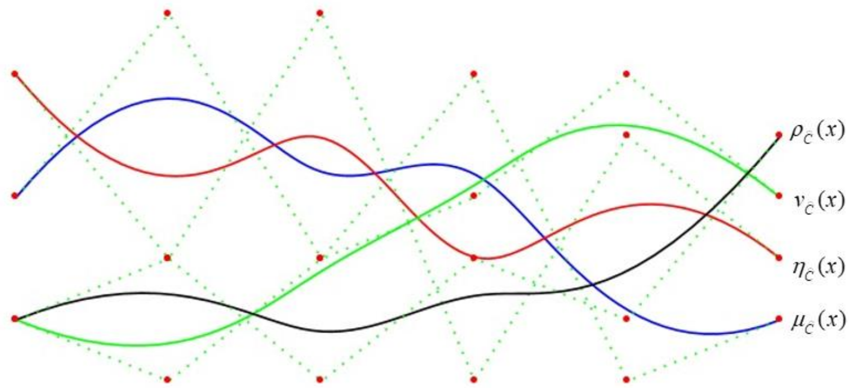

Fig. 5 Five PFBSC approximation using appropriate picture control polygon data point.

Figure 5 shows a approximation of picture B-spline curve for all membership in one axes, where blue curve representing membership of positive, red curve representing membership of neutral, green curve representing membership of negative and black curve representing membership of refusal. To determine the points or degree of each membership, it determined through characteristic of PFS, which is followed from Eq. (2).

V. CONCLUSION

Picture fuzzy B-spline curve approximation is specialized research that holds significance in various practical and research context. This is because, picture fuzzy set provides an advanced framework which is consist of four membership degree (positive, neutral, negative, refusal) to handle uncertainty and vagueness in data provide. By combining this with B-spline curves, the approximation can achieve higher accuracy and flexibility especially in scenarios where data is imprecise or noisy. In modelling, the combination of picture fuzzy set and B-spline curve can improve the quality of object modelling, image reconstructions, and animation leading to more realistic and visually appealing result. By introducing PFCP, this study has introduced PFBSC approximation model. Because the approximate PFBSC model has a positive membership function, neutral membership function, negative membership function, and refusal function, it is the best method for modelling data with picture features. These functions will be used to process and analyse all of the provided data in relation to a smooth B-spline curve. While combined with B-spline visualization, the data of picture fuzzy can provide comprehensive analysis and description of the types of problems under study, along with their explanation. There are some potential approaches and applications such as in uncertainty computer graphics, image processing, product design, manufacturing, quality control, supply chain management, databases, remote sensing, data mining, real-time tracking, the stock market, managerial decision-making, economics, routing, wireless sensor networks and financial analysis.

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