Uluslararası İleri Doğa Bilimleri ve Mühendislik Araştırmaları Dergisi
Sayı 8, S. 1-7, 9, 2024
© Telif hakkı IJANSER'e aittir
Araştırma Makalesi

IJANSER
https://as-proceeding.com/index.php/ijanser
ISSN:2980-0811

International Journal of Advanced Natural Sciences and Engineering Researches
Volume 8, pp. 1-7, 9, 2024
Copyright © 2024 IJANSER
Research Article

Free vibration analysis of sandwich composite plate based on First-order shear deformation theory using p-element formulation

A. BELAIDI*[1,3], S.M.N. SERDOUN*[2,3], S. A. BELALIA*[3,3]

[1] *University of Abou Bekr Belkaid, Depatement of Mechanical Engineering,Tlemcen BP Chetouan,13000 Tlemcen, Algeria*

[2]*Higher School of Applied Sciences, Department of Technology, Tlemcen, BP 165 RP Bel horizon, 13000 Tlemcen, Algeria*

[3]*MECACOMP Laboratory*

**ayoubjobbi@gmail.com*

(Received: 26 September 2024, Accepted: 01 October 2024)

(6th International Conference on Applied Engineering and Natural Sciences ICAENS 2024, 25-26 September 2024)

ATIF/REFERENCE: Belaidi, A., Serdoun, S.M.N. & Belaidi, S. A. (2024). Free vibration analysis of sandwich composite plate based on First-order shear deformation theory using p-element formulation, *International Journal of Advanced Natural Sciences and Engineering Researches*, 8(9), 1-7.

Abstract **–** This paper investigates the free vibration responses of sandwich composite plates, employing a high-level programming language code based on Mindlin's first-order shear deformation theory (FSDT) to model the plates. Initially, the validity of the proposed model is established, demonstrating favorable convergence. Subsequently, several numerical examples are solved and discussed Investigate the impact of parameters such as thickness ratio, aspect ratio, boundary conditions, and number of layers on natural frequencies. The results demonstrate the effectiveness of FSDT in predicting the dynamic behavior of sandwich composite plates with low computational cost.

Keywords – Free vibration, sandwich composite, plate, Finite element method, FSDT

I. INTRODUCTION

fiber-reinforced plastic sandwich plates find extensive application in diverse engineering contexts due to their exceptional strength-to-weight ratio, rigidity, and design versatility. The investigation of their free vibration behavior is of significant importance in structural mechanics and engineering innovation. As engineering structures become increasingly complex, understanding the dynamic response of sandwich plates under free vibration is paramount.

The researchers have used a combination of experimental, analytical, and numerical methods. In the field of free vibration analysis of sandwich plates, several theories have been employed over the years. The need for enhanced accuracy has led researchers towards Higher-Order Shear Deformation Theory (HSDT). Kolarevic [1] applied the theory to isotropic plate assemblies, demonstrating its versatility and accuracy. Canales [2] extended the application to laminated beams, considering arbitrary boundary conditions, and Li [3] introduced a simple quasi-3D HSDT for functionally graded plates on elastic foundations Serdoun [4] methodology involves the use of Reddy's higher-order shear deformation theory, development of a new C 1 -HSDT p-element, implementation of trigonometric hierarchical shape functions, and investigation of the effects of various parameters on natural frequencies through numerical analysis. further enhancing the theory's applicability. These studies collectively highlight the effectiveness of HSDT in free vibration analysis across a range of structural configurations. However, this increased accuracy comes at the cost of higher computational complexity.

The Classical Plate Theory (CPT) and the First-Order Shear Deformation Theory (FSDT) are computationally less expensive compared to HSDT because they involve fewer variables and simpler equations. Liew [5] and Ngo-Cong [6] applied FSDT to the analysis of shear-deformable and laminated composite plates, respectively, using different mesh-free methods. Both studies demonstrated the effectiveness of FSDT. Mantari [7] further extended this work by proposing a simplified FSDT for sandwich plates and laminated composite, achieving good accuracy in predicting fundamental frequencies.

Based on the preceding review, the present study aims to analyze the dynamic behavior of a specific fiber-reinforced plastic (FRP) sandwich plate with a relatively rigid core material poly-vinyl chloride (PVC) foam with good accuracy and mitigating high computational complexity, using a high-level programming language.

The validation of free vibration responses will be conducted against existing data from the published literature, contributing to the ongoing discourse on sandwich plate dynamics.

II. THEORY AND FORMULATION

A. Energy Formulation

In the present study, a sandwich plate composed of two laminated faces of FRP composite and a rigid core of thickness h_c. The plate is assumed to have a length of a and width of b and a total thickness of h, as shown in Figure 1.

The displacement of the plate is

$$\begin{aligned} u &= u_0 + z\vartheta_x \\ v &= v_0 + z\vartheta_y \\ w &= w_0 \end{aligned} \tag{1}$$

Where u, v, w are the displacement components of any point in the layer along x,y,z, and u_0, v_0, and w_0 are the displacements of the middle surface of the plate, O_x and O_y are rotations of transverse normal about y-axis and x-axis of the plate respectively. The constitutive equations for a kth layer, in the orthotropic local coordinate derived from Hook's law for plane stress given by

$$\{\sigma\}^k = [C]^k\{\varepsilon\}^k \tag{2}$$

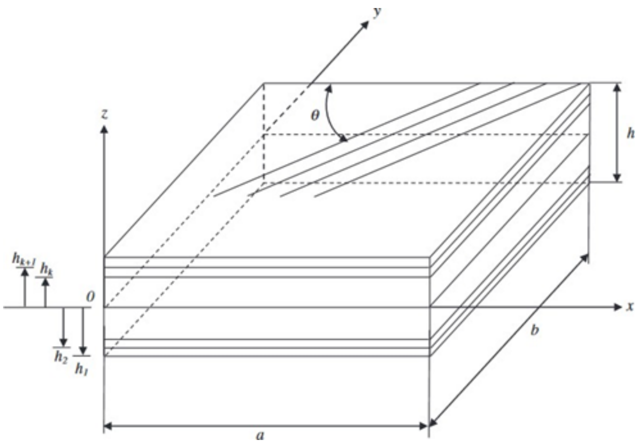

Fig 1: Laminate Geometry

where C_{ij} are engineering constants. By performing a proper coordinate transformation, The stress-strain relations in theglobal (x, y, z) coordinate system can be written as

$$\{\sigma\}^k = [Q]^k\{\varepsilon\}^k \tag{3}$$

The expressions for the strain energy V and the kinetic energy T are mentioned by Reddy J [8].

B. Hierarchical finite element formulation

A four-node rectangular hierarchical finite element with five degrees of freedom per node is used based on FSDT. Trigonometric hierarchical functions are used as shape functions. The model requires C_0 continuity for u_0, v_0, w_0 , ϑ_x and ϑ_y.

The displacements and rotations of the rectangular plate p-element are expressed in terms of the trigonometric hierarchical shape functions which are

$$\begin{aligned} f_1 &= 1 - \xi \\ f_2 &= \xi \\ f_{n+2} &= \sin(\delta r \xi) \\ \delta r &= r\pi \\ r &= 1, 2, 3, \cdots \end{aligned} \tag{4}$$

The displacements and rotations can be expressed in matrix form

$$\begin{pmatrix} \mathrm{u0} \\ \mathrm{v0,} \\ \mathrm{w0} \\ \theta x \\ \theta y. \end{pmatrix} = [N] \times \{q\} \tag{5}$$

Where [N] is the matrix of the shape functions and {q} in the vector of the generalized displacement

.

The equations of motion of free vibration of composite plates are expressed as

$$(K - \omega^2 M)q = 0 \tag{6}$$

Where K is the stiffness matrix, M is the mass matrix, and the parameter ω refers to the corresponding frequency.

III. RESULTS AND DISCUSSION

This study investigates the dynamic responses of a specific fiber-reinforced plastic (FRP) sandwich plate based on the First- Order Shear Deformation Theory (FSDT) using the finite element method. A 4-noded p-element is used to model the plate. Initially, shear correction factor K_c is determined. A convergence study is then conducted, and the results are validated to verify the accuracy of the present analysis. The study investigates the influence of changes in the thickness ratio and aspect ratio on the fundamental frequencies, and the effect of different boundary conditions. and the results are compared with those available in previously published data

A. *Determination of shear correction factor*

The accuracy of the first-order shear deformation especially for sandwich plates highly depends on the shear correction factor K_c used to adjust the transverse shear stiffness of the plate, introduced in the stiffness matrix this section the aim is to determine an adequate K_c factor, as is it know that The shear correction factors are dependent upon the core material and need to be determined for each study. a simply supported square sandwich plate made of HEREX C70.130 closed-cell foam core and two laminated FRP faces is used with The following material properties: Face sheet :

$$E_1 = 24.51\text{GPa}, E_2 = 7.77\text{GPa}, G_{12} = 3.34\text{GPa}, G_{13} = 3.34\text{GPa}, G_{23} = 1.34\text{GPa}, \nu_{12} = 0.078, \nu_{21} = 0.24\rho = 1800\text{kg/m}^3 \quad (7)$$

Core properties: HEREX C70.130:

$$E_c = 103.63\text{MPa}, G_c = 50\text{MPa}, \nu_{12} = 0.32\rho = 130\text{kg/m}^3 \quad (8)$$

. By comparing the results obtained for different shear coefficient value presented in Table I. It can be seen that for the FRP sandwich plate considered, a value of 0.2 can be chosen for K_c, the values are also quite close to the data of the analytical solution [9] values, especially for lower modes.

Table I: Effect Of Shear Correction Factor K With A/H = 10

Mode	HSDT[9]	K	Present	Difference %
1	15.28	0.833	19.151	20.21304
		0.195	15.161	0.784909
		0.2	15.255	0.163881
2	28.69	0.833	40.358	28.91124
		0.195	27.76	3.350144
		0.2	28.011	2.424048
3	30.01	0.833	40.358	25.64052
		0.195	30.348	1.113747
		0.2	30.631	2.027358
4	38.86	0.833	41.191	5.659003
		0.195	38.458	1.045296
		0.2	38.844	0.04119

the convergence study is performed on [0/90/0/core/0/90/0] cross-ply square plate with simply supported (SSSS) edges, Figure:3 shows that good convergence and accuracy of the first four frequency parameters are obtained by increasing the number of hierarchical terms to 12 trigonometric functions4.

The validation study is made in two sections. First, the results (non-dimensional frequency) computed for sandwich plates with different thicknesses and aspect ratios are compared with the results published by A.K Nayak [10]. The comparison is presented in Table . Then, the non-dimensional frequency with different numbers of layers and boundary conditions obtained is compared with the results published by R.K Khare R [11]. The geometrical and material properties are kept the same as reported in the previous paper. The results are presented in Table III. In both of the validation studies, it is found that the present values of the non-dimensional frequencies are very close to the reference values.

B. *Effect of thickness ratios and aspect ratios on the dimensionless fundamental frequency*

The effect of aspect ratio and side-to-thickness ratio on the non-dimensional fundamental frequency for stacking sequences of [0°/90°/0°] is adopted for the faces of the sandwich plates, laid symmetrically about the mid-plane of the core, The ratio of the thickness of the core to the total thickness is assumed to be equal to 0.88. the effect of the aspect ratios a/b on the natural frequencies for different length-to-thickness ratios is examined in Table III. The present FSDT results show good agreement with the finite element results of FEM-HSDT [10]. It is observed that the dimensionless fundamental frequency increases with the increase in the side-to-thickness ratio as well as the increase of the aspect ratio. this can be explained by A plate with a higher aspect ratio is generally stiffer in the direction perpendicular to its length, which can lead to a higher fundamental frequency.

C. *Effect of number of layers and boundary conditions on the dimensionless fundamental frequency*

Various boundary conditions are applied to the analysis: simply supported on all sides (SSSS), fully clamped on all sides (CCCC), and combined conditions such as clamped-free-clamped-free (CFCF) and clamped-supported-clamped-supported (CSCS). The outcomes of this analysis are presented in Table **??**. This section examines how boundary conditions and the number of layers affect the frequency parameters of a square sandwich plate. Two layup configurations are considered: (0/90/core/0/90) and (0/90/0/core/0/90/0). The first five frequency values obtained are compared against the published findings of Nayak et al. [10].

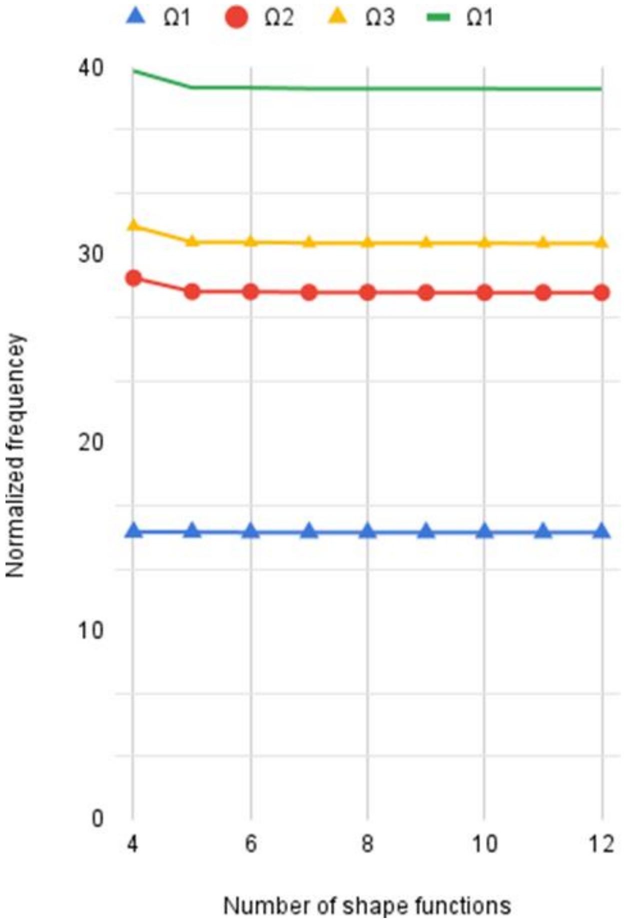

Fig. 2: Convergence of the frequency parameters for simply-supported sandwich plate

Table II: Effect Of Thickness Ratios And Aspect Ratios On The Dimensionless Fundamental Frequency With H_c/H_f = 16

	a/h	20		10	
a/b	Mode	Present	FEM-HSDT[10]	Present	FEM-HSDT[10]
0.5	1	13.915	13.85	11.4926	11.2
	2	19.2516	19.24	15.264	15.05
	3	29.026	29.16	20.179	20.41
1	1	19.2412	19.23	15.2598	15.04
	2	41.3949	41.7	28.023	28.1
	3	46.0081	44.88	30.6446	29.2
1.5	1	28.8668	28.97	21.1642	21.08
	2	52.1445	51.12	34.1549	32.86
	3	70.3776	71.17	40.358	40.82
2	1	41.3632	41.65	28.0146	28.07
	2	61.0626	60.24	38.8522	37.73
	3	80.716	81.65	40.358	40.82

Table III: Effect of number of layers and boundary conditions on the dimensionless fundamental frequency with h_c/h_f = 16

Nbr of layers		Mode	SSSS	CSCS	CCCC	CFCF
0/90/0/core/0/90/0	Present	1	15.25	17.58	20.17	13.13
		2	28.01	30.09	31.18	14.82
		3	30.63	32.34	33.66	26.75
		4	38.84	40.35	41.85	27.67
		5	40.35	40.95	45.30	29.26
	FEM-HSDT[10]	1	15.04	17.54	20.01	13.52
		2	28.1	30.23	32.23	15.21
		3	29.2	31.14	33.34	26.83
		4	37.76	39.96	42.27	28.32
		5	40.82	40.82	48.16	30.55
0/90/core/90/0	Present	1	15.26	17.92	20.18	14.06
		2	29.29	31.27	32.40	15.62
		3	29.42	31.53	32.49	27.05
		4	38.84	40.36	41.86	28.61
		5	40.36	41.08	47.73	30.88
	FEM-HSDT[10]	1	15.03	17.56	19.98	13.83
		2	28.43	29.85	32.53	15.45
		3	28.78	31.5	32.88	26.4
		4	37.66	39.94	42.14	28.85
		5	40.82	40.82	48.72	30.98

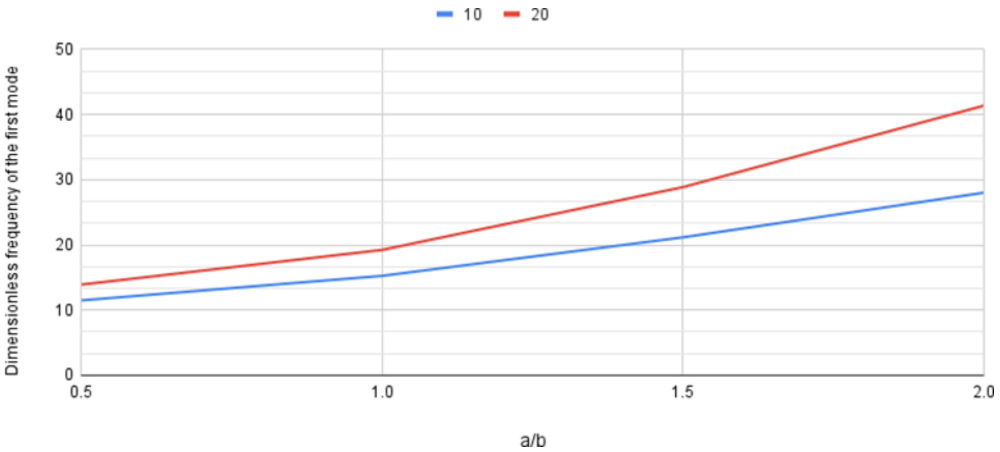

Fig. 3: Convergence of the frequency parameters for simply-supported sandwich plate

IV. Conclusion

This study has conducted a free vibration analysis of FRP sandwich plates using a high-level programming language code based on the C0 finite element formulation of Mindlin's First-Order Shear Deformation Theory (FSDT). The accuracy was validated by solving a variety of problems and comparing the results with both analytical solutions and high-order shear deformation theory results from the previous literature. Parametric studies were performed to investigate the effects of various factors such as the length-to-thickness ratio, aspect ratio, number of layers, and boundary conditions on the frequencies and mode shapes of the sandwich plates. Our results demonstrate the effectiveness of our approach in predicting the dynamic response of thin to moderately thick composite plates. The results demonstrate the effectiveness of the developed code in predicting the dynamic response of the plates, offering good accuracy while maintaining efficient computational costs.

REFERENCES

[1] N. Kolarević, M. Marjanović, M. Nefovska-Danilović, and M. Petronijević, "Free vibration analysis of plate assemblies using the dynamic stiffness method based on the higher order shear deformation theory," J Sound Vib, vol. 364, pp. 110–132, 2016

[2] F. G. Canales and J. L. Mantari, "Buckling and free vibration of laminated beams with arbitrary boundary conditions using a refined HSDT," Composites Part B-engineering, vol. 100, pp. 136–145, 2016

[3] M. Li, C. Soares, and R. Yan, "Free vibration analysis of FGM plates on Winkler/Pasternak/Kerr foundation by using a simple quasi-3D HSDT," Compos Struct, vol. 264, p. 113643, 2021

[4] S. Serdoun and S. M. H. Cherif, "Free vibration analysis of composite and sandwich plates by alternative hierarchical finite element method based on Reddy's C1 HSDT," Journal of Sandwich Structures & Materials, vol. 18, pp. 501–528, 2016

[5] K. M. Liew, J. C. Wang, T. Y. Ng, and M. J. Tan, "Free vibration and buckling analyses of shear-deformable plates based on FSDT meshfree method," J Sound Vib, vol. 276, pp. 997–1017, 2004

[6] D. Ngo-Cong, N. Mai-Duy, W. Karunasena, and T. Tran-Cong, "Free vibration analysis of laminated composite plates based on FSDT using one-dimensional IRBFN method," Comput Struct, vol. 89, pp. 1–13, 2011

[7] J. L. Mantari and M. C. D. Ore, "Free vibration of single and sandwich laminated composite plates by using a simplified FSDT," Compos Struct, vol. 132, pp. 952–959, 2015

[8] J. N. Reddy, "Mechanics of Laminated Composite Plates and Shells: Theory and Analysis, Second Edition", 1996

[9] Meunier M, Shenoi RA. Free vibration analysis of composite sandwich plates. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science. 1999;213(7):715-727.

[10] A. K. Nayak, S. S. J. Moy, and R. A. Shenoi, "Free vibration analysis of composite sandwich plates based on Reddy's