Uluslararası İleri Doğa Bilimleri ve Mühendislik Araştırmaları Dergisi
Sayı 7, S. 18-25, 10, 2023

Araştırma Makalesi

https://alls-academy.com/index.php/ijanser
ISSN: 2980-0811

International Journal of Advanced Natural Sciences and Engineering Researches
Volume 7, pp. 18-25, 10, 2023

Research Article

An approach to solving real-life problems using normal distribution. Approximation of results using the Lagrange-Euler method

Shkelqim Hajrulla*, Taylan Demir[2], Vasil Lino[3] and Loubna Ali[4]

[1]*Computer Engineering Department/ Epoka University, Tirana, Albania*
[2]*Department of Mathematics/Çankaya University, Ankara, Turkey*
[3]*Department of Economics/Epoka University, Tirana, Albania*
[4]*Department of Information Technology/University of Tartous, Tartous, Syria*

*(*demir.taylan96@gmail.com*)

(Received: 15 October 2023, Accepted: 22 October 2023)

(2nd International Conference on Recent Academic Studies ICRAS 2023, October 19-20, 2023)

ATIF/REFERENCE: Hajrulla, S., Demir,T., Lino, V. & Ali, L. (2023). An approach to solving real-life problems using normal distribution. Approximation of results using the Lagrange-Euler method. *International Journal of Advanced Natural Sciences and Engineering Researches*, 7(10), 18-25.

Abstract – We deal with the normal distribution as an overview of real-life problems. Our paper focuses on analyzing the normal distribution and its special cases by solving real-life problems and examining them through detailed calculations and graphs. Several research papers have reported that the normal distribution of a random variable has an enormous contribution in analyzing and comparing the data with each other, making the process easier for other methods in real-life applications. Using numerical methods, we get the approximations and the error during the different cases.

In this research paper, we will discuss how the Euler equation will be used to solve the same problem in a more efficient way. The Euler method is used during the approximation process and with some widely studied models, including the standard formulas for each method, simulations, and graphs.

Keywords – Normal Distribution, Real Life, Time Charging, Lagrange Equation, Approximation

I. Introduction

The normal distribution model states that the averages of independent, identically distributed random variables do have approximately normal distributions. A normal distribution (Gaussian distribution) is a probability distribution that is symmetric about the mean. This indicates that the data approaching the mean are more frequently occurring than data placed far from the mean.

The general characteristics of the normal distribution are conected with tha fact of symmetry (mean = median = mode) and dealing with the graph the shape of the normal distribution function will be nearly flat on top, quickly decreasing toward the x-axis and at a point, it will slowly decrease towards the "tails" of the distribution.

This describes the relationship between the mean and values close to it which will be relatively frequent in occurrence and values far away from the mean that will have an infrequent occurrence. Frequency distribution describes a random variable but does not fit the normal distribution graph.

Probability theory states that the distribution variable approximates a normal distribution as the sample size becomes larger as shown in figure 1, assuming that all samples are identical in size, and regardless of the population's actual distribution shape. Efforts to find solutions to problems were continued through Matlab and Python software. Simulations and results are improved ([4], [5]). Comparison of Numerical Methods are applied. Distributed energy resource [1], distributed energy of heat transfer problem are connected with with associated Electric power systems interfaces [3].

Our research gives a survey and statistical data of computational math results were applied to get the predicted results to measure student understanding by comparing the results of groups created for this research. We will use a probability distribution ([15], [16]), a normal distribution that tells the distribution of results. The formula of the probability density function of the normal distribution:

$$f(x) = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}} \qquad (1.1)$$

where μ is the location parameter and σ is the scale parameter.

A special case [5] of the normal distribution is the standard normal distribution. For the standard normal distribution, the mean value is equal to zero (μ=0), and the standard deviation is equal to 1 (σ=1). The general formula follows:

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \qquad (1.2)$$

II. Materials and Method

The purpose of this paper is to evaluate the probabilities between a range of outcomes, find a critical outcome value at a given probability, and use the normal distribution to calculate expected values and standard deviations. 2.2 there are some examples of this distribution presented, at Analysis we solve two real-life examples of normal distribution, and at the end, we explain conclusions.

2.1 Standardizing a normal distribution function.

We introduce the standardization of normal distribution. To standardize a normal distribution function, we change the random variable X to Z. To do this we use the formula:

$$z = \frac{x - \mu}{\sigma} \qquad (2.1)$$

This formula shifts all the data so that they can give a mean of 0 rather than μ. As we discussed above in standard normal distribution μ = 0, where x is a particular measurement, μ is the population mean, and σ is the population standard deviation. The benefit of standardization is that it allows different data sets to be directly compared. (Data from one year to another.) Compare to some previous results [6], we simulate the given data of heat wave equation with logarithmic source term. Unimodular matrix on wave theory is a good theory to get application of unimodularity through matrix method ([5], [14]). We improved two simulations using matrix 3x3 and 5x5 introducing two different case of heat transfer problem [10].

2.2 Introducing applications of Normal Distribution in several fields like intelligence quotient (IQ).

Standardized intelligence quotient tests create a ‘bell curve’ when constructing the data distribution in the general population with an average of 100. This curve has a peak in the middle where most people score and tapering ends where only a few people score. The area under the curve between scores corresponds to the % of the population between those scores. The scores on this IQ bell curve are color-coded in standard deviation units. A standard deviation is a measure of the spread of the distribution – the bigger the standard deviation, the more spread out the scores in the population.

Using Python Code, figure 2 explains the distribution of IQ (intelligence quotient) scores and the result generation of normal distribution data.

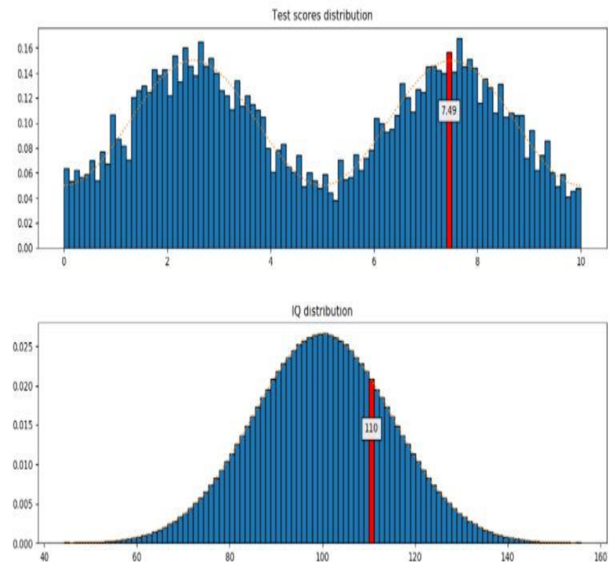


Fig. 1 Normal Distribution of IQ generation through Python code

III. RESEARCH AND APPLICATION

At the end of the research, a scientific committee tested the results.

3.1 Application in Technical Sock Market

The Stock Market is characterized by a rise and fall in prices of the shares. When changes happen in the log values of Forex rates, price indices, and stock prices returns often form a bell-shaped curve. If returns are normally distributed, more than 99 percent of the returns are expected to fall within the deviations of the mean value. A Statistical Method to Estimate An Unkonown Price in Financial Markets heps to understand that statistical appliacation [7]. A Survey and Statistical Data of Math Economics Applications International Relations Triple-Purpose explaine dgraphically with normal distribution ([2], [8]) allow analysts and investors to make statistical inferences about the expected return and risk of stocks.

3.2 Income Distribution in Finance

The normal distribution is utilized to devise quantitative and qualitative financial decisions based on the mathematical nature of normal distributions. This implies that normal distributions tend to follow certain similarities, such as the combination of distribution toward the mean, among other things like the standard deviation from the mean.

Generally, fractional order differential equations are used on engineering, medicine, biology, chemistry, physics and other fields commonly. Similarly, local fractional [11] order differential equations are used with heat transfer, dynamical systems, control systems modelling, population dynamics and other fields commonly [9]. Simulations and results are refered [10].

Due to this and various other trends, by the statistical patterns underlying the data, numerical forecasting is a bit more validated. Thus, ascertaining whether certain financial events are normally distributed can prove to be useful because those events may be more likely to follow probabilistic patterns in the future. Logically, we use constructive mathematics and approach in Mathematics [12] for better understanding.

IV. RESULTS AND DISCUSSION

4.1 Dell laptop battery using probability distribution table

The probability that Dell laptop battery in the time period between 50 and 70 hours. For some laptops, the time between charging the Dell laptop battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. We need to calculate the probability that the time period will be between 50 and 70 hours.

In figure 3, assume y as the random variable that indicates the time period.

If we consider $x = 50$, then $z = (50 – 50) / 15 = 0$. If we consider $x = 70$, then $z = (70 – 50) / 15 = 1.33$

$P (50 < x < 70) = P (0 < z < 1.33)$ = area to the left of $z = 1.33$ area to the left of $z = 1.33$ – area to the left of $z = 0$ area to the left of $z = 0$.

From the standard normal distribution shown in Table 1, we get the values, such as;

Table 1. Z value correspondence

Z	0.00	0.01	0.02	0.03	0.04
0.0	0.5000	0.5040	0.5080	0.5120	0.5160
0.1	0.5398	0.5438	0.5478	0.5517	0.5557
0.2	0.5793	0.5832	0.5871	0.5910	0.5948
0.3	0.6179	0.6217	0.6255	0.6293	0.6331
0.4	0.6554	0.6591	0.6628	0.6664	0.6700
0.5	0.6915	0.6950	0.6985	0.7019	0.7054
0.6	0.7257	0.7291	0.7324	0.7357	0.7389
0.7	0.7580	0.7611	0.7642	0.7673	0.7704
0.8	0.7881	0.7910	0.7939	0.7967	0.7995
0.9	0.8159	0.8186	0.8212	0.8238	0.8264
1.0	0.8413	0.8438	0.8461	0.8485	0.8508
1.1	0.8643	0.8665	0.8686	0.8708	0.8729
1.2	0.8849	0.8869	0.8888	0.8907	0.8925
1.3	0.9032	0.9049	0.9066	0.9082	0.9099
1.4	0.9192	0.9207	0.9222	0.9236	0.9251

P $(0 < z < 1.33) = 0.9082 - 0.5 = 0.4082$. This is the final result of the probability of a Dell laptop during a time period between 50 and 70 hours.

4.2 Lagrange method applied

Lagrange's is a tool that determines the important properties of mechanical systems. We can approach to this method by some linear expressions that describe the dynamics of the system in a neighborhood sufficiently close to the point of interest.

We can generate faster systems by using the Lagrange equation in the applications that were previously generated by Hamilton. Lagrangian and Hamiltonian mechanics: Lagrangian mechanics describe the difference between kinetic and potential energies, whereas Hamiltonian mechanics describe the sum of kinetic and potential energies. But both of them measure energy in quantum systems. We study the measurement of the Hamiltonian equation for motion in quantum mechanics we receive two first-order differential equations.

$$q = \frac{\partial H}{\partial p} \tag{4.1}$$

$$p = -\frac{\partial_1 H}{\partial q} \tag{4.2}$$

Then, the Hamiltonian equation also provides a measurement for the total energy of the system:

$$H = \sum_i pq - L \tag{4.3}$$

On the other has Euler-Lagrange has the following equation:

$$\frac{\partial L}{\partial q}\frac{d}{dt}\frac{\partial L}{\partial q} = 0 \tag{4.4}$$

If one particle is moving somewhere where it also has some potential energy V(x), the Lagrangian is simply:

$$L = \frac{1}{2}mx^2 - V(x) \tag{4.5}$$

A phase portrait graph of a dynamical system depicts the system's trajectories (with arrows) stable steady states (with dots) and unstable steady states (with circles) in a state space

We can observe by equation that Hamilton can be generated by Lagrange and thus we can receive the same results for both equations. Langrange as we observed above uses only one second order of differential equation and Hamilton uses two, which makes the system go through many calculations and makes it slower.

$$H = px - \left(\frac{1}{2}mx^2 - V(x)\right)$$

$$P = \frac{\partial \alpha}{\partial x} = P = \frac{\partial}{\partial x}\left(\frac{1}{2}mx^2 - V(x)\right) = mx \tag{4.6}$$

The Lagrange equations are used to derive the equations of motion of a solid mechanics problem, including damping, in matrix form. Using an unimodular matrix, we perform our results.

We will analyze by a simple equation how Lagrange is a simplified method of Hamilton.

Let us take a simple example of how Hamilton solves a problem: To optimize a system using the Hamilton equation we get two ground states to encode the function. V_G is measured to be the market state and the K_G vector, equivalent to the state, will be the diagonal basis. To find the interpolation of the equation we use Hamiltonian interpolation to find a spectral gap:

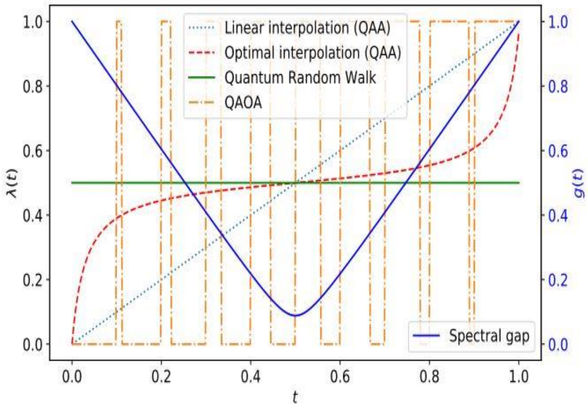


Fig.3 Interpolation of the equation using Hamiltonian interpolation

4.3 Euler method applied. Solutions of Euler's System of Equations and Performance comparison for heat transfer problem

For a general initial value problem and mixed initial boundary value problem remain largely open with limited success in some particular cases. We introduce the results in heat transfer problem [15]. We will compare different algorithms used and show which one of them performs better under our test conditions. The program will simulate the heat transfer of a single heat source in a closed environment. The results of the simulations will be presented in graphs and demonstrated in visual settings. In the end, we will provide our conclusions on the performance of the numerical methods

One of the approaches that has gained much attention in recent times is the use of aproximations. [1–4]. Nonlinearities in the problem make solving and analyzing initial boundary value problems. The different frequencies, amplitudes and phase angles of sinusoidal heating are used as for understanding the heat behavour problems and standard for Interconnection of distributed energy resources is studied

After giving the code of both methods we need to analyze which one of these methods performs better in our test conditions. The result that we got are shown by a tabular representation. Furthermore, the algorithm would be extended from first-order temporal accuracy to second-order temporal accuracy. Of course, using numerical methods, appropriate algorithms and simulations, we can achieve our goal of future studies related to heat transfer problems.

We have considered four test cases. After performing the measurements, we ended up with the following results:

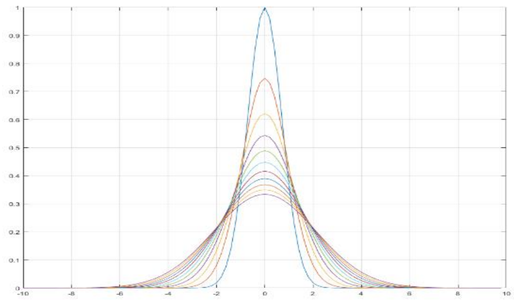


Fig. 4 Heat distribution for the first simulation

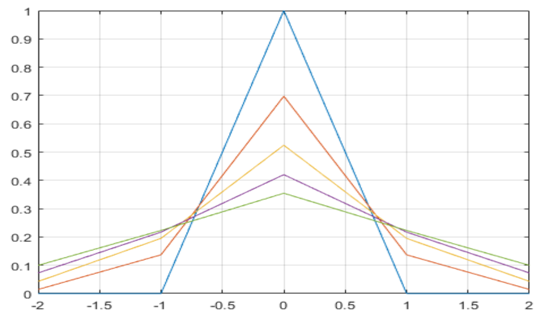
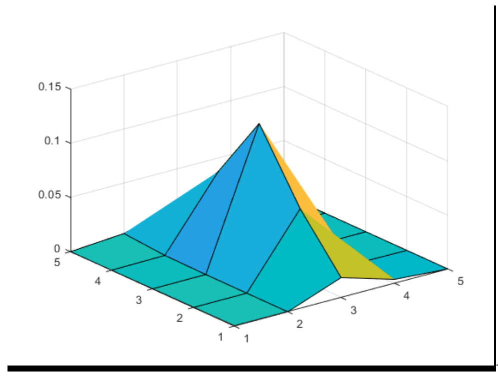


Fig. 5 Heat distribution for the second simulation



A

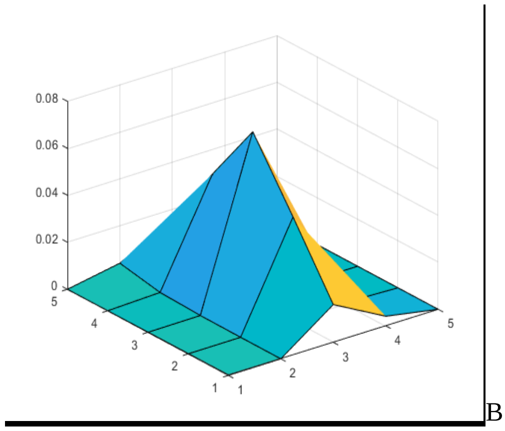


Fig. 6 A, B (from the left to the right): Comparison of heat distribution in different conditions

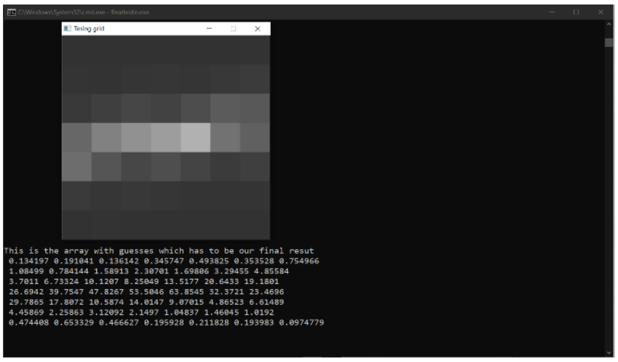


Fig. 7 The first simulation with a 3 by 3 matrix

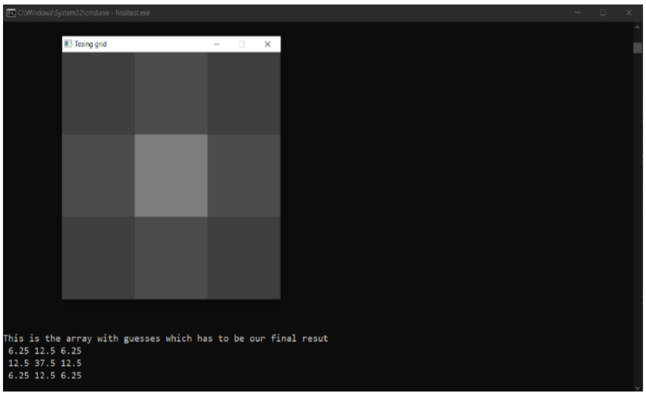


Fig. 8 The second simulation with a 5 by 5 matrix

Table 2. For the Euler method

Case	1	2	3	4
Time	1899000 mls	2702000 mls	26000000 mls	9890000 mls
Nr of iterations	13	5	15	16

Table 3. For the Lagrange method

Case	1	2	3	4
Time	1104000 mls	812300032 mls	192023186 mls	37000000 mls
Nr of iterations	8	2003	8	8

On the other hand, we see that Euler method has a better time in almost every case compared to Lagrange method. An important fact to be mentioned is that we have taken as a constant number of iterations the time it takes the LU decomposition algorithm to perform the decomposition process.

Unimodularity through matrix is applied [14]. Unimodular matrix simulation in different size gives some interesant results.

Furthermore, the algorithm would be extended from first-order temporal accuracy to second-order temporal accuracy. Of course, using numerical methods, appropriate algorithms and simulations, we can achieve our goal of future studies related to heat transfer problems.

V. CONCLUSION

Our research indicates how to write exactly in the subjects of science. We have introduced the importance of practical interactions of laboratories, and projects.

The evidence that Normal Distribution has several applications in real life is clear. Throughout this paper, we have introduced how Normal Distribution has contributed to several fields and how it makes the data to be easier for an analytical approach.

We have also standardized the normal distribution function in the application of Dell's battery charging time. Through the calculations, we also proved that standardization would make the data comparison easier and faster.

On the other hand, we analyzed how normal distribution predicts the failure of a product in terms of time and predicting the time interval for the next products to fail.

Through normal distribution, we have proved that data distribution in relationship with each other is comparable and easier to read and analyze.

Additionally, the data presented through normal distribution are more accurately described for several natural phenomena. We have also observed through the tables a symmetry in the mean for the real-valued random variables under fair conditions.

In this case study paper, we have gathered all the significant information needed to receive an exact solution for this topic. It is clarified through the graphs and equations that Lagrange is a one-level equation methodology, while Hamilton is a two-level one.

In the methodology of working, we have tried to compare both methods, and seems that Euler Lagrangian is shorter at first glance. So, we proceed with simplifying both equations we realize that the same happens even in the last equation that we get.

By considering this application, we get the result that to calculate several qubit states and their energy it is more helpful to use fewer variables and calculations. Moreover, by observing both methods the graph represented by Lagrange is far easier to be understood compared to that of Hamilton. So, the procedure is easier to get conclusions and more readable.

Moreover, we have provided an example in the analysis section that concentrates on interpreting the graph of interpolation equation by both methods. As we can see, Lagrange converges faster than Euler.

Additionally, the numerical methods are used to solve the heat transfer problem, considering the methods and comoutional operations, approximation and simulation model.

If we consider states and qubit states, they act faster under the Lagrangian interpolation rather than Euler's one as qubit is a two-dimensional object, and applying one summarizing equation on them will simplify the answer to only one result rather than two.

Therefore, with the aid of computers, we can avoid these errors and the long amount of time needed to perform the calculations. Numerical methods help us to perform complex calculations of complex heat-transfer problems.

The numerical method provides theoretical and technical support to accelerate resolving heat diffusion, heat transport problems, approximation of solutions and expected results.

REFERENCES

[1] Feynman, *"IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces"*, IEEE-SASB Coordinating Committees: Piscataway, NJ, USA, 2018.

[2] Hajrulla, S., Demir, T., Bezati, L., & Hajrulla, D. 5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS.

[3] P Mahardika and F F Haryani 2020 *"Numerical analysis of one dimensional heat transfer"*, Journal of Physics: Conference Series, *J. Phys.: Conf. Ser.* 1591 012105 *J. Phys.: Conf. Ser.* 1511 012049

[4] Bezati, L., Hajrulla, S., & Hamzallari, B. Comparison of Numerical Methods for SWW Equations, 2018.

[5] Hajrulla Sh. Bezati L. Demir. T, Desantila H, *"Through Unimodular Matrix on SLE using LaTex. "* pp. 45-55 December 2021, Istanbul. International Conference on Mathematics

[6] Higher Order Wave Equation with Logarithmic Source Term S Hajrulla, L Bezati, F Hoxha - 2017

[7] A Statistical Method to Estimate An Unkonown Price in Financial Markets, S Hajrulla, D Osmani, V Lino, D Avdiu, D Hajrulla - PROCEEDINGS BOOK, 2022

[8] A Survey and Statistical Data of Math Economics Applications International Relations Triple-Purpose S HAJRULLA, G HAJRULLA - PROCEEDINGS BOOK, 2021

[9] Rusul Mohammed Hussein Alshmary 2020, *"Application of Partial Differential Equations"*, Journal of Physics: Conference Series, *J. Phys.: Conf. Ser.* 1591 012105

[10] Simulations and Results for the Heat Transfer Problem S Hajrulla, A Uka, T Demir - European Journal of Engineering Science and …, 2023

[11] Local Fractional Differential Operators on Heat Transfer Modelling, T Demir, MH Cherif, S Hajrulla, G Erikçi

[12] Bhowmik, M. Constructivism Approach in Mathematics Teaching and Assessment of Mathematical Understanding. Basic Research Journal of Education Research and Review ISSN 2315-6872 Vol 4(1) pp 08- 12 January 2015.

[13] Exponential Decay of Solutions for a Higher Order Wave Equation with Logarithmic Source Term, E Pişkin, T Uysal, S Hajrulla - 2018

[14] Unimodular matrix on shallow water wave theory. Unimodularity through matrix method_S Hajrulla, A Uka, T Demir, F Hoxha, D Hajrulla… - New Trends in Mathematical Sciences, 2022

[1**5**] Numerical Methods and Approximations for the Heat Transfer Problem,_S Hajrulla, A Uka, L Ali, T Demir - Proceedings of The International Conference on …, 2022

[16] Lognormal distribution of Toxocara canis (n/e/g/f) at the street dogs. A Bocari, B Bizhga, R Kosova, N Mehmett - Albanian Journal of Agricultural Sciences, 2014