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Research Article

Feedback Control over Quantum Sensing Based on Bose-Einstein Condensate Trapped in Two-Dimensional Ring Potential

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Abstract – Recent great progress in studying theoretical and especially experimental properties of Bose-Einstein condensate (BEC) creates a new era in developing and designing principally novel types of quantum devices, including quantum sensors for measuring nonlinear interactions, external electrical and magnetic fields, and other physical characteristics with high sensitivity. Here we discuss the application of feedback control over a quantum sensor based on the Bose-Einstein condensate trapped in two-dimensional ring potential. For a weakly interacting regime, the dynamics of such a system are modeled by three coupled complex differential master equations containing the parameter of interaction and the chemical potential parameter. The last one plays the role of control variable in sensing protocol for two-body interaction. The goal of control is to minimize the effects of the higher energy levels in BEC by driving their corresponding matrix density elements. The control algorithm is designed as Kolesnikov's feedback forming an artificial target attractor in the dynamical system. We re-formulate Kolesnikov's approach in the operator form to adapt it to quantum engineering processes. The control approach proposed here can be efficiently extended to different sensing protocols for detecting external magnetic fields, rotational components, and other physical characteristics of BEC interacting with the environment.

Keywords – Bose-Einstein Condensate, Ring Potential Trapping, Weakly Interacting Regime, Feedback Control, Target Attractor

I. Introduction

Recent great progress in studying theoretical [1], [2], [3], [4] and especially experimental [5], [6], [8], [9] properties of Bose-Einstein condensate (BEC) creates a new era in developing and designing principally novel types of quantum devices [10], [11], including quantum sensors for measuring nonlinear interactions, external electrical and magnetic fields and other physical characteristics with high sensitivity [12], [13].

Here we discuss the application of feedback control over a quantum sensor based on the Bose-Einstein condensate trapped in two-dimensional ring potential. For a weakly interacting regime, the dynamics of such a system are modeled by three coupled complex differential master equations containing the parameter of interaction U and the chemical potential parameter Δ [14]. In our approach, the last one plays the role of control variable in sensing protocol for two-body interaction. The goal of control is to minimize the effects of the higher energy levels in BEC by driving their corresponding matrix density elements. The control algorithm is designed as Kolesnikov's feedback [15], [16] forming an artificial target attractor in the dynamical system. Such an approach already has been successfully applied to different quantum systems [17], [18], including BEC [19]. We re-formulate Kolesnikov's approach in the operator form to adapt it to modeling quantum engineering processes.

II. Mathematical Model for BEC-Based Quantum Sensing Protocol

We develop the quantum sensing protocol proposed in [14], we shall call it the Pelegrí-Mompart-Ahufinger (PMA) protocol, see Figure 1.

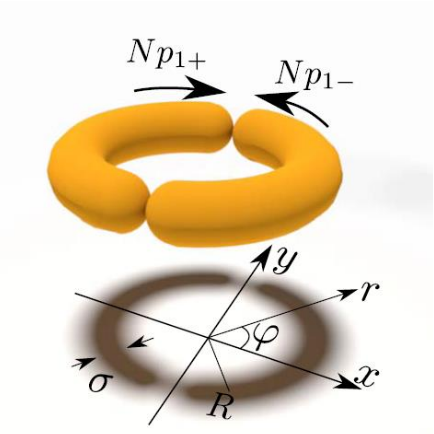

Fig. 1. Principal scheme of Pelegrí-Mompart-Ahufinger BEC-based quantum sensing setup [14].

In Fig.1 a BEC is trapped in a two-dimensional ring potential with the radial harmonic shape. It has the radius R and the width σ.

The initial state of BEC is prepared as an imbalanced superposition of two quantum states |1,+> and |1,−>, with the population imbalance number $n = p_{1+} - p_{1-}$. Two counter-rotating modes in Fig,1 provide a minimum line in the probability density due to their interference.

A. Model Master Equations

The dynamics of the BEC sensor is described by the von Neumann system of three differential equations for the complex density matrix elements:

$$\begin{aligned} i\frac{d\rho_1}{dt} &= Up_{1-}(\rho_1+\rho_3+2\rho_2) - Up_{1+}(\rho_1+\rho_2+2\rho_3)\,; \\ i\frac{d\rho_2}{dt} &= Up_{1+}(\rho_1+\rho_2+2\rho_3) + \Delta\rho_2\,; \\ i\frac{d\rho_3}{dt} &= -Up_{1-}(\rho_1+\rho_3+2\rho_2) - \Delta\rho_3\,. \end{aligned} \tag{1}$$

The Planck constant in (1) is equal to 1. Eqs (1) corresponds to a weakly interacting regime, where, apart from the initial states, only |3,+> and |3,−> higher energy states make a contribution to the system dynamics, for further details see Appendix A in [14].

We use a simplified notation for the density matrix elements: $\rho_1 = \rho_{1+,1-}$, $\rho_2 = (\rho_{1+,3+})^*$, and $\rho_3 = \rho_{1-,3-}$. The elements $p_{1+} = \rho_{1+,1+}$ and $p_{1-} = \rho_{1-,1-}$ are assumed to be constants. U corresponds to the nonlinear term $g_2|\Psi|^2$ in the two-dimensional Gross-Pitaevskii potential [20], [21], with the 2D two-body interaction constant g_2 and the radial part of the ring potential ground state $f(r)$, such that:

$$U = g_2 \int |f(r)|^4\, d^2 r\,. \tag{2}$$

Also, $\Delta = \mu_3 - \mu_1$ is the difference of two chemical potentials.

The following inequality is held for the BEC:

$$U \ll \Delta\,, \tag{3}$$

it must be satisfied for a weakly interacting regime [22], [23].

B. Sensing Protocol for Two-Body Interaction

Here we focus on the PMA sensing protocol for two-body interaction [14], the similar procedure can be developed for external magnetic field sensing [24], [25].

In BEC the rotational frequency Ω of the minimum density nodal line [26] can be measured by direct imaging in real time of the density distribution [5], [14].

For the model (1), the measured interaction parameter is given by [14]:

$$U = \frac{2\Omega}{n - \dfrac{2\Omega}{\Delta}}\,. \tag{4}$$

Thus, by (4) one can define experimentally *ab initio* the values of frequency Ω and the imbalance number n, and then using Δ as a control variable, evaluate the interaction parameter U.

III. Feedback Control Algorithm for BEC-Based Sensing

To provide an efficient application of the PMA sensing protocol in the form (4) by using Δ as a control parameter, one should minimize the effects of the higher energy states via the minimization of their population, i.e. the absolute values of the density matrix elements ρ_2 and ρ_3 in our case.

A. Operator Form of Kolesnikov's Algorithm

First of all, one can derive from (1) the property of our dynamical system:

$$i\frac{d}{dt}(\rho_1+\rho_2+\rho_3)=\Delta(\rho_2-\rho_3)\ . \qquad (5)$$

Our goal is to minimize the density matrix elements ρ_2 and ρ_3. Reformulating the Kolesnikov 'synergetic' algorithm in the operator form [15], let's demand:

$$\frac{d}{dt}(\rho_2+\rho_3)=-\frac{1}{T}(\rho_2+\rho_3)\ ; \qquad (6)$$

with a positive constant T. Eq.(6) provides the exponential decay of the absolute values for the corresponding complex matrix elements.

By substitution the derivatives for ρ_2 and ρ_3 one can obtain:

$$\begin{aligned}&-\frac{i}{T}(\rho_2+\rho_3)=Up_{1+}(\rho_1+\rho_2+2\rho_3)-\\&-Up_{1-}(\rho_1+\rho_3+2\rho_2)+\Delta(\rho_2-\rho_3)\ .\end{aligned} \qquad (7)$$

Defining the feedback control signal u as:

$$u=\Delta(\rho_2-\rho_3)\ , \qquad (8)$$

finally by (7) we get:

$$\begin{aligned}&u=-\frac{i}{T}(\rho_2+\rho_3)-Up_{1+}(\rho_1+\rho_2+2\rho_3)+\\&+Up_{1-}(\rho_1+\rho_3+2\rho_2)\ .\end{aligned} \qquad (9)$$

By (5) and (6) the first equation in the system (1) becomes:

$$i\frac{d\rho_1}{dt}=\frac{i}{T}(\rho_2+\rho_3)+u\ . \qquad (10)$$

In the limit (3), RHS(10) tends to 0, i.e. control drives the matrix element ρ_1 to stabilization.

B. Control over Parameter of Interaction

The control algorithm proposed here is valid only in the weakly interacting regime (3). That implies the constraint over the parameters in the system (1). Particularly, using (4), one can get:

$$n=p_{1+}-p_{1-}>>\frac{4\Omega}{\Delta}\ . \qquad (11)$$

Eq.(11) can be interpreted as the upper limit for the control signal parameter (chemical potential difference):

$$\Delta<<\left\{\frac{\Omega}{p_{1+}},\frac{\Omega}{p_{1-}}\right\}. \qquad (12)$$

One should consider (12) while constructing the target attractor control (8).

The scale of constant T is defined by (7). Due to (3), it must be compared with Δ and provide the fast exponential decay toward the artificial attractor. Thus, $T << 1/\Delta$.

IV. Results

The feedback target attractor control algorithm is presented in the novel operator formulation to drive efficiently matrix density elements of multi-level quantum systems.

V. Discussion

The control approach proposed here can be efficiently extended for different sensing protocols for detecting external magnetic fields, rotational components, and other physical characteristics of BEC interacting with the environment.

The system (1) is a particular version of qudit, for that reason, the algorithm developed here could be extremely useful for different types of qubit- and qudit-based quantum sensors.

VI. CONCLUSION

Control over particular elements of the density matrix improves the efficiency of the sensing protocol and opens a door for new types of mathematical algorithms for modeling quantum devices.

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