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Portfolio Optimization with Multi-Objective Optimization Algorithms

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Abstract – One of the critical issues in financial management is the investment decision-making process, and one of the main goals of investment management is optimal stock portfolio selection. In this context, there are various criteria and methods for optimal stock portfolio selection in the literature. This article first calculates investment return and investment risk using data from 6 companies such as Amazon, Yahoo, Microsoft, IBM, Apple, and Google for a one-month period from January 2014 to November 2014. Investment is calculated by 4 classical methods (mean-variance, mean-semi variance, mean absolute deviation, conditional value at risk). As a result of these calculations, 0.05706, 0.028409, 0.028871, and 0.032995 with maximum ROI (0.0142 respectively) and Risk are calculated for each classical method. Then, meta-heuristic methods (PSO, NSGA-II) are used for optimal selection of the portfolio. As a result of the calculations, it can be seen that the NSGA-II meta-heuristic algorithm tends to achieve the highest return on investment with a lower risk. These results suggest that the integration of advanced computational methods, such as multi-objective optimization algorithms, may be important to improve the precision and efficiency of portfolio selection in financial management. This can provide valuable insights for investors and financial analysts.

Keywords – Modern Portfolio Theory, Markowitz Mean-Variance, Mean Semi Variance, Mean Absolute Deviation, Conditional Value at Risk, Portfolio Optimization, Particle Swarm Optimization, Non-Dominated Sorting Genetic Algorithm-II

I. INTRODUCTION

Selecting the right portfolio is one of the most common problems and complex issues that different investors face with different levels of capital in the world of finance [1]. There for, one of the most important issues in modern capital markets and financial management is asset allocation for portfolio optimization [2].

The most important parameters in portfolio optimization issues are expected return and risk. Investors generally prefer to maximize returns and minimize risk. However, higher returns often require increased risk. Return and risk can be considered as two influential and important variables in the subject of investment. On the other hand, due to fluctuations in financial markets such as the stock market, investing will be associated with uncertainty and risk. Therefore, choosing a portfolio that has less risk and higher returns, as well as measuring portfolio risk is of particular importance to investors [3]. To solve this problem, Harry Markowitz in 1952, proposed a mean variance model for selecting the appropriate portfolio. The solution to the problem in the mean variance model is to choose portfolios of the best variation between the expected return (mean) and the risk measured by variance. While the expected return is maximized, the variance is minimized at the same time. Finding these exchange portfolios requires solving a multi-objective nonlinear optimization problem [4].

In recent years, stock investment is not only heavily traded by organizations, but it has become quite common for household investors to invest in the stock market as well. Investors usually do not like and avoid risk-taking and always look to invest in commodities and stocks of assets that have the highest returns and the least risk for them. In other words, return on investment is considered as a favorable factor and risk variance is considered as an undesirable element. In portfolio optimization issues, the main issue is the optimal selection of assets and securities that can minimize risk and maximize return on investment. There are many ways to create an optimal stock [5]. The concepts of portfolio optimization and diversity are useful in developing and understanding financial markets and decision making in this area. The publication of Harry Markowitz's portfolio theory was also the main and most important achievement in this field. Markowitz suggested that investors accept risk and return together and choose the amount of capital allocation between different investment opportunities based on the interaction between the two [6].

One of the methods that has been used in recent years to solve many optimization problems is the use of heuristic algorithms. Innovative methods that have been introduced to address the shortcomings of classical optimization methods with a comprehensive and random search, greatly guarantee the possibility of better results. In this paper, meta-heuristic methods and algorithms (PSO and NSGA-II) have been used to select the appropriate portfolio. In reviewing the results obtained from portfolios, meta-heuristic methods have less risk and higher returns than classical methods.

II. Materials and Method

A. *Classic Methods for Portfolio Optimization*

Harry Markowitz's work formed the basis of what is now known as the Modern Portfolio Theory. Modern portfolio theory (MPT) is an investment framework for selecting and building an investment portfolio based on maximizing the expected return on the portfolio while minimizing investment risk [7]. The problem of selecting the optimal portfolio was proposed in 1952 by Harry Markowitz with the aim of maximizing the expected returns, provided the variance is limited from above [8].

In generally, parametric risk can be measured using various mathematical formulas and through the concept of diversification, which aims to select the correct weighted set of investment assets, which together are less risky factors than investing in any single asset or asset class. Diversity is the main concept of modern portfolio theory [7]. Markowitz showed that under certain conditions, investor portfolio selection can be reduced by balancing the expected portfolio return and portfolio risk (variance). Given the potential for diversity risk reduction, portfolio investment risk, measured as its variance, depends on both the variances of individual asset returns and the "covariance" of asset pairs [9]. In this part of the article, we review the classical methods used in optimal portfolio selection.

1) Mean Variance Model

The issue of portfolio allocation in financial subjects is of great theoretical and practical importance. The main goal of investors is to divide their capital among different assets in the best way. The first fundamental solution to this problem was proposed by Harry Markowitz. Markowitz considered the portfolio selection process as a matter of optimizing the mean variance, the basic idea of which is to balance risk and return [10]. In the old approach to portfolio selection, the investor must estimate the expected return on stocks at $t = 0$ and then invest in the stocks with the highest expected returns. According to Markowitz's theory, this decision is irrational because the investor, in addition to maximizing the expected return, also wants to ensure the return as safe as possible, so the investor should seek to balance maximizing the expected return and reducing investment uncertainty. Markowitz also suggested that investors should consider risk and return together and allocate their budgets between investment options based on risk-return swaps [11]. The Markowitz model assumes that investors make their decision to build a portfolio by selecting assets that maximize the return on their portfolio at the end of the investment period. The mean variance of the Markowitz portfolio can be expressed mathematically as follows [4]:

$$\mu_p = \sum_{i=1}^{n} w_i \mu_i = W^T \mu \quad (1)$$

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j\, \sigma_{ij} = W^T \Sigma\, w \quad (2)$$

$$s.t \begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \end{cases} \quad (3)$$

Where w_i is weight or proportion of asset i in in the portfolio p, μ_i is expected return of asset i, μ_p is the expected return of the portfolio, σ_{ij} is covariance between asset i and j, if $i = j$, it is variance of asset i and σ_p^2 is variance of the portfolio assets.

The problem of portfolio optimization is formulated as maximizing the expected return by considering the upper limit for variance of the investment portfolio (equation 4 and 5) or minimizing the variance by considering the lower limit for the expected return (equation 6 and 7) [11]:

$$\max \quad \mu_p = \sum_{i=1}^{n} w_i \mu_i = W^T \mu \quad (1)$$

$$s.t \begin{cases} \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j\, \sigma_{ij} = W^T \Sigma\, w \leq \sigma_{p_0}^2 \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \end{cases} \quad (5)$$

$$\min \quad \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j\, \sigma_{ij} = W^T \Sigma\, w \quad (2)$$

$$s.t \begin{cases} \mu_p = \sum_{i=1}^{n} w_i \mu_i = W^T \mu \geq \mu_{p_0} \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \end{cases} \quad (3)$$

2) Mean Semi Variance Model

The mean variance method for portfolio optimization has been widely criticized by researchers in this field. Markowitz later criticized the use of mean-variance as a measure of risk in portfolio management. This criterion is more acceptable than variance because it is applicable even when the distribution of return on assets shows a wider sequence [4]. Mao [12] supports the fact that investors are only interested in downside risks and that the semi-variance criterion is more appropriate for use than the average variance criterion. The semi variance defined as [4]:

$$\mu_p = \sum_{i=1}^{n} w_i \mu_i = W^T \mu \quad (4)$$

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j\, \sigma_{i-}\sigma_{j-}\rho_{ij} = W^T \Sigma_-\, w \quad (5)$$

$$s.t \begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \end{cases} \quad (6)$$

Where w_i is weight or proportion of asset i in in the portfolio p, μ_i is expected return of asset i, μ_p is the expected return of the portfolio, σ_p^2 is variance of the portfolio assets.

The following formulas are used to calculate downside risk:

$$\sigma_p^2 = var\{R_p(t)\} = \mathbb{E}\left\{\left(R_p(t) - \mu_p\right)^2\right\} \quad (7)$$

$$\sigma_{p-}^2 = \mathbb{E}\left\{\left(R_p(t) - \mu_p\right)^2 \mid R_p(t) < \mu_p\right\} \quad (8)$$

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij} \Rightarrow \sigma_{ij-} = \sigma_{i-}\sigma_{j-}\rho_{ij} \quad (9)$$

Where σ_{p-}^2 is semi variance of the portfolio assets and $R_p(t) < \mu_p$ is the downside risk.

3) Mean Absolute Deviation Model

The mean absolute deviation (MAD) approach has been proposed by Konno and Yamazaki [13]and is now widely used by experts to solve a portfolio optimization problem on a very large scale. The MAD model uses absolute deviation of the portfolio rate of return instead of variance as a risk measure. These two criteria are mathematically equivalent to each other. However, they are computationally different because the former can be reduced to a linear programming problem, while the latter leads to a convex quadratic programming problem [14]. MAD leads to a linear programming model that has been proven to be equivalent to the Markowitz model but much more computationally tractable [15]. Mean absolute deviation defined as [13]:

$$s_i = \mathbb{E}\{|r_i(t) - \mu_i|\} \quad (10)$$

$$s_p = \sum_{i=1}^{n} w_i s_i = w^T s \tag{11}$$

Minimizing s_i is equivalent to minimizing σ_i if $(r_1, r_2, \ldots, r_n)$ is multivariate and normally distributed, leading to the following mean absolute deviation model [13]:

$$\min s_i = \mathbb{E}\{|r_i(t) - \mu_i|\} \tag{12}$$

$$s.t \begin{cases} r_i(t) \geq r_{i_0}(t) \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \end{cases} \tag{13}$$

Where $r_{i_0}(t)$ is the minimum return set by the investor.

4) Conditional Value at Risk Model

One of the most well-known risk measures is the measurement of risk value (VaR). At a given confidence level of α, var_α represents the maximum expected loss over a given period of time. Although this criterion is widely used by researchers, VaR has also been widely criticized as an incompatible risk criterion [16]. Also, using VaR in optimization is difficult because it requires solving a non-convex problem [17]. Alternatively, Rockafellar and Uryasev [18] introduced conditional VaR (CVaR), which was defined as the conditional expectation of losing a basket at least equal to VaR. Formally, to distribute the probability of a stable return on assets, CVaR is defined at the $\alpha\%$ confidence level for a portfolio with x composition as follows [19]:

$$CVaR_\alpha(x) = \frac{1}{1-\alpha} \int_{f(x,r) \geq \alpha_\beta(x)}^{\infty} f(x,r) p(r) dr \tag{14}$$

where r is the vector of random assets' returns, $p(r)$ is the associated probability density function, $f(x,r)$ denotes the portfolio loss function, and $\alpha_\beta(x)$ denotes the VaR_β threshold for the portfolio weights x.

The portfolio composition x, which optimizes CVaR at the β confidence level, while having the minimum expected yield level μ^*, is obtained by solving the following linear program:

$$\min \alpha + \frac{1}{(1-\beta)T} 1y \tag{15}$$

$$s.t \begin{cases} \alpha + y + Rx \geq 0 \\ \hat{\mu} \geq \mu^* \\ 1x = 1, x, y \geq 0, \alpha \in R \end{cases} \tag{16}$$

Here α is a decision variable representing VaR_β, and $y = \{y_1, \ldots, y_T\}$ is a vector of decision variables denoting losses that are at least equal to VaR_β.

B. *Intelligent Methods for Portfolio Optimization*

While the main problem of Markowitz theory can be solved using quadratic programming, metaheuristic algorithms have been used significantly to solve this optimization problem [20]. The classical branch of the portfolio optimization problem can be solved Considered as a one-objective optimization problem in which the investor minimizes his risk exposure provided the minimum expected return is achieved, or the investor maximizes the expected return for a certain level of risk [21]. In this article, we use the Particle Swarm Optimization algorithm to optimize the single-objective modern portfolio theory.

While Single-Objective Optimization methods consider the minimum risk for a given return or the maximum risk for a given expected return, or a goal function that weighs two goals and therefore must be performed several times with the corresponding weights [22], Multi-Object Optimization Methods use two or more sets of Pareto solutions while balancing the objective function simultaneously [23]. In this article, we use the Non-Dominated Genetic Algorithm-II to optimize the multi-objective modern portfolio theory.

1) Particle Swarm Optimization

Particle swarm optimization is an evolutionary computational method proposed by Kennedy and Eberhart in 1995 [24]. The particle swarm optimization algorithm simulates animal social behavior, including insects, swarms, birds, and fish. These groups participate in a collaborative way to find food, and each member of the herd continues to change their search pattern according to their own learning experiences and those of other members [25].

Particle swarm optimization algorithm is a swarm-based search process in which each individual is called a particle, as a potential solution to the optimized problem in the next search space D, and can remember the speed as well as the optimal position [26]. Congestion and itself In each generation, particle information is combined to adjust the velocity of each dimension and used to calculate the new position of the particle. Particles are constantly changing their state in multidimensional search space until they reach equilibrium or optimality, or beyond computational constraints. The unique connection between the different dimensions of the problem area is achieved through the objective functions. The general flowchart of the particle swarm optimization algorithm is shown in Figure 1 [25].

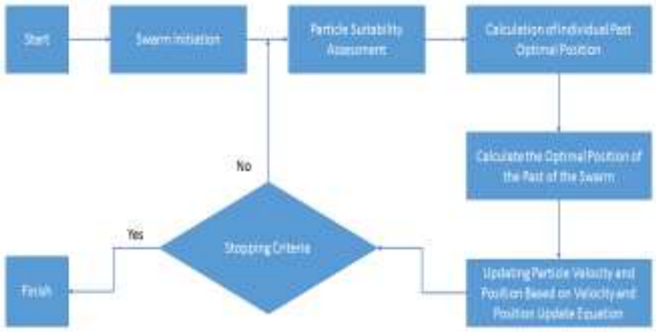

Fig. 1 General flowchart of particle swarm optimization algorithm

There are parameters in the particle swarm optimization algorithm that can affect its performance. For each optimization problem, some values and options of these parameters have a large effect on the efficiency of the particle swarm optimization method, and other parameters have little or no effect. The basic parameters of particle swarm optimization are particle swarm size or number, number of iterations, velocity components and acceleration coefficients [27].

1) Non-Dominated Sorting Genetic Algorithm-II

The genetic algorithm was created and developed thanks to the theory of biological evolution and the theory of genetics. It is a random search algorithm that mimics natural biological selection and natural genetic machinery [28]. However, there are some improved versions of the genetic algorithm. One of them, the NSGA (Non-Dominant Genetic Algorithm) was proposed by Srinivas and Deb in 1995 [29] and an improved version [30] was introduced. This algorithm uses the density and density comparison operator and transfers good people to the next generation through the elite strategy method. Thus, population levels are increasing rapidly [31].

The NSGA-II procedure begins by establishing a population of individuals. It then ranks each individual according to a rule of dominance. It then applies the evolutionary operators i.e. crossover and mutation to find a new population of offspring. After creating two equally sized populations, it combines parents and descendants to share half of the newly combined population on Pareto fronts. To ensure the diversity of the front, the NSGA-II adds a crowding distance to each individual. This ensures the diversity of the population and improves the exploration of the fitness environment [32]. Figure 2 clearly explains the general flowchart of the NSGA-II algorithm.

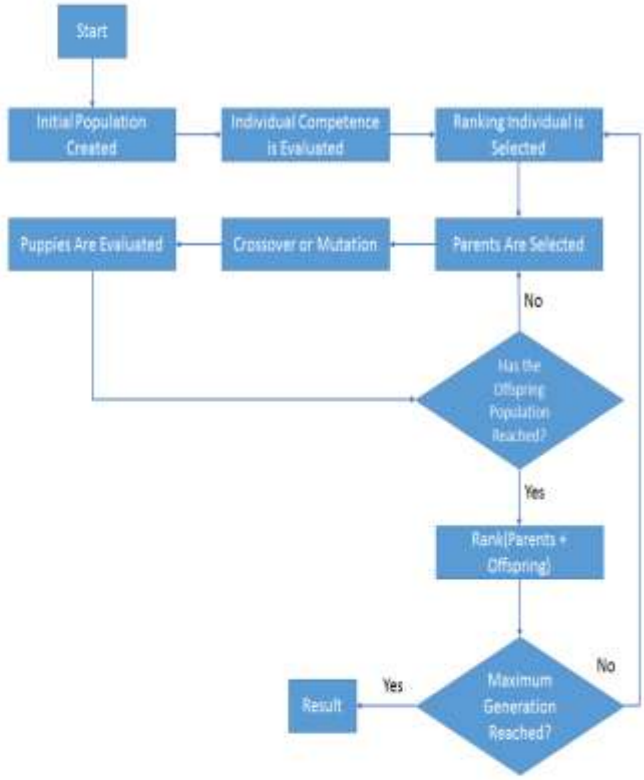

Fig. 2 General flow chart of NSGA II algorithm.

III. Results

In this part of the article, we review the results of classical and intelligent methods for portfolio optimization.

A. Results of Classical Methods for Portfolio Optimization

In this section, we tried to find the optimum portfolio by using the classical methods determined earlier. The data we use is the closing prices of 6 different companies in February 2014. As a result of the study, it was proven that the mean semi variance model performed better as seen in Figure 3. Also, when we look at Table 1, we found the highest return expectation (0.01423), with a risk of 0.02841 when we run the mean semi variance model. therefore, as stated in the above sections, it is seen that the mean semi variance model gives better results.

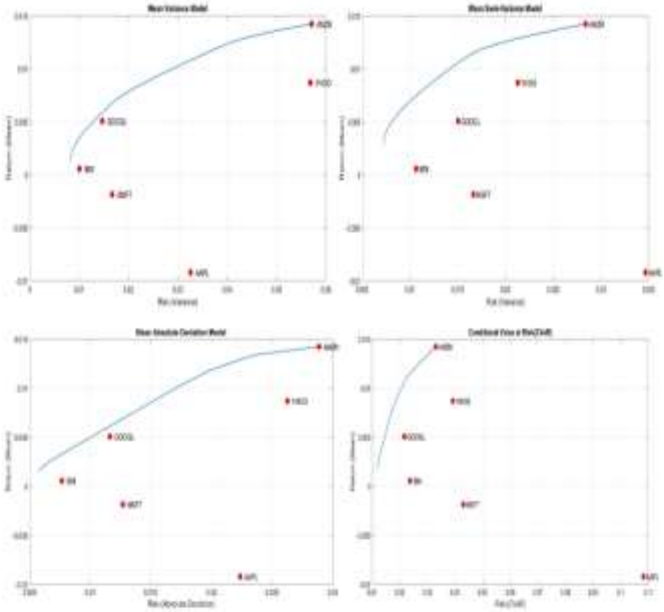

Fig. 3 Approximated pareto front for the mean variance (MV), mean semi variance (MSV), mean absolute deviation (MAD) and conditional value at risk (CVaR)

Table 1. Weight Values, Return and Risk for classical methods: mean variance (MV), mean semi variance (MSV), mean absolute deviation (MAD) and conditional value at risk (CVaR)

	MV	MSV	MAD	CVaR
IBM	0.00056	0.00056	0.00056	0.00056
GOOGL	0.00507	0.00507	0.00507	0.00507
MSFT	-0.00184	-0.00184	-0.00184	-0.0018
AAPL	-0.00920	-0.00920	-0.00920	-0.0092
YHOO	0.00869	0.00869	0.00869	0.00869
AMZN	0.01423	0.01423	0.01423	0.01423
Return	0.01423	0.01423	0.01423	0.01423
Risk	0.05706	**0.02841**	0.02889	0.03300

B. Results of Intelligent Methods for Portfolio Optimization

In this part, portfolio optimization is performed using single-objective meta-heuristic algorithms (particle swarm optimization). For single-objective optimization, the objective function was designed to minimize risk and the expectation return was desired as 0.0060. Particle swarm optimization was run with 100 iterations and 40 populations, and when looking at the results, pso-msv (particle swarm optimization-mean semi variance) found the lowest risk (0.0090). Therefore, looking at the Figure 4, it is seen that the best cost graph belongs to the mean variance model, and when looking at Table 2, it is seen that the lowest risk belongs to the mean semi variance model. As stated in the classical models, the mean semi variance model works better.

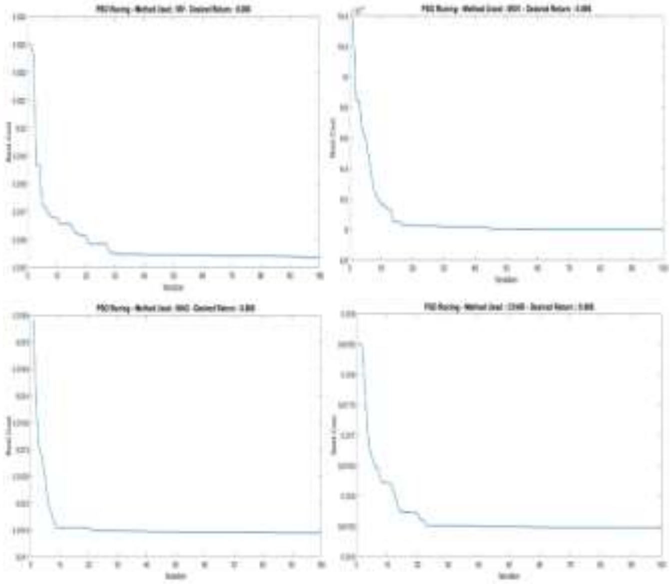

Fig. 4 The best cost graph by desired expectation return (0.0060) ,100 iterations and 40 population for pso-mv, pso-msv, pso-mad and pso-cvar

Table 2. Weight Values, Return and Risk for intelligent methods (particle swarm optimization) by desired expectation return (0.0060), 100 iteration and 40 population

	PSO-MV	PSO-MSV	PSO-MAD	PSO-CVaR
IBM	0.2274	0.4006	0.3620	0.3379
GOOGL	0.4857	0.0653	0.3068	0.3941
MSFT	3.3849e-05	0.0166	0.0029	3.0328e-04
AAPL	0	0	0	0
YHOO	0.1219	0.3410	0.0771	0
AMZN	0.1650	0.1764	0.2511	0.2677
Return	0.0060	0.0060	0.0060	0.0060
Risk	0.0153	**0.0090**	0.0114	0.0155

In this part, portfolio optimization is performed using multi-objective meta-heuristic algorithms (Non-Dominated Sorting Genetic Algorithm-II). The difference between this method and the single-objective method is to solve the problem as it is. Therefore, in this method, there is no limit on the objective function and both forms are considered minimum and maximum. NSGA-II was run with 100 iterations and 40 populations, and when looking at the Figure 4, Table 3 and Table 4 minimum risk results, NSGA-II-MSV and NSGA-II-MAD found the lowest risk respectively 0.0073 and 0.0060. when looking at the maximum return (0.0142) results, NSGA-II-MSV found the lowest risk 0.0284.

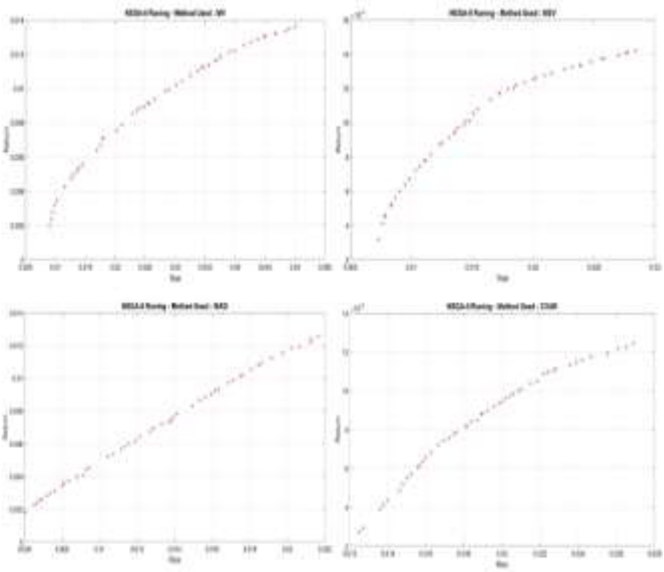

Fig. 5 Approximated pareto front for the NSGA-II-MV, NSGA-II-MSV, NSGA-II-MAD and NSGA-II-CVaR

Table 3. Weight Values, Return and Risk for minimum risk by intelligent methods (Non-Dominated Sorting Genetic Algorithm): NSGA-II-MV, NSGA-II-MSV, NSGA-II-MAD and NSGA-II-CVaR

	NSGAII -MV	NSGAII -MSV	NSGAII -MAD	NSGAII -CVaR
IBM	0.4636	0.5579	0.7083	0.5233
GOOG L	0.2889	0	0.1496	0.4767
MSFT	0.1728	0.1060	0.0489	0
AAPL	0.0050	0.0089	0.0075	0
YHOO	0.0696	0.2739	0.0763	0
AMZN	0	0.0533	0.0094	0
Return	0.0020	**0.0032**	0.0018	0.0027
Risk	0.0090	**0.0073**	**0.0060**	0.0125

Table 4. Weight Values, Return and Risk for maximum return by intelligent methods (Non-Dominated Sorting Genetic Algorithm): NSGA-II-MV, NSGA-II-MSV, NSGA-II-MAD and NSGA-II-CVaR

	NSGAII -MV	NSGAII -MSV	NSGAII -MAD	NSGAII -CVaR
IBM	0	0	0	0
GOOG L	0	0	0.0880	0.1953
MSFT	0	0	0.0046	0
AAPL	0	0	0	0
YHOO	0.1136	0	0.1403	3.9407e-04
AMZN	0.8864	1	0.7670	0.8043
Return	0.0136	**0.0142**	0.0126	0.0124
Risk	0.0501	**0.0284**	0.0217	0.0269

IV. CONCLUSION

One of the major concerns of financial managers today is to make high-speed, optimal decisions amid large volumes of stock and capital market information and data. Especially when the diversity of investments in the investment portfolio increases, optimal decisions are very important given the constraints on expected returns and the level of risk and liquidity of assets and other variables. Portfolio optimization makes it possible to attract more investors, because with the emergence of a proper investment process, fixed capital is attracted to the community. Therefore, in this article, classical and intelligent methods for portfolio optimization were examined. The results of this article are generally as follows:

1. According to the obtained results, the mean semi variance method with lower risk (0.0284) and higher return (0.0142) than the other classical methods has acted on the specified data.
2. According to the results obtained in single-objective meta-heuristic methods (Particle Swarm Optimization), with a desired return value of 0.0060 and risk minimization, the particle swarm optimization-mean semi variance method with a risk of (0.0090) has the lowest risk among other methods.
3. According to the results obtained in multi-objective meta-heuristic methods (Non-Dominated Sorting Genetic Algorithm-II), in minimizing the risk, the lowest value of risk was (0.0060) with a return of (0.0018) by the NSGA-II-MAD method, also a NSGA-II-MSV method with a risk of (0.0073) and a higher return of (0.0032) It was more efficient than other methods. But in maximizing the expected value of return, the highest return value is (0.0142) with a lower risk of (0.0284) belonging to the NSGA-II-MSV method.

According to the results, it is clear that the use of intelligent methods can provide less risk with more returns for financial investors. Therefore, in this article, by reviewing the results, it can be said that multi-objective meta-heuristic algorithms can help financial investors to choose the right portfolio.

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