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Functional diagnosis and state estimation for nonlinear systems represented by Takagi-Sugeno multi-models

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Abstract **–** In this paper, a method of state estimation and diagnostics is implemented for the non-linear systems. These systems are modeled using a Takagi-Sugeno multi-model with measurable decision variables in order to be able to use a classical technique of bank of observers. First, we present the approximation of the nonlinear model by a multi-model, and then the development of a multi-observer allows estimating the states of the system. This type of observer is then used in bank of observers which generate residues its analysis makes it possible to reveal the occurrence of sensor defects. Finally, this diagnostic strategy is applied to a hydraulic system to illustrate the effectiveness of the proposed method.

Keywords – Nonlinear System, Takagi-Sugeno Multi-Model, Multi-Observer, Diagnosis, Identification, Levenberg-Marquardt Algorithm, State Estimation, Linear Matrix Inequalities (LMI).

I. INTRODUCTION

The diagnosis based on observers is a technique that has been the subject of many developments. This is based on a model of good operation of the system, to make a state estimation from the knowledge of the inputs and outputs of the system and to use the error of estimation of the output as a residual. In normal operation, this residual must be substantially zero (to modeling errors and measurement errors) and deviate significantly from zero when a fault occurs on the system. The detection of the occurrence of defects is in general quite easy; on the other hand, its location (the determination of the input or output grandeur on which it has intervened) is more delicate. So, we use frequently, a technique based on the development of bank of observers piloted by different grandeurs. The analysis of the different residues generated by these observers coupled to decision logic then allows the localization of the defects. These methods were first developed for linear models. They have subsequently been extended to systems described by nonlinear models. However, in this case, the design of observer is much more delicate and the works developed has focused on particular classes of nonlinear systems, for example, Lipschitz systems [1], [2], [3] or systems. LPV [4]. The modeling of systems using Takagi-Sugeno models [5], [6] is an interesting way to represent the behavior of nonlinear systems [7]. This is based on the use of a set of linear models and an interpolation mechanism of these models. In certain cases, this type of model permits to describe, in an exact way, the nonlinear behavior of a system by rejecting, in the interpolation functions, all the nonlinearities of the system [7]. The major interest of this formulation resides in its simplicity. This type of modeling allows to transpose, to nonlinear systems, some results obtained for linear systems. Mention for example the works relating to the study of the stability or stabilization of systems [6], [8], [9], [10] where the authors propose sufficient stability conditions developed using the techniques applied to linear models.

II. THE MULTI-MODEL

The multi-models [11] are an interesting alternative and a powerful tool in modelling non-linear systems. The multi-model approach is based on the decomposition of the dynamic behaviour of the non-linear system into a number L of operating domains, each domain being characterized by a linear sub-model. Figure (1) illustrates this principle in a two-dimensional case the set of system operating points of the coordinate $x(t) = (x_1(t), x_2(t))$, has been decomposed into three operating local domains noted$D_1, D_2 and D_3$. The overall domain of operating is then defined by the meeting of the local domains$D = D_1 \cup D_2 \cup D_3$. On each of the local domains, or sub domains, can be built a local model. The output of each sub-model contributes more or less to the approximation of the overall behaviour of the nonlinear system. The contribution of each sub-model is defined by a weighting function. These different local models can then be combined using an interpolation technique to obtain a global representation, or multi-model, valid on the global operating domain D.

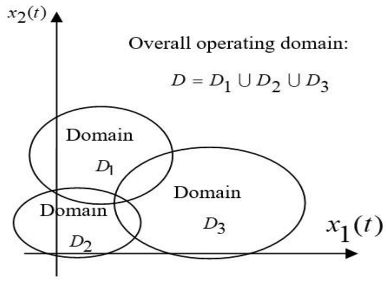

Fig. 1 Principle of the multi-model approach

Several structures permit to interconnect the different sub-models in order to generate the global output of the multi-model. Two essential structures of multi-models can be distinguished, one where the sub-models share the same state vector (Takagi-Sugeno multi-model), the other where the sub-models are decoupled, each sub-model then having its own state vector (decoupled multi-model). The Takagi-Sugeno multi-model is currently the most commonly used.

A. Conception a structure of multi-model

Three distinct methods can be used to obtain a multi-model by identification, by linearization around different operating points (in this case, they are affine local models due to the presence of the linearization constant) or by convex polytopic transformation. In the first situation, from inputs and outputs data, we can identify the parameters of the local model corresponding to the various operating points. In the second and third situation, we assume to have a nonlinear mathematical model. In this document we present the second method. To illustrate this method, we consider a nonlinear mathematical model (1) of the physical process that is linearized around various judiciously chosen operating points, for which we seek to determine a multi-model representation allows to describe the behaviour of this system.

$$\dot{x}(t) = f(x(t), u(t)) \qquad (1)$$

With $f(.) \in C^1$ is a nonlinear function, $x(t) \in \Re^n$ is the state vector and $u(t) \in \Re^m$ is the input vector. Suppose we have a set of N local models $f_i(x(t), u(t))$, $i \in \{1, \dots, N\}$describing the behavior of the system in different areas of operation, each local model built by the linearization of the system (1) around an arbitrary operating points $(x_i, u_i) \in \Re^n \times \Re^m$:

$$f_i(x(t), u(t)) = A_i(x(t) - x_i) + B_i(u(t) - u_i) + f(x_i, u_i) \quad (2)$$

That can be rewritten in the form:

$$f_i(x(t), u(t)) = A_i x(t) + B_i u(t) + d_i \quad (3)$$

The local validity of each mode f_i is indicated by a validity function $w_i(\xi(t))$ for $i \in \{1, \dots, N\}$

The global model is obtained in the following way:

$$\dot{x}_m(t) = \frac{\sum_{i=1}^{N} w_i(\xi(t)) f_i(x(t),u(t))}{\sum_{j=1}^{N} w_j(\xi(t))} \quad (4)$$

We pose:

$$\mu_i(\xi(t)) = \frac{w_i(\xi(t))}{\sum_{j=1}^{N} w_j(\xi(t))} \quad (5)$$

By combining equations (4) and (5), we obtain the general expression of a structure multi-model:

$$\dot{x}_m(t) = \sum_{i=1}^{N} \mu_i(\xi(t)) f_i(x(t),u(t)) \ (6)$$

We replace the equation (3) in (6), we obtain:

$$\dot{x}_m(t) = \sum_{i=1}^{N} \mu_i(\xi(t))(A_i x(t) + Bu(t) + d_i) \ (7)$$

The activation function $\mu_i(\xi(t)), i \in \{1,\dots,N\}$ determines the degree of activation of the associated i^{ih} local model, this function indicates the more or less important contribution of the corresponding local model in the global model (multimodel). It ensures a gradual transition from this model to neighboring local models. These functions are generally triangular, sigmoidal or Gaussian, and must satisfy the following properties:

$$\begin{cases} \sum_{i=1}^{N} \mu_i(\xi(t)) = 1 \\ 0 \le \mu_i(\xi(t)) \le 1 \end{cases} \quad (8)$$

B. Parametric optimization

The Parametric optimization consists in estimating the parameters of the activation functions and those of the local models, these parameters must be optimized by an iterative procedure because of the non-linearities of the global model (multi-model) to its parameters. The Parametric identification methods are generally based on the minimization of a functional of the difference between $x_m(t)$ estimated by the multi-model and $x(t)$ estimated by the system (1). The criterion most often used is the criterion which represents the quadratic difference between the two indicated outputs.

$$J(\theta) = \frac{1}{2}\sum_{t=1}^{M} \varepsilon(t,\theta)^2 = \frac{1}{2}\sum_{t=1}^{M}(x_m(t) - x(t))^2 \ (9)$$

Where M is the observation horizon and θ is the parameter vector of the local models and those of the activation functions. Among the iterative optimization methods of the Quasi-Newton type, the Marquardt method, which is considered one of the most efficient resolution methods, does not require long calculations or large memory space.

C. Marquardt algorithm

If n is iteration index of the Marquardt algorithm and $\theta^n$the value of the solution at iteration n, the update of the estimate is done as follows:

$$\theta^{n+1} = \theta^n - [G(\theta^n)^T G(\theta^n) + \mu_n D^2(\theta^n)]^{-1} G(\theta^n)^T \varepsilon(t,\theta) \quad (10)$$

Where: $G(\theta^n)$: represents the jacobian matrix

$D^2(\theta^n)$: is the diagonal matrix containing the elements of the diagonal of $G^T G$. To remedy the case where the elements of the diagonal are null, we take:

$$D^2(i,i) = G^T G(i,i) + 1$$

μ_n: is a parameter of Marquardt and which is chosen in such a way that: $J(\theta^{n+1}) < J(\theta^n)$ (11)

III. APPLICATION TO THE THREE TANK SYSTEM

To approach a nonlinear dynamic system by a multi-model we have chosen to study the system of the three tanks because we know relatively well its mathematical description.

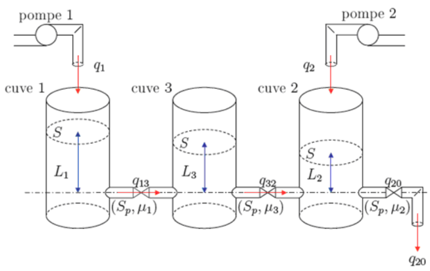

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Fig. 2 Schematization of the 3-tank system

A. *System description*

The benchmark considered consists of three cylindrical vessels of identical section S. The tanks are connected by two cylindrical pipes of section$S_p$whose viscosity coefficient is $\mu_1 = \mu_3$. the output of the system is located at the tank 2, it is also characterized by a section $S_p$and a viscosity coefficientμ_2. Two pumps controlled by DC motors feed tanks 1 and 2 with flow rates q1 (t) and q2 (t). The three tanks are equipped with pressure sensors to measure the liquid level$(L_1(t), L_2(t)$ et $L_3(t))$.

B. *Mathematical model of the system*

By writing the equations of the conservation of the volume of liquid, we obtain:

$$\sum NL: \begin{cases} S\frac{dL_1(t)}{dt} = q_1(t) - q_{13}(t) \\ S\frac{dL_2(t)}{dt} = q_2(t) - q_{32}(t) - q_{20}(t) \\ S\frac{dL_3(t)}{dt} = q_{13}(t) - q_{32}(t) \end{cases} \quad (12)$$

Where $q_{ij}(t)$ is the flow rate of liquid from the tank i to the tank $j(i, j = 1,2,3 \forall i \neq j)$
which can be expressed using Torricelli's law by:

$$q_{ij} = \mu_i . S_p . sign(L_i(t) - L_j(t)) . \sqrt{2g\left|L_i(t) - L_j(t)\right|} \quad (13)$$

And $q_{20}(t)$ represents the output flow, with:

$$q_{20} = \mu_2 . S_p . \sqrt{2gL_2(t)} \quad (14)$$

we consider the system as the levels verify the following inequalities$L_1(t) > L_3(t) > L_2(t)$, we assume that the system of the three tanks is perfectly described using the defined nonlinear model (15)

$$\sum NL: \begin{cases} \dot{x}_1(t) = -2C_1\sqrt{x_1(t) - x_3(t)} + u_1(t)/S \\ \dot{x}_2(t) = 2C_3\sqrt{x_3(t) - x_2(t)} - 2C_2\sqrt{x_2(t)} + u_2(t)/S \\ \dot{x}_3(t) = 2C_1\sqrt{x_1(t) - x_3(t)} - 2C_3\sqrt{x_3(t) - x_2(t)} \\ y_1(t) = x_1(t) \\ y_2(t) = x_2(t) \\ y_3(t) = x_3(t) \end{cases} \quad (15)$$

With $x_i(t)$ is the level of liquid in the tank i and $C_i = (1/2).(1/S).\mu_i.S_P.\sqrt{2g}$. The two control signals $u_1(t), u_2(t)$ are respectively the two input flow rates $q_1(t)$ and$q_2(t)$.

C. *Representation of the nonlinear model by a multi-model*

We consider a multi-model composed from three coupled local models:

$$\begin{cases} \dot{x}_m(t) = \sum_{i=1}^{3} \mu_i(\xi(t))(A_i x_m(t) + B_i u(t) + D_i) \\ y_m(t) = Cx_m(t) \end{cases} \quad (16)$$

with $\dot{x}_m(t) = [\dot{x}_{m1}(t)\dot{x}_{m2}(t)\dot{x}_{m3}(t)]^T$

The activation functions μ_i were constructed as follows:

$$w_i(u(t)) = exp(\frac{-(u_1(t) - u_i)^2}{2\sigma_i^2}); \mu_i(u(t)) = \frac{w_i(u_1(t))}{\sum_{i=1}^{3} w_i(u_1(t))}$$

The index i corresponds to the ith local model. each local domain i have an operating point $p_i(x_{1i}, x_{2i}, x_{3i}, u_{1i}, u_{2i})$ such that $i = 1,2,3$

The different operating point coordinates are obtained by the resolution of the system (15)

$$\begin{cases} -2C_1\sqrt{x_1(t) - x_3(t)} + u_1(t)/S = 0 \\ 2C_3\sqrt{x_3(t) - x_2(t)} - 2C_2\sqrt{x_2(t)} + u_2(t)/S = 0 \\ 2C_1\sqrt{x_1(t) - x_3(t)} - 2C_3\sqrt{x_3(t) - x_2(t)} = 0 \end{cases} \quad (17)$$

The numerical values of the operating points are:

Table 1. The three operating points

i	x_{1i}	x_{2i}	x_{3i}	u_{1i}	u_{2i}
1	7.476	3.182	5.413	0.2480	0.0600
2	3.635	2.629	3.152	0.1200	0.1600
3	18.008	5.917	12.195	0.4160	0.0040

We will identify the parameters of the activation functions σ_i from the minimization of the criterion $J(\theta)$ defined as follows: $J(\theta) = \frac{1}{2}\sum_{t=1}^{3}[(x_{is}(t) - x_{im}(t))^2]$ (18)

We minimize the criterion (18) by the algorithm of Marquardt, after the optimization, we have found:

$\sigma_1 = 0.1274 .\ 10^{-4}; \sigma_2 = 0.1251.\ 10^{-4} \sigma_3 = 0.0277.\ 10^{-4}$

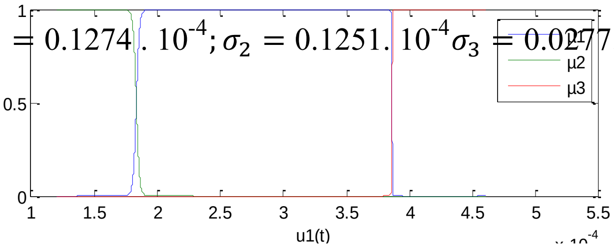


Fig. 3 The evolution of the three activation functions

To evaluate the simulation results, we simulate two models in parallel: the multi-model (16), and the nonlinear model (15). The figure (4) shows the superposition of the output vector components of the nonlinear model and their approximation by the multi-model.

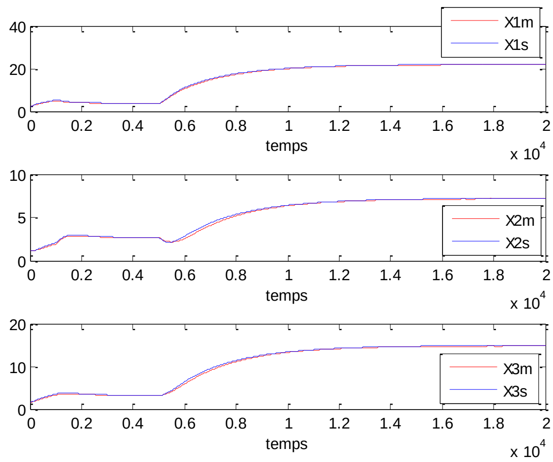


Fig. 4 The state variables of the nonlinear model "X1m, X2m X3m,", and those of the multi-model "X1s, X2s X3s".

D. *Conclusion 1 : we can conclude that the multi-model approach is a useful technique in approximating nonlinear models by local models. And offers the possibility of extending control and diagnostic techniques from linear systems to non-linear systems.*

IV. STATE ESTIMATION OF NON-LINEAR SYSTEMS REPRESENTED BY MULTI-MODELS

A. Method of designing a multi-observer

Consider a nonlinear dynamic system represented by a multi-model, composed of M local models, described by the following equations:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + D_i) \\ y(t) = Cx(t) \end{cases} \quad (19)$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector and $y(t) \in \Re^p$ represents the output vector. The matrices A_i, B_i, D_i, C are of appropriate size. $\mu_i(\xi(t))$ are the activation functions of local models and $\xi(t)$ represents the vector of decision variables that can depend on the state, outputs or inputs.

To design a multi-observer, it is assumed that local models are locally observable, that is all pairs (A_i, C) are observable. To design the multi-observer, we associate with each local model a local observer, the multi-observer (global observer) is a sum of local observers weighted by activation functions identical to those associated with local models of the multi-model [12]. The equations governing the multi-observer are as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i\hat{x}(t) + B_i u(t) + D_i + G_i(y(t) - \hat{y}(t))) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \quad (20)$$

Where$\hat{x}(t)$ represents the state vector estimated by the multi-observer,$\hat{y}(t)$ is the estimated output vector and $G_i \in \Re^{n\times p}$are the gains of the local observers. The state estimation error is defined by the following equation: $e(t) = x(t) - \hat{x}(t)$ (21)

The dynamics of the state estimation error is explained:

$$\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i - G_i C)e(t) \quad (22)$$

If the state estimation error (22) converges asymptotically to zero, the estimation of the state and output vectors (20) converges asymptotically to the state and output vectors of the multi-model (19) respectively.

B. Application to the three tank system

Consider the nonlinear three-tank system represented by the multi-model described by equation (16), Initial values of state variables and estimated variables are:

$$x(0) = [1.8 \;\; 1.2 \quad 1.6]^T et \hat{x}(0) = [111]^T$$

Using the multi-observer (20), the gains $G_i$are chosen so as to ensure the observer's stability (pole placement technique).

$$G_1 = \begin{bmatrix} 0.0087 & -0.0000 & 0.0039 \\ -0.0000 & 0.0058 & 0.0036 \\ 0.0039 & 0.0036 & 0.0051 \end{bmatrix}, G_2 = \begin{bmatrix} 0.0045 & 0.0000 & 0.0081 \\ -0.0000 & 0.0017 & 0.0075 \\ 0.0081 & 0.0075 & -0.0029 \end{bmatrix}, G_3 = \begin{bmatrix} 0.0103 & 0.0000 & 0.0023 \\ 0.0000 & 0.0081 & 0.0022 \\ 0.0023 & 0.0022 & 0.0081 \end{bmatrix}$$

Figures (5), (6) respectively represent the outputs y (t) and their estimates by the multi-observer and the evolution of the estimation errors of the outputs.

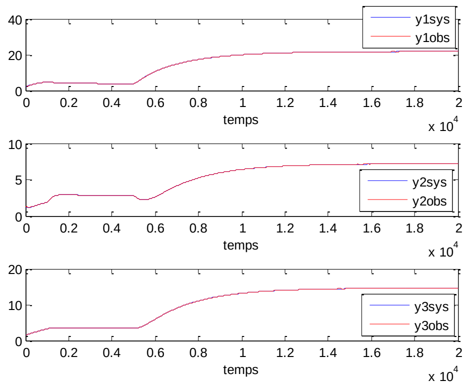


Fig. 5 Evolution of outputs and their estimates

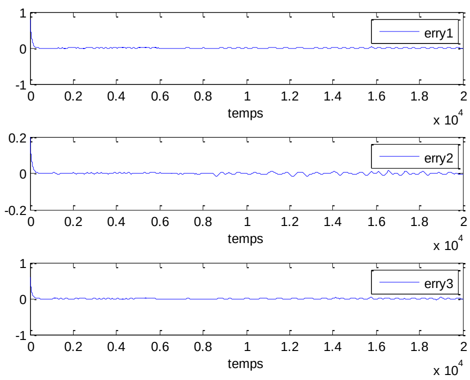


Fig. 6 Evolution of estimation errors of outputs

We note that the quality of the estimate is satisfactory. This type of observer can be used in the field of diagnosis to detect sensor faults as we will present in the following paragraph.

V. DETECTION AND LOCATION OF SENSOR FAULTS APPLICATION TO THE THREE-TANK SYSTEM

It is possible to exploit the proposed state observer from a perspective of diagnosis and location of sensor faults of a nonlinear system described by the multi-model. For the detection of sensor faults, it is possible to use an observer bank according to the DOS architecture (Dedicated Observer Scheme) presented in Fig. (7). The ith observer is controlled by the ith output and all the inputs; the output of this ith observer is insensitive to the faults of the unused outputs so each residue from an observer is sensitive to a single sensor fault which permit to detect and locate the faults even when they occur simultaneously.

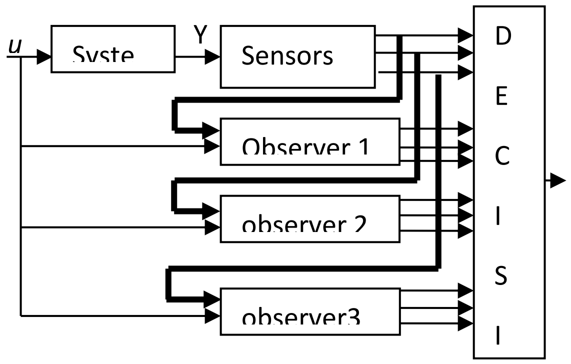


Fig. 7 Sensor fault detection by DOS structure

The three-tank system contains three outputs (y1, y2 and y3), the number of multi-observers that can be developed is 3.

The of the j[ih] multi-observer is explicit:

$$\begin{cases} \dot{\hat{x}}^j = \sum_{i=1}^{3} \mu_i(u(t))(A_i \hat{x}^j(t) + B_i u(t) + D_i + G_i(y^j(t) - \hat{y}^j(t)) \\ \hat{y}^j = \sum_{i=1}^{3} C\hat{x}(t) \end{cases} \quad (23)$$

Where $\hat{x}^j(t)$ (respectively $\hat{y}^j(t)$) represents the estimated state vector (respectively the estimated output vector) by the multi-observer jih. The of observers permits to generate different residues:

$$r_{ij}(t) = y_i(t) - y_i^j(t), pouri \in \{1,2,3\} etj \in \{1,2,3\} \quad (24)$$

A. Development of the signature table

The table of the signatures is elaborate starting from the following considerations:

1. Multi-observer 1 reconstructs the output of the multi-model using only the output y1 and all the inputs of the system. If this output has a fault, then there is a poor estimate of the states and the residues r_{i1} can be affected. It should be emphasized, however, that it is difficult to predict the evolution of the state estimate in the presence of a fault on the output y1.
2. If the output y1 does not present a fault then the state estimation is correctly carried out. Therefore, in the presence of a fault on the output y2 the residuer_{21} is removed from zero (sensitivity to the fault δ_2) while the residue r_{11} remains insensitive to this same fault. It is then possible to draw a positive conclusion on the presence of a fault δ_2 if the residues r_{i1} simultaneously present the signature:$r_{11} = 0$and $r_{21} = 1$

A similar approach is adopted to develop the signatures of observers 2 and 3. We draw all the possible cases of sensor faults. We define a binary function of the residues

$$z_{ij}(t) = \begin{cases} 0 si r_{ij}(t) = 0 \\ 1 si r_{ij}(t) \neq 0 \end{cases} \quad (25)$$

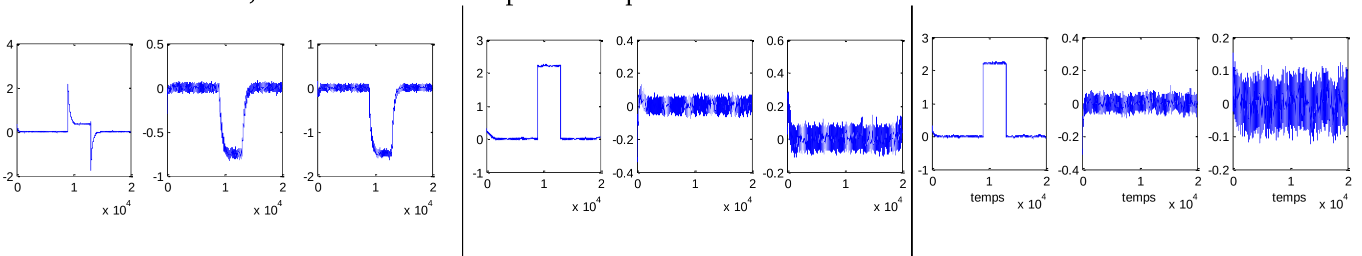
Table 2. Theoretical signature of sensor faults

	$z_{11}(t)$	$z_{21}(t)$	$z_{31}(t)$	$z_{12}(t)$	$z_{22}(t)$	$z_{32}(t)$	$z_{13}(t)$	$z_{23}(t)$	$z_{33}(t)$
δ_1	1	1	1	1	0	0	1	0	0
δ_2	0	1	0	1	1	1	0	1	0
δ_3	0	0	1	0	0	1	1	1	1

Note δ_ia variable associated with the sensor iit can take two values 0 or 1, see Table (2).

- $\delta_i = 1 \Rightarrow$ the sensor i is faulty,
- $\delta_i = 0 \Rightarrow$normal operating,

From Table (2), we notice that the failure signatures are independent. Thus, it is theoretically possible to detect and locate sensor faults even if they appear simultaneously on tow outputs. Consider now the case of the appearance of a fault affecting the measurement y1 (t) appears s at the at time t = 90 sec and disappears at time t = 130 sec, with a constant amplitude equal to 2.2.



The residues $r_{i1}(j=1)$ $and\ i = 1,2,3$	The residues $r_{i2}(j=2)$ $and\ i = 1,2,3$	The residues $r_{i3}(j=3)$ $and\ i = 1,2,3$

Fig. 8 Residues $r_{ij}(t)$ obtained by the bank of multi-observers in the presence of faults

B. Evaluation and analysis of the residues

The generation of the experimental signature matrix consists of associating each residue with the value 0 or 1 according to whether or not it is assigned by default. In a simplified manner, the detection of defects at a residue is similar to the following logical test:

if $|r_{ij}(t)| \leq \tau_{ij}$ then no fault affects the residue

if $|r_{ij}(t)| > \tau_{ij}$ then the residue is affected by a fault

where the variable τ_{ij} represents the threshold associated, its value, can for example, being determined starting from the following expression: $\tau_{ij} = \alpha\sqrt{Var(r_{ij})}$ (26)

Where α is a parameter of adjustment of the sensitivity of detection and $Var(r_{ij})$ is the empirical variance of the residue $r_{ij}(t)$ under normal functioning.

At the end of the test of each of the residues $r_{ij}(t)$, the experimental binary signature, noted z_{ij}^{*}, is generated at each instant t, as follows: $z_{ij}^{*} = \begin{cases} 0 si |r_{ij}(t)| \leq \tau_{ij} \\ 1 si |r_{ij}(t)| \geq \tau_{ij} \end{cases}$ (27)

C. Location of faults

The location of the faults is based on the comparison, at each moment, of the experimental fault signature with the different theoretical signatures.

the analysis of the residues in figure (8) gives the experimental signature of defect as follows: (111100100), According to the comparison of the resulting experimental signature with the theoretical signatures of the output's faults, we find that the first sensor is at default.

VI. CONCLUSION

In This paper, a strategy for state estimation and diagnosis of nonlinear systems is presented. The proposed methodology uses Takagi Sugeno models this type of model permits to design of multi-observer which allows the estimation of states, and the detection and location of sensor faults using bank of observer. In order to test their efficiencies, we applied on a nonlinear system "three-tank system". The simulation results showed the ability of multi-models in the approximation of nonlinear models by local linear models, and the performance of multi-observer for state estimation and for fault detection and localization. The multi-observer bank according to the DOS architecture was created for the detection and localization of sensor faults.

REFERENCES

[1] R. Rajamani. Observers for Lipschitz nonlinear systems. IEEE Transactions on Automatic Control, 43 :397–401, March 1998.

[2] S. Raghavan and J. K. Hedrick. Observer design for a class of nonlinear systems. International Journal of Control, 59(2) :515–528, 1994.

[3] A.M. Pertew, H.J. Marquez, and Q. Zhao. LMI-based sensor fault diagnosis for nonlinear lipschitz systems. Automatica, 43 :1464–1469, 2007.

[4] G. Balas, J. Bokor, and Z. Szabo. Failure detection for LPV systems - a geometric approach. In Proceedings of the American Control Conference, 2002.

[5] T. Takagi and M. Sugeno. Fuzzy identification of systems and its applications to modeling and control. IEEE Transactions on Systems, Man, and Cybernetics, 15 :116–132, 1985.

[6] K. Tanaka, T. Ikeda, and H.O. Wang. Fuzzy regulators and fuzzy observers: Relaxed stability conditions and lmi-based designs. IEEE Transactions on Fuzzy Systems, 6(2) :250–265, 1998.

[7] K. Tanaka and H.O. Wang. Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. Hardcover, 2001.

[8] T.M. Guerra, A. Kruszewski, L. Vermeiren, and H. Tirmant. Conditions of output stabilization for nonlinear models in the Takagi-Sugeno’s form. Fuzzy Sets and Systems, 157(9) :1248–1259, May 2006.

[9] A. Kruszewski. Lois de commande pour une classe de mod`eles non lin´eaires sous la forme Takagi-Sugeno : Mise sous forme LMI. PhD thesis, Universit´e de Valenciennes et du Hainaut-Cambresis, 2006, (In french).

[10] M. Chadli. Analyse des systèmes non linéaires décrits par des multi-modèles. PhD thesis, Institut National Polytechnique de Lorraine, 2002.

[11] R. Murray-Smith et T.A. Johansen. Multiple model approaches to modelling and control. Taylor & Francis, London, 1997.

[12] R. J. Patton et J. Chen, Observer-based fault detection and isolation: Robustness and applications, Control Engineering Practice, Vol. 5 (5), pp. 671-682, 1997.

[13] 3R. J. Patton et J. Chen ,Neural networks in nonlinear dynamic systems fault diagnosis, Engineering simulation, Vol. 13 (6), pp. 905-924, 1996.

[14] A. Akhenak, Conception d’observateurs non linéaires par approche multimodèle, Doctorat de L’Institut National Polytechnique de Lorraine, 2003

[15] S. Boyd, L. El Ghaoui, E. Feron et V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Philadelphia : SIAM, 1994.