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Modeling of a Hydraulic System Using the Takagi-Sugeno Model and Parametric Optimization with the Levenberg-Marquardt Algorithm

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Abstract **–** Process industries have long faced the challenge of liquid level control. The performance of a control system largely depends on the accuracy of the mathematical model used to predict its dynamic behavior. This paper presents the development of a Takagi-Sugeno fuzzy model for a coupled-tank system, based on a linearization technique. Furthermore, the Marquardt-Levenberg optimization algorithm was employed to identify the parameters of the Takagi-Sugeno model. Finally, a comparison between the nonlinear model and the identified model was conducted, demonstrating satisfactory results.

Keywords – Nonlinear System, Takagi-Sugeno Multi-Model, Identification, Levenberg-Marquardt Algorithm, Hydraulic System, Coupled-Tank System.

I. INTRODUCTION

Liquid level control plays a crucial role in various process industries, particularly in the petrochemical, biochemical, spray coating, wastewater treatment, beverage, and pharmaceutical sectors [1]. Efficient management of these processes is essential to ensure product quality, optimize resource consumption, and maintain the proper functioning of industrial systems.

To achieve these objectives, modeling of hydraulic systems is a fundamental step. It allows for analyzing and understanding the dynamic behavior of processes, facilitating the design of effective control and diagnostic strategies [2]. The nonlinear nature of these systems requires advanced models capable of capturing complex process dynamics.

In this paper, we focus on the modeling of a three-tank hydraulic system using a Takagi-Sugeno multi-model approach [3]. This type of model is particularly well-suited for complex systems as it combines several local linear models, defined by fuzzy rules, to represent an overall nonlinear behavior. To enhance the accuracy and robustness of the model, we employed the Marquardt-Levenberg algorithm to optimize its parameters [4]. This algorithm, widely used for nonlinear model optimization, effectively adjusts model parameters by minimizing the error between predictions and actual data [5].

Due to its advantageous mathematical properties, the Takagi-Sugeno model, optimized using the Marquardt-Levenberg algorithm, provides an efficient trade-off between accuracy and complexity. It also enables the extension of control and diagnostic techniques from linear to nonlinear systems, paving the way for improved management of industrial hydraulic processes.

II. THE MULTI-MODEL

The Multimodels [6] are an interesting alternative and a powerful tool in modeling non-linear systems. The multimodel approach is based on the decomposition of the dynamic behavior of the non-linear system into a number L of operating domains, each domain being characterized by a linear sub-model.

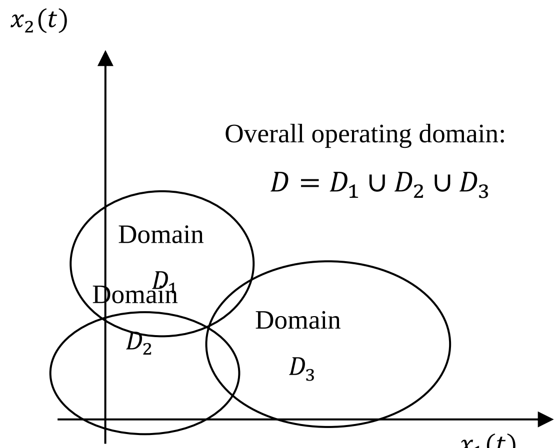

Fig. 1 Principle of the multi-model approach

Figure (1) illustrates this principle in a two-dimensional case the set of system operating points of the coordinate$x(t) = (x_1(t), x_2(t))$, has been decomposed into three operating local domains noted$D_1, D_2 and D_3$. The overall domain of operating is then defined by the meeting of the local domains$D = D_1 \cup D_2 \cup D_3$. On each of the local domains, or sub domains, can be built a local model. The output of each sub-model contributes more or less to the approximation of the overall behavior of the nonlinear system. The contribution of each submodel is defined by a weighting function. These different local models can then be combined using an interpolation technique to obtain a global representation, or multimodel, valid on the global operating domain$D$.

Several structures permit to interconnect the different sub-models in order to generate the global output of the multimodel. Two essential structures of multimodeles can be distinguished, one where the submodels share the same state vector (Takagi-Sugeno multimodel), the other where the submodels are decoupled, each submodel then having its own state vector (decoupled multimodel). The Takagi-Sugeno multimodel is currently the most commonly used.

A. Conception a structure of multimodel

Three distinct methods can be used to obtain a multimodel by identification, by linearization around different operating points (in this case, they are affine local models due to the presence of the linearization constant) or by convex polytopic transformation. In the first situation, from inputs and outputs data, we can identify the parameters of the local model corresponding to the various operating points. In the second and third situation, we assume to have a nonlinear mathematical model. In this document we present the second method. To illustrate this method, we consider a nonlinear mathematical model (1) of the physical process that is linearized around various judiciously chosen operating points, for which we seek to determine a multimodel representation allows to describ the behavior of this system.

$$\dot{x}(t) = f(x(t), u(t)) \qquad (1)$$

With $f(.) \in C^1$ is a nonlinear function, $x(t) \in \Re^n$ is the state vector and $u(t) \in \Re^m$ is the input vector.

Suppose we have a set of N local models $f_i(x(t), u(t))$, $i \in \{1, \dots, N\}$describing the behavior of the system in different areas of operation, each local model built by the linearization of the system (1) around an arbitrary operating points $(x_i, u_i) \in \Re^n \times \Re^m$:

$$f_i(x(t), u(t)) = A_i(x(t) - x_i) + B_i(u(t) - u_i) + f(x_i, u_i) \qquad (2)$$

That can be rewritten in the form:

$$f_i(x(t), u(t)) = A_i x(t) + B_i u(t) + d_i \quad (3)$$

With

$$A_i = \frac{\partial f(x,u)}{\partial x}\Big|_{\substack{x=x_i \\ u=u_i}}, B_i = \frac{\partial f(x,u)}{\partial u}\Big|_{\substack{x=x_i \\ u=u_i}}, d_i = f(x_i, u_i) - A_i x_i - B_i u_i$$

The local validity of each mode f_i is indicated by a validity function $w_i(\xi(t))$ for $i \in \{1, \dots, N\}$
The global model is obtained in the following way:

$$\dot{x}_m(t) = \frac{\sum_{i=1}^{N} w_i(\xi(t)) f_i(x(t), u(t))}{\sum_{j=1}^{N} w_j(\xi(t))} \quad (4)$$

We Pose :

$$\mu_i(\xi(t)) = \frac{w_i(\xi(t))}{\sum_{j=1}^{N} w_j(\xi(t))} \qquad (5)$$

By combining equations (4) and (5), we obtain the general expression of a structure multi-model:

$$\dot{x}_m(t) = \sum_{i=1}^{N} \mu_i(\xi(t)) f_i(x(t), u(t)) \quad (6)$$

We replace the equation (3) in (6), we obtain :

$$\dot{x}_m(t) = \sum_{i=1}^{N} \mu_i(\xi(t))(A_i x(t) + Bu(t) + d_i) \qquad (7)$$

The activation function $\mu_i(\xi(t)), i \in \{1, \dots, N\}$ determines the degree of activation of the associated i^{ih} local model, this function indicates the more or less important contribution of the corresponding local model in the global model (multimodal). It ensures a gradual transition from this model to neighboring local models. These functions are generally triangular, sigmoidal or Gaussian, and must satisfy the following properties:

$$\begin{cases} \sum_{i=1}^{N} \mu_i(\xi(t)) = 1 \\ 0 \le \mu_i(\xi(t)) \le 1 \end{cases} \qquad (8)$$

And $\xi(t)$ is the vector of the decision variables, depending on the measurable state variables and eventually the command$u(t)$. We note that in this case, the number of local models (N) depends on the desired modeling accuracy, the complexity of the non-linear system and the choice of the structure of the activation functions.

B. Parametric optimization

The Parametric optimization consists in estimating the parameters of the activation functions and those of the local models, these parameters must be optimized by an iterative procedure because of the non-linearities of the global model (multimodel) to its parameters. The Parametric identification methods are generally based on the minimization of a functional of the difference between $x_m(t)$ estimated by the multimodel and$x(t)$ estimated by the system (1). The criterion most often used is the criterion which represents the quadratic difference between the two indicated outputs.

$$J(\theta) = \frac{1}{2}\sum_{t=1}^{M} \varepsilon(t, \theta)^2 = \frac{1}{2}\sum_{t=1}^{M}(x_m(t) - x(t))^2 \ (9)$$

Where M is the observation horizon andθ is the parameter vector of the local models and those of the activation functions. Among the iterative optimization methods of the Quasi-Newton type, the Marquardt method, which is considered one of the most efficient resolution methods, does not require long calculations or large memory space.

III. MARQUARDT ALGORITHM

If n is iteration index of the Marquardt algorithm and θ^n the value of the solution at iteration n, the update of the estimate is done as follows:

$$\theta^{n+1} = \theta^n - [G(\theta^n)^T G(\theta^n) + \mu_n D^2(\theta^n)]^{-1} G(\theta^n)^T \varepsilon(t,\theta) \quad (10)$$

Where: $G(\theta^n)$: represents the jacobian matrix

So:

$$G = \begin{bmatrix} \frac{\partial\varepsilon_1(t,\theta)}{\partial\theta_{(1)}} & \frac{\partial\varepsilon_1(t,\theta)}{\partial\theta_{(2)}} & \cdot\cdot & \frac{\partial\varepsilon_1(t,\theta)}{\partial\theta_{(n)}} \\ \frac{\partial\varepsilon_2(t,\theta)}{\partial\theta_{(1)}} & \frac{\partial\varepsilon_2(t,\theta)}{\partial\theta_{(2)}} & \cdot\cdot & \frac{\partial\varepsilon_2(t,\theta)}{\partial\theta_{(n)}} \\ \cdots\cdot & & & \\ \cdots\cdot & & & \\ \frac{\partial\varepsilon_m(t,\theta)}{\partial\theta_{(1)}} & \frac{\partial\varepsilon_m(t,\theta)}{\partial\theta_{(2)}} & \cdot\cdot & \frac{\partial\varepsilon_m(t,\theta)}{\partial\theta_{(n)}} \end{bmatrix}$$

$D^2(\theta^n)$: is the diagonal matrix containing the elements of the diagonal of $G^T G$. To remedy the case where the elements of the diagonal are null, we take:

$$D^2(i,i) = G^T G(i,i) + 1$$

μ_n: is a parameter of Marquardt and which is chosen in such a way that:

$$J(\theta^{n+1}) < J(\theta^n) \qquad (11)$$

IV. APPLICATION TO THE THREE-TANK SYSTEM

To approach a nonlinear dynamic system by a multimodel we have chosen to study the system of the three tanks because we know relatively well its mathematical description.

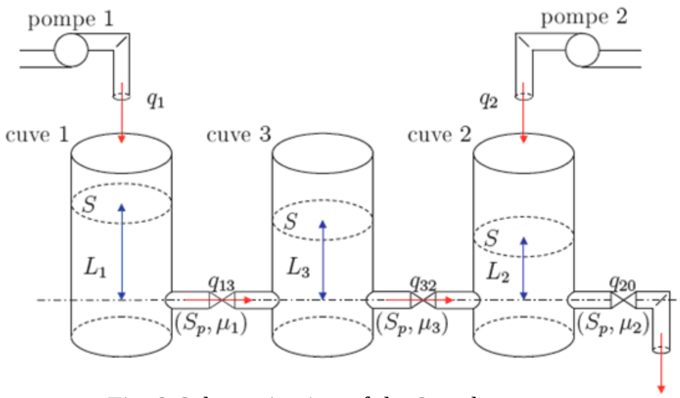

Fig. 2 Schematization of the 3-tank system

A. *System description*

The benchmark considered consists of three cylindrical vessels of identical section S, the tanks are connected by two cylindrical pipes of section $S_p$whose viscosity coefficient is $\mu_1 = \mu_3$. the output of the system is located at the tank 2, it is also characterized by a section S_p and a viscosity coefficient μ_2. Two pumps controlled by DC motors feed tanks 1 and 2 with flow rates $q_1(t)$ and $q_2(t)$. The three tanks are equipped with pressure sensors to measure the liquid level ($L_1(t), L_2(t)$ et $L_3(t)$).

B. *Mathematical model of the system*

By writing the equations of the conservation of the volume of liquid, we obtain:

$$\sum NL:\begin{cases} S\frac{dL_1(t)}{dt} = q_1(t) - q_{13}(t) \\ S\frac{dL_2(t)}{dt} = q_2(t) - q_{32}(t) - q_{20}(t) \\ S\frac{dL_3(t)}{dt} = q_{13}(t) - q_{32}(t) \end{cases} \quad (12)$$

Where $q_{ij}(t)$ is the flow rate of liquid from the tank i to the tank $j(i,j = 1,2,3 \forall i \neq j)$ which can be expressed using Torricelli's law by:

$$q_{ij} = \mu_i.S_p.sign(L_i(t) - L_j(t)).\sqrt{2g\left|L_i(t) - L_j(t)\right|} \quad (13)$$

And $q_{20}(t)$ represents the output flow, with:

$$q_{20} = \mu_2.S_p.\sqrt{2gL_2(t)} \quad (14)$$

Without restricting our study, we consider the system as the levels verify the following inequalities$L_1(t) > L_3(t) > L_2(t)$. Also, we consider a particular sense of inter-tank flow rates $(q_{ij}(t))$. With these equations, we assume that the system of the three tanks is perfectly described using the defined nonlinear model (15)

$$\sum NL:\begin{cases} \dot{x}_1(t) = -2C_1\sqrt{x_1(t) - x_3(t)} + u_1(t)/S \\ \dot{x}_2(t) = 2C_3\sqrt{x_3(t) - x_2(t)} - 2C_2\sqrt{x_2(t)} + u_2(t)/S \\ \dot{x}_3(t) = 2C_1\sqrt{x_1(t) - x_3(t)} - 2C_3\sqrt{x_3(t) - x_2(t)} \\ y_1(t) = x_1(t) \\ y_2(t) = x_2(t) \\ y_3(t) = x_3(t) \end{cases} \quad (15)$$

With $x_i(t)$ is the level of liquid in the tank i and $C_i = (1/2).(1/S).\mu_i.S_P.\sqrt{2g}$. The two control signals $u_1(t), u_2(t)$ are respectively the two input flow rates $q_1(t)$ and $q_2(t)$.

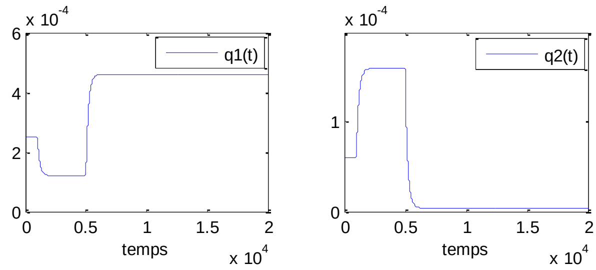

Fig.3 flow rates q1(t) et q2(t)

C. *Representation of the nonlinear model by a multimodel*

We consider a multimodel composed from three coupled local models :

$$\begin{cases} \dot{x}_m(t) = \sum_{i=1}^{3}\mu_i(\xi(t))(A_i x_m(t) + B_i u(t) + D_i) \\ y_m(t) = Cx_m(t) \end{cases} \quad (16)$$

with

$$\dot{x}_m(t) = [\dot{x}_{m1}(t)\dot{x}_{m2}(t)\dot{x}_{m3}(t)]^T$$

The activation functions μ_i were constructed as follows:

$$w_i(u(t)) = exp(\frac{-(u_1(t) - u_i)^2}{2\sigma_i^2}); \mu_i(u(t)) = \frac{w_i(u_1(t))}{\sum_{i=1}^{3} w_i(u_1(t))}$$

the index *i* corresponds to the *i*th local model, each local domain *i* have an operating point

$$p_i(x_{1i}, x_{2i}, x_{3i}, u1_i, u2_i) \rightleftarrows \text{ such that } i = 1,2,3$$

The different operating point coordinates are obtained by the resolution of the system (19)

$$\begin{cases} -2C_1\sqrt{x_1(t) - x_3(t)} + u_1(t)/S = 0 \\ 2C_3\sqrt{x_3(t) - x_2(t)} - 2C_2\sqrt{x_2(t)} + u_2(t)/S = 0 \\ 2C_1\sqrt{x_1(t) - x_3(t)} - 2C_3\sqrt{x_3(t) - x_2(t)} = 0 \end{cases} \quad (17)$$

The numerical values of the operating points are:

Table 1. The three operating points

i	x_{1i}	x_{2i}	x_{3i}	u_{1i}	u_{2i}
1	7.476	3.182	5.413	0.2480	0.0600
2	3.635	2.629	3.152	0.1200	0.1600
3	18.008	5.917	12.195	0.4160	0.0040

The numerical values of the different matrices are :

A_1 = [-3.9 0 3.9 ; 0 -6.7 3.6 ; 3.9 3.6 -7.5] $* 10^{-3}$
A_2 = [-8.0 0 8.0 ; 0 -10.9 7.4 ; 8.0 7.4 -15.5] $* 10^{-3}$
A_3 = [-2.3 0 2.3 ;0 -4.4 2.1 ; 2.3 2.1 -4.4] $* 10^{-3}$

$B_1 = B_2 = B_3 =$ [64.9351 0 ;0 64.9351 ; 0 0]

$$D_1 = \begin{bmatrix} -0.0081 \\ -0.0019 \\ 0 \end{bmatrix}, D_2 = \begin{bmatrix} -0.0039 \\ -0.0052 \\ 0 \end{bmatrix}, D_3 = \begin{bmatrix} -0.0135 \\ -0.0001 \\ 0 \end{bmatrix}$$

We identify the parameters of the activation functions σ_i from the minimization of the criterion $J(\theta)$ defined as follows:

$$J(\theta) = \frac{1}{2}\sum_{t=1}^{3}[(x_{is}(t) - x_{im}(t))^2] \quad (18)$$

We minimize the criterion (18) by the algorithm of Marqurdt, after the optimization, we have found:

$$\sigma_1 = 0.1274 . 10^{-4}; \sigma_2 = 0.1251. 10^{-4} \sigma_3 = 0.0277. 10^{-4}$$

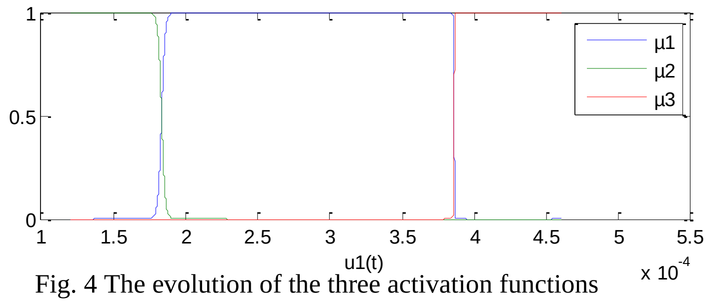

Fig. 4 The evolution of the three activation functions

To evaluate the simulation results, we simulate two models in parallel: the multimodel (16), and the nonlinear model (15). The figure (5) shows the superposition of the output vector components of the nonlinear model and their approximation by the multimodel.

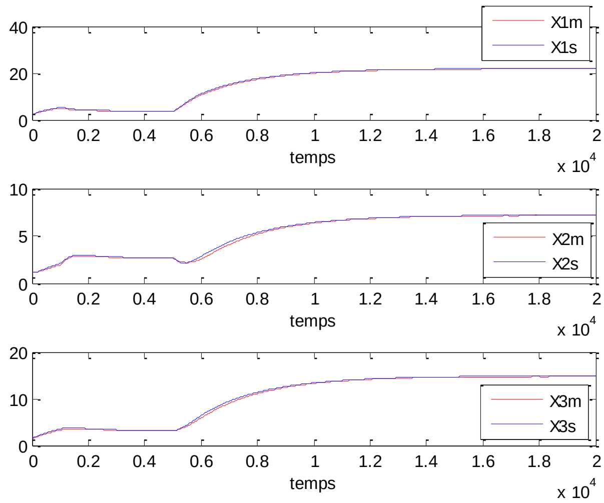

Fig. 5 The state variables of the nonlinear model "X1m, X2m X3m,", and those of the multimodel "X1s, X2s X3s".

V. CONCLUSION

In this work, we developed a Takagi-Sugeno (TS) fuzzy model based on three local linear models and three Gaussian membership functions for a hydraulic system. The optimization of the local model parameters and membership functions was performed using the iterative Marquardt-Levenberg algorithm. The results obtained show that the TS model outperforms the nonlinear analytical model in terms of both accuracy and simplicity.

More generally, the multimodel approach has proven to be an effective technique for approximating nonlinear systems using local models. It thus enables the extension of control and diagnostic techniques from linear systems to nonlinear systems, paving the way for future improvements, particularly through the integration of new optimization methods and the use of neuro-fuzzy controllers.

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