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Research Article

"Learning by doing" the PageRank algorithm for ranking nodes in a graph

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Abstract – Introducing numerical techniques and algorithms to bachelor physics students is often quite challenging, especially in the first and second year. In our previous works, we have implemented and tested the "Learning by Doing" method with the Physics Master students and we have clearly seen that the student performance has been increased significantly in two important subjects: Elementary Particle Physics and Computational Physics. In this work we present an application of the PageRank and PowerMethod Algorithms for the teaching based ranking of the Departments of the Faculty of Natural Sciences and some results of the students performance in understanding ranking and eigen-values algorithms, using the same method: "Learning by Doing". We have tested the performance of the students for these algorithms during the Midterm exam. Then the students worked together on the project of teaching based ranking of the departments using the PageRank algorithm. We have considered the teaching connections of nine departments between them, in ten bachelor programs offered by the Faculty of Natura Sciences, UT in Albania and used the PageRank and PowerMethod algorithms for finding the most teaching based ranked department. After the project, we tested the students performance on these algorithms in the final exam. The compared results clearly demonstrated that the students performance increase significantly.

Keywords –PageRank Algorithm, Powermethod, Learning By Doing, Ranking, Departments, Nodes, Graphs.

I. INTRODUCTION

Introducing numerical techniques and algorithms to bachelor physics students is often quite challenging, especially in the first and second year. In our previous works, we have implemented and tested the "Learning by Doing" method with the Physics Master students and we have clearly seen that the student performance has been increased significantly in two important subjects: Elementary Particle Physics and Computational Physics [17]. In this work we wanted to see if the same method would have any effect on the students performance in the Bachelor in Physics program, regarding numerical techniques and algorithms topics. In the second year of the Bachelor in Physics program, one of the courses that students have a moderate interest is Programming (Lecturer R.Osmanaj), where students are introduced to numerical techniques and algorithms, implemented in Octave/Matlab, Python and C++, needed for

solving physics problems and complex situations. In this subject are planned 10 laboratory works and a project, with a variable topic each year. Every year, what we observe is the fact that students have difficulties especially in understanding the eigen-values and eigen-vectors algorithms and also the ranking algorithms, reflected even the in results of the midterm and final exams. So, we decided the project of the last year would be focused on the eigen-values, vectors and ranking algorithms: Rank the departments of the FNS, UT, based only on the teaching activity, as explained in the Material and Methods section.

The Faculty of Natural Sciences, University of Tirana, Albania, is one of the oldest faculties in Albania. It has nine departments: Mathematics (Math), Physics (Phy), Biology (Bio), Biotechnology (BTCh), Chemistry(Chem), Industrial Chemistry (ICh), Applied Mathematics (AM), Informatics (Info) and the Flora and Fauna Center (FFC) [1]. Each one of the departments has its own research, educational and teaching performance. The idea of our work was to perform a teaching based ranking of the departments, using PageRank algorithm, as an application of the PowerMethod algorithm, such as ranking internet pages, soccer player performance or ranking the football teams [2, 3,4]. This was chosen as an example for a project for the students of the Bachelor in Physics program, in the subject of Programming, after explaining eigenvalues, eigenvectors and PowerMethod. We have considered the teaching connections of eight departments between them, in ten bachelor programs offered by the Faculty of Natural Sciences, UT. The PageRank algorithm measures the importance of each node (department) within the graph, based on the number incoming/outgoing relationships (teaching connections) and the importance of the corresponding source nodes [4].

So, the first thing to do was building the model, the graph with the nodes and the incoming/outgoing connections between them. Then build the matrix with the points for each node of the graph and use PowerMethod for finding the eigenvalues of this matrix. The results of the final exam, regarding these topics were analysed too. The methodology, the analysis, results and conclusions are presented in the next sections.

II. MATERIALS AND METHOD

The Programming course is an annual 2nd year course in the Bachelor in Physics program, where are treated the most important numerical techniques and algorithms needed in physics and not only. As we mention in the Introduction Section, students have problems in the assimilation of the eigen values/vectors and ranking algorithms. So we tried to the "Learning by doing" method and see if any students performance change will be seen. Two tests regarding these topics were done with the same group of students: 20 students of the 2nd year of the Bachelor in Physics program, the Midterm exams and in the final one. In between, a project regarding the ranking of the departments of the Faculty of Natural Sciences was prepared by the same students, with the help of the lecturers of the course. Both tests results were analysed.

Let's remember few important things and then clarify what we did.

- *"Learning by doing"* is a theory that emphasize the student engagement and is a hands-on, task-oriented, process to education [1]. This theory refers to the process in which students participate very actively in practical ways of learning. Learning-by-doing is related to some types of learning such as action learning, cooperative learning, experiential learning, peer learning, service-learning, and situated learning.

- *PageRank* (PR) is an algorithm used by Google Search to rank web pages in their search engine results, founded by Larry Page. It is a link analysis algorithm and it assigns a numerical weighting to each element of a hyperlinked set of documents, such as the World Wide Web, with the purpose of "measuring" its relative importance within the set. The algorithm may be applied to any collection of entities with reciprocal quotations and references. [12]

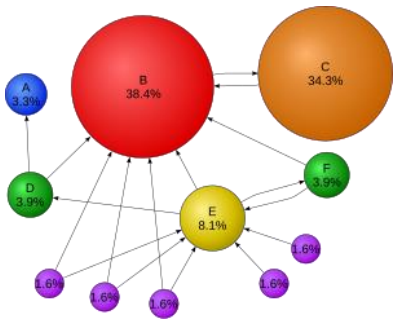

Figure 1. Illustration of the PageRank algorithm

- *The power method* is an iterative algorithm that can be used to determine the largest eigenvalue of a square matrix. The algorithm works by starting with a random initial vector, and then iteratively applying the matrix to the vector and normalizing the result to obtain a sequence of improved approximations for the eigenvector associated with the largest eigenvalue. [18]

How the project was organized and the model was builded?

A. Building the matrix

After we have taken the proper information from the webpage of our faculty, we have studied the connections between the nodes (departments) of the graph. We have consider as incoming connections those departments where we teach (offer) and outgoing those we want resources for teaching (required). These connections are presented in the figure 2.

LESSONS OFFERED AND REQUIRED BY DEPARTMENTS
INCOMING AND OUTGOING CONNECTIONS

Deparment	Incoming connections	Outgoing connections
Mathematics	Physics **Phy** Biology **Bio** Biotechnology **BTCh** Chemistry **Chem** Informatics **Info** Applied Mathematics **AM** Industrial Chemistry**ICh**	Phy, AM, Info
Physics	Math, Bio, BTCh, Chem, Info, AM ,ICh	Math, AM, Info, Chem
Biology	BTCh, Chem, FFC	Math, Phy,BTCh,Chem,Info
Biotechnology	Bio, FFC	Phy, Bio, Chem, ICh
Chemistry	Phy,Bio,BTCh,ICh	Math, Phy,Info
Industrial Chemistry	BTCh	Math,Phy,Bio,Chem,Info,AM
Applied Mathematics	Phy,Info,Math, ICh	Math, Phy,Info
Flora and Fauna Center	-	Bio, BTCh
Informatics	Math, Phy,Bio,ICh,AM,Chem	Math, Phy, AM

Figure 2. Incoming and outgoing connections between departments

With the figure's 2 data we have make a graph with the nodes and the connections and after that, taking into account the PageRank algorithm we build the matrix with the points of each node (department). As we see, in the figure 3, is represented a simply graph with our nine nodes, which correspond to our nine departments and the corresponding connections.

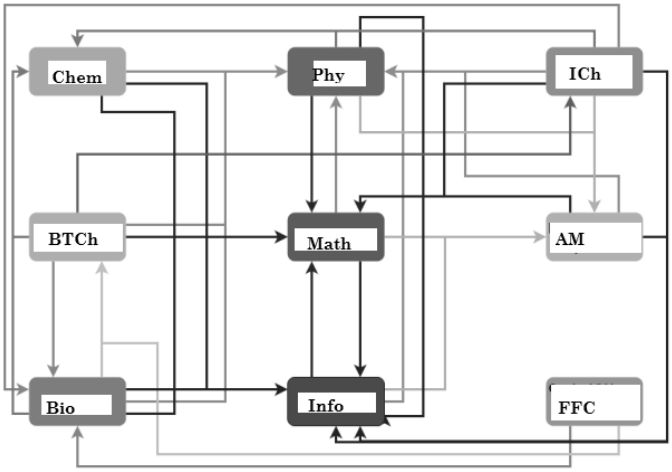

Figure. 3 Simple graph of the departments and the teaching connections

After that, let's build our matrix with the points corresponding to each department, using PageRank [12]. First of all, let be x_1 to x_9 the points taken from each department as below:

x_1 - Math
x_2 - Phy
x_3 - Bio
x_4 - BTCh
x_5 - Chem
x_6 - ICh
x_7 - AM
x_8 - FFC
x_9 - Info

Considering the PageRank Algorithms[12, 14], the accumulated points are for each node:

.

$$
\begin{aligned}
X_1 &= \frac{x_2}{4} + \frac{x_3}{5} + \frac{x_4}{5} + \frac{x_5}{3} + \frac{x_6}{6} + \frac{x_7}{3} + \frac{x_9}{3} \\
X_2 &= \frac{x_1}{3} + \frac{x_3}{5} + \frac{x_4}{5} + \frac{x_5}{3} + \frac{x_6}{6} + \frac{x_7}{3} + \frac{x_9}{3} \\
X_3 &= \frac{x_4}{5} + \frac{x_6}{6} + \frac{x_8}{2} \\
X_4 &= \frac{x_3}{5} + \frac{x_8}{2} \\
X_5 &= \frac{x_2}{4} + \frac{x_3}{5} + \frac{x_4}{5} + \frac{x_6}{6} \\
X_6 &= \frac{x_4}{5} \\
X_7 &= \frac{x_1}{3} + \frac{x_2}{4} + \frac{x_6}{6} + \frac{x_9}{3} \\
X_8 &= \\
X_9 &= \frac{x_1}{3} + \frac{x_2}{4} + \frac{x_3}{5} + \frac{x_5}{3} + \frac{x_6}{6} + \frac{x_7}{3}
\end{aligned}
$$

So, now we can build easily the matrix of the points taken by each departments and then applying the PowerMethod, finding the eigenvalues and eigenvectors. The corresponding matrix is the one presented in figure 4.

$$\begin{bmatrix} 0 & 1/4 & 1/5 & 1/5 & 1/3 & 1/6 & 1/3 & 0 & 1/3 \\ 1/3 & 0 & 1/5 & 1/5 & 1/3 & 1/6 & 1/3 & 0 & 1/3 \\ 0 & 0 & 0 & 1/5 & 0 & 1/6 & 0 & 1/2 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 1/4 & 1/5 & 1/5 & 0 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/5 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/4 & 0 & 0 & 0 & 1/6 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/4 & 1/5 & 0 & 1/3 & 1/6 & 1/3 & 0 & 0 \end{bmatrix}$$

Figure. 4 The points matrix for the teaching connections of the departments

B. PageRank and PowerMethod

The PageRank algorithm for determining the "importance" of Web pages has become a central technique in Web search [12]. The central idea of the PageRank algorithm involves computing the principal eigenvector of the Markov matrix representing the hyperlink structure of the Web. One way to compute the stationary distribution of a Markov chain is by explicitly computing the distribution at successive time steps, using $x(k) = Ax(k-1)$, until the distribution converges [13]. This leads us to the Power Method for computing the principal eigenvector of A. The Power Method is the oldest method for computing the principal eigenvector of a matrix, and is at the heart of both the motivation and implementation of the original PageRank algorithm. We have implemented this method in Matlab, in a simple code as below:

```
function [v,lambda,rr]=PowerMethod(A);
tol=1e-13;
n=max(size(A));
v=rand(n,1); v=v/norm(v);
rr=[];
while 1,
u=A*v;
lambda=v'*u;
r=v-u/lambda;
rnorm=norm(r), rr=[rr;rnorm];
if rnorm<tol, break, end
v=u/norm(u);
end
```

III. RESULTS

A. Analysing the results for the ranking of the nodes (Departments)

Let's summarize that after studying the teaching connections between the departments of our faculty, considering what each department offer and what required from others, we build the graph with nine nodes and the corresponding connections and then applying the PageRank we arrived to the final matrix.

$$\begin{bmatrix} 0 & 1/4 & 1/5 & 1/5 & 1/3 & 1/6 & 1/3 & 0 & 1/3 \\ 1/3 & 0 & 1/5 & 1/5 & 1/3 & 1/6 & 1/3 & 0 & 1/3 \\ 0 & 0 & 0 & 1/5 & 0 & 1/6 & 0 & 1/2 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 1/4 & 1/5 & 1/5 & 0 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/5 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/4 & 0 & 0 & 0 & 1/6 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/4 & 1/5 & 0 & 1/3 & 1/6 & 1/3 & 0 & 0 \end{bmatrix}$$

Applying the PowerMethod to this matrix, we take as a result the vector:

```
ans =
    0.9375
    1.0000
    0.0000
    0.0000
    0.2500
    0.0000
    0.8750
         0
    0.9375
```

As we can see, it's clear that the ranking of the department, now can be:
1- x_2 - Physics
2 - x_1/x_9 - Mathematics/Informatics
3 - x_7 - Applied Mathematics
4 - x_5 - Chemistry
5/6/7/8 - Biology, Biotechnology, Flora and Fauna Center

B. Student's Test Performance

Table 1. Results of the Midterm exam/ Standart Methodology Lessons

Number of students	**Eigenvalues algorithms Points**	**Ranking algorithms Points**
3	7/10	8/10
5	6/10	6/10
8	5/10	4/10
4	Below 5	Below 4

Table 2. Results of the Final exam/ Learning by doing method

Number of students	**Eigenvalues algorithms Points**	**Ranking algorithms Points**
7	9/10	10/10
9	8/10	9/10
4	7/10	6/10
0	Below 7	Below 6

As it can be seen, the taken points during the final test, after the implementation of the project by the students, regarding the eigenvalues and ranking algorithms are increased, and also the number of students that had better understood the topic is increased compared to the first test (the Midterm test and the same topics).

IV. CONCLUSION

In this work we presented a naive teaching based ranking of the departments of the Departments of the Faculty of Natural Sciences, UT, Albania and also tested the results of two different teaching methodology. We used the PageRank algorithm and the PowerMethod for finding the eigenvalues and eigenvectors of the points matrix, which was build considering the teaching connections between the departments. What was interesting to see was the fact that Physics was the first teaching based ranked department, because of the interactions it has with all subjects and departments in the Natural Sciences' Faculty. Let's remember, that it was just a naive evaluation, and was performed for making students understand ranking algorithms. We also confirmed again that the performance of the students increases significantly when they do things and projects by their selves, a result that make us believe that even in the higher education the method "Learning by doing" has to be one of the most important teaching methodologies.

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