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Research Article

Geometric Modeling of Closed Steiner Chain in Micro-Scale Structures

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Abstract – Many functional properties of living and non-living materials around us depend on the size, shape and therefore geometric models of their smaller parts. The geometric models of these parts are factors that affect their distribution and settlement patterns in the area where they are located. The settlement patterns of geometric models of materials are not simple and random, and most of them are concepts that we can define with formulas. We can detect geometric models of some materials that we can observe with the naked eye. However, it is more difficult to detect geometric models that make up materials at micro scales. Such studies will provide information about the efficiency of the whole of these materials by knowing their microstructures. At the same time, when the micro mathematical properties and geometric structures of these materials are known and taken into consideration, the efficiency parameters at the macro scale can also be easily determined. In this study, it was tried to determine the definitions of geometric structures based on mathematical rules in some microstructures that we can only observe with a microscope. In our research, different plant micro geometric modeling was used to define geometric models. In mathematical evaluations, mathematical concepts determined in the light of literature were used to describe the geometric models of microstructures.

Keywords – Geometric Modeling, Steiner Chain, Micro-Structures, Mathematical Formulas.

I. INTRODUCTION

The macro structures that we can see with the naked eye on our planet or microstructures that we can only see with a microscope; they have a certain structure that allows them to settle and perpetuate in the environment they live in. These structures are not random, most of them have a certain geometric modeling, they are located in a certain order. Plant structures often exhibit excellent mechanical properties. This feature is largely controlled by the geometrical structures of their micro structures [1].

In this study, it was tried to determine the definitions of geometric models based on mathematical rules in some micro structures that we can only observe with a microscope. We observed that some of the micro structures we examined have closed steiner chain models. Steiner chains are named after Jakob Steiner, who defined them in the 19th century and discovered many of their properties. In geometry, given two circles with one interior to the other, if small tangent circles can be inscribed around the region between the two circles such that the final circle is tangent to the first, the circles form a Steiner chain. The centers of the circles in a Steiner chain lie on an ellipse [2]. The lines of tangency passing through the contact points of neighboring circles in the chain are concurrent in a point. Furthermore, this is the same point at which the lines through the contact points of the inner and outer circles also concur [3].

A Steiner chain is a set of n circles, all of which are tangent to two given non-intersecting circles (Figure 1), where n is finite and each circle in the chain is tangent to the previous and next circles in the chain. In the usual closed Steiner chains, the first and last (n-th) circles are also tangent to each other; by contrast, in open Steiner chains, they need not be. The given circles α and β do not intersect but otherwise are unconstrained; the smaller circle may lie completely inside or outside of the larger circle. In these cases, the centers of Steiner-chain circles lie on an ellipse or a hyperbola, respectively [2].

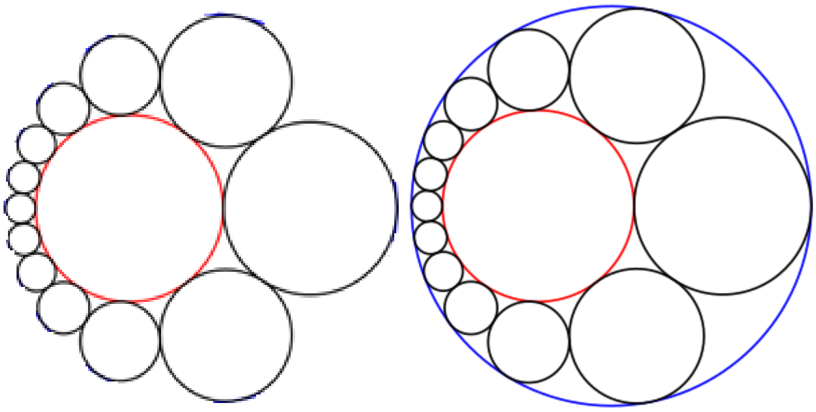
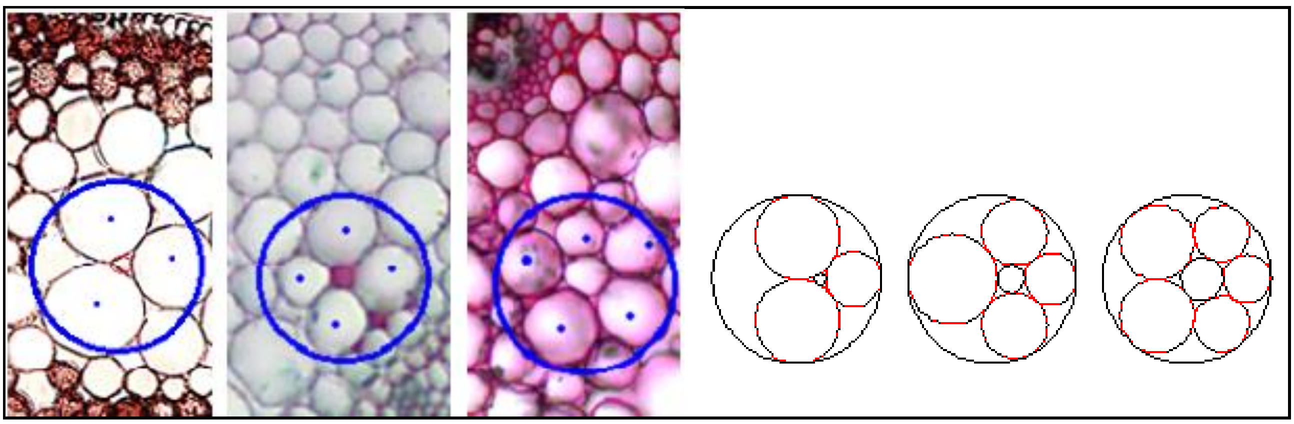


Fig. 1 A Steiner chain of twelve black circles (n = 12) [2], [3].

II. RESULTS

In our research, we determined that the microstructures of the plant cells examined have closed Steiner chain of n circles (Figure 2,3,4).

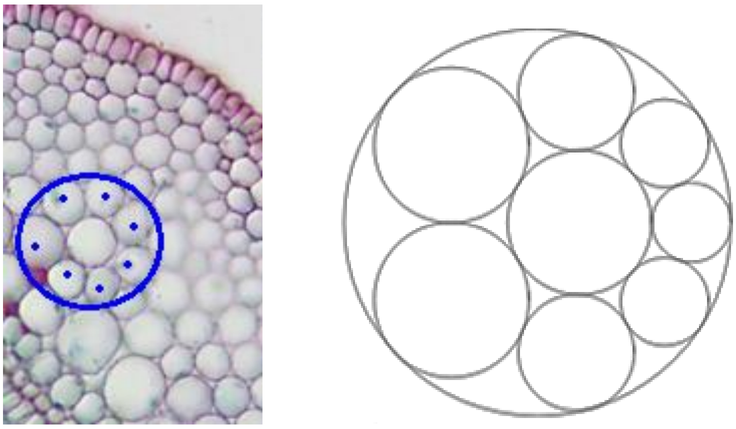


(n = 3) (n = 4) (n = 5) (n = 3) (n = 4) (n = 5)

a **b**

Fig. 2 Types of Steiner chain is a closed chain of *n* circles

a: Original micro modeling of plant cell **b:** Geometric diagram modeling of microstructure [2], [3].



a b

Fig. 3 Type steiner chain with n=7
a:Original micro modeling of plant cell b: Geometric diagram model of microstructure[2], [3].

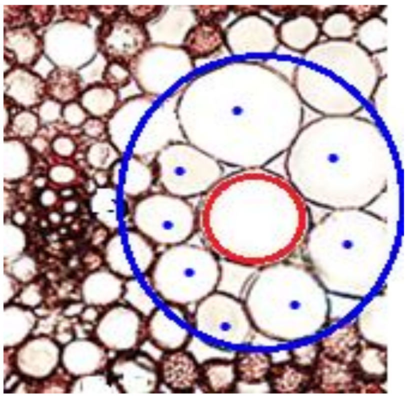
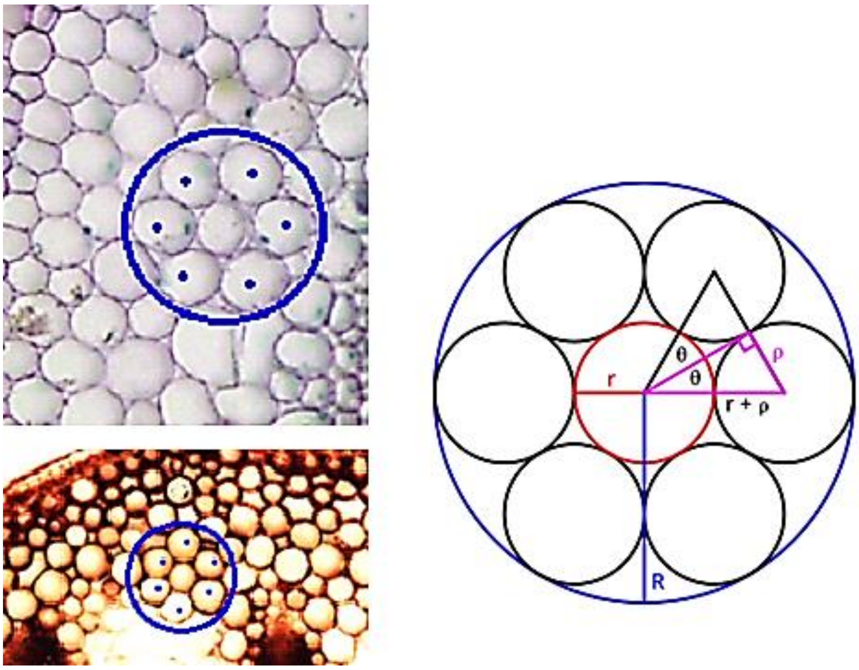


Fig..4 Steiner chain of eight circles (n = 8). Original micro modeling of cells.

The simplest type of Steiner chain is a closed chain of n circles of equal size surrounding an inscribed circle of radius r; the chain of circles is itself surrounded by a circumscribed circle of radius R. The inscribed and circumscribed given circles are concentric, and the Steiner-chain circles lie in the annulus between them.



a **b**

Fig. 5 Steiner chain of six circles (n = 6)
a. Original microscopic view of micromodeling **b.** Geometric model of micro modeling [2], [3].

In figure 5, the radius of the Steiner circles is ρ whereas those of the inner and outer given circles are r and R, respectively. The distance from the center of the inner circle to the center of a Steiner circle is r + ρ (hypotenuse of pink triangle).
By symmetry, the angle 2θ between the centers of the Steiner-chain circles is $360°/n$. Because Steiner chain circles are tangent to one another, the distance between their centers equals the sum of their radii, here twice their radius ρ. The bisector creates two right triangles, with a central angle of $\theta = 180°/n$. The sine of this angle can be written as the length of its opposite segment, divided by the hypotenuse of the right triangle

$$\sin\theta = \frac{\rho}{\tau + \rho}$$

Since θ is known from n, this provides an equation for the unknown radius ρ of the Steiner-chain circles

$$\rho = \frac{\tau \sin\theta}{1 - \sin\theta}$$

The tangent points of a Steiner chain circle with the inner and outer given circles lie on a line that pass through their common center; hence, the outer radius $R = r + 2\rho$.
These equations provide a criterion for the feasibility of a Steiner chain for two given concentric circles. A closed Steiner chain of n circles requires that the ratio of radii R/r of the given circles equal exactly

$$\frac{R}{\tau} = 1 + \frac{2\sin\theta}{1 - \sin\theta} = \frac{1 + \sin\theta}{1 - \sin\theta} = [\sec\theta + \tan\theta]^2$$

As shown below, this ratio-of-radii criterion for concentric given circles can be extended to all types of given circles by the inversive distance δ of the two given circles. For concentric circles, this distance is defined as a logarithm of their ratio of radii

$$\delta = \text{In}\frac{R}{\tau}$$

Using the solution for concentric circles, the general criterion for a Steiner chain of n circles can be written

$$\delta = 2\text{In}(\sec\theta + \tan\theta).$$

If a multicyclic annular Steiner chain has n total circles and wraps around m times before closing, the angle between Steiner-chain circles equals

$$\theta = \frac{m}{n}180^{\circ}$$

In other respects, the feasibility criterion is unchanged [3].

III. MATERIALS AND METHOD

Microstructure of plant cells were used as study material. Sections measuring 10-20 µm were taken to obtain the microstructures to be used in defining the geometric models of these samples. In mathematical evaluations, mathematical concepts determined in the light of literature were used to describe the geometric models of microstructures. In the study, literature information about geometric structures and their mathematical formulas was evaluated. [1],[2], [3], [4], [5], [6], [7]. In addition, the structures of the geometric models obtained from these definitions were shown with figures (Figure 1-5).

IV. DISCUSSION

With this study, the geometric modeling of the cells in the microstructures of plants was tried to be revealed by using the definitions corresponding to the geometrical rules. It has been seen that some micro structures have properties that can be defined in geometry and expressed with parametric formulas.

All living and nonliving materials are made up of many micro structure that make up their whole. The coming together of the micro structures are not random, they are placed in a certain order. The properties of the micro structure depend not only on a single micro structure, but also on the connections, locations and interactions between they components. The geometric models of these micro structure are factors that affect their distribution and settlement patterns in the area where they are located. In this study, different perspectives were provided by evaluating the micro structures of some cells mathematically. Thus, it creates a new comparison opportunity for future research on the relevant subject. should explore the significance of the results of the work. Such mathematical studies of micromorphological structures are very limited in the literature [8],[9],[10], [11].

V. CONCLUSION

With this study, it was provided to examine some micro structures a different perspective. In the literature, there are studies defined with the help of geometric models and mathematical formulas. However, these studies are mostly on visible structures. In our studies, we tried to describe microscopic structures with geometric modeling. We think that our study will bring a different perspective for future researchers.

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