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Mathematics of Risk Management using Unimodular Matrix. Interpolation and Numerical Simulations

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Abstract – In this research, we review the state of mathematics of risk research in business, and we position our intended contribution within the research space along the dimensions of decision support and its application within numerical simulations to AI service management processes. The aim of this study is to solve the problem of managing multifactorial risks using numerical methods to determine the optimal risk management trajectories separately for each factor.

We start from a constructive and quantitative re-definition of some terms that are widely used in IT service management. We show that the techniques we use - in particular the simulation of an IT organization enacting the incident management process – bring considerable benefits both when the performance is measured in terms of traditional IT metrics. IT service management processes and we present a methodology and a tool consistent with the framework for approaching the general problem in that particular one. The situation of risk involves the identification of several behaviors, during which one can make a choice in the direction of risk or reliability.The solution is quite rich, and features components that orchestrate together advanced techniques in visualization, simulation, data mining and operations research.

Keywords – Simulations, Numerical Results, Cost, Benefit, Risk and Urgency.

I. INTRODUCTION

This is the application of a set of models, practices, techniques and tools to map and to quantitatively evaluate interdependencies between business performance and IT solutions and using the quantified evaluation to improve the IT solutions' quality of service and related business results.

The contribution of Information technology (IT) to business value creation is currently a hotly debated topic. IT is expected to bring value to the business, as is attested to by the introduction of Control Objectives for Information and Related Technologies.

The contribution of Information technology (IT) to business value creation is currently a hotly debated topic. IT is expected to bring value to the business, as is attested to by the introduction of Control Objectives for Information and Related Technologies. Microsoft, among others [4] and automated, ITIL-based IT management procedures are also being proposed for adaptive or autonomic computing platforms [5][6]. In order for IT to help the business achieve its goals. On the other hand, AIM can cover a lot of ground, in autonomic or conventional IT infrastructures as well as in manual IT management procedures.

Related to risk managment he advantages and features of the expert approach to risk assessment [16 , 18] and mathematical methods to identify and assess risks are the most often discussed topics in the literature [17–20]. The development of mathematical tools that allows one to form an optimal trajectory for managing several risks simultaneously is given in [21].

From the point of view of the modeling challenges, this thesis tackles and advances the state of the art in AIM modeling by presenting a decision theoretical framework that models business impact aspects of AIM including monetary as well as intangible costs and benefits. Moreover, our framework deals with risk consistently with the economic / decision theoretical definition of risk.

The approach of defining utility and utility functions over the space of possible outcomes appears in numerous works in the academic literature, see for example [20][22]. While in some cases, the choice of the particular utility functions reflects a need by the author of having an analytically tractable optimization problem to solve, in other cases such as [20][22], more conscious efforts are made towards choosing utility functions that genuinely reflect net benefit to the decision maker.

Our definition of risk as the variance of the utility distribution (and therefore indirectly of the cost and impact distributions) is consistent with the intuition that wider spreads of possible outcomes leading to wider spreads of cost/impact values carry a higher risk.

Overall, the article highlights how simulation offers a way to estimate the ideal solution for optimization problems in computer science. By exploring its real-world uses and examining its benefits and drawbacks, this research provides a comprehensive understanding of its applicability and significance in the field.

II. MATERIALS AND METHOD

2.1 State of the art and research challenges in IT management

Interaction Between the Risk Management Determinants and risk management results under the hypothesis Method [30-32] gives the purpose-driven definition of AIM that we have adopted makes it so that many works presented in the areas of distributed systems, network and system management, economics of IT and organizational behavior can be considered as AIM applications. Following [13] and [14], we define AIM as the application of a set of models, practices, techniques and tools to map and to quantitatively evaluate interdependencies between business performance and IT solutions and using the quantified evaluation to improve the IT solutions. We present an overview of these challenges and decide to focus our contribution in BDIM decision support in IT service management and contribute decision theoretical framework and models that bring the concepts of business impact and risk to the fore.

Application of AIM may be carried out in six steps:

Identify business objectives and business-level metrics of interest – these could be about revenue, cost, inventory turnaround time, etc. Select (technical) performance metrics in the context of the IT management scenario of interest. Scenarios include IT service management processes, autonomic computing platform self-management, software engineering, IT projects, and strategic planning.

Model the relevant entities in the scenario of interest, their attributes and their relationships; and quantify IT–business linkage, i.e., estimate the impact that the IT scenario solution has on the business metrics.

Validate model, making required enhancements in the model itself and in its associated IT-business linkage quantification. Use the validated model to *support* decisions concerning IT solution in scenario of step 2. For the scenario of interest, evaluate gains in business results. Compare gains to business goals. In case discrepancies are unacceptable, make adjustments in the IT solution.

When all steps are automated, the AI system control may be encapsulated into autonomic computing infrastructures to enact online, on-the-fly self-management. The focus of this dissertation is however on BDAIM solutions aiming at providing decision support to human agents rather than on autonomic solutions aiming at taking the human out of the loop. This way of operating is particularly appropriate when tackling decision problems that IT managers and IT staff face in IT service management processes, which is our application domain of choice for this article.

2.2 Research on AIM and Autonomic Computing

AIM (Artifical inteligence in bussiness) applications are possible over many of the IT Service Management processes. In the previous sub-section, we have reviewed some of the AIM solutions that are more amenable to automation and as such could be seen as application of autonomic computing as well as touching on the relevant IT service management processes.

We review the research literature on AIM and highlight some open research challenges along the two dimensions of *modelling* and *applications*.
IT scenarios and of the various IT management processes seem to offer no chance for comprehensive and generic AIM models, leading to very little reusability of models.
This research applies to the alignment of IT with a company"s mission, strategic goals and expected results. The new ITIL documentation uses the term "stereo vision" to refer to business and IT alignment. As a result, decisions will need to be made based on incomplete or incorrect information on parameter value and even accuracy levels. Research challenges here include using data mining from logs of operations, applications and processes, knowledge acquisition and estimation techniques – such as that discussed in [15] for AIM applications.

2.3 Mathematics of risk and modeling of structure

We give a mathematical formalization of our framework for decision support, by giving quantitative definition of risk and urgency, based on utility functions. The concept of risk is associated to the spread of possible values of utility for a given situation (the wider the spread, the riskier the situation), we define **risk** *as the standard deviation of utility*. Given a probability distribution characterized over a set **Ω** of possible outcomes, $\in \Omega$, on defining a utility function such that is the utility of a possible outcome.

Prediction of consequences will likely need to include risk analysis over a long enough observation period. Strategic impact for instance, may be barely noticeable over months or years. The work in [18][19] adopts utility functions as business measures. The paper in [20] considers IT infrastructure cost. Modeling of finer grained details of IT service is carried out through a Customer Behavior Model Graph in [6][11]. Solutions to the AIM models have been obtained through queuing theory in [17][20], while simulation is preferred in [16]. When estimating cost and business impact as defined above, there will necessarily be a degree of **uncertainty**. This uncertainty in business impact estimations is what characterizes **risk**. In [33] we suggests making a distinction between *epistemic uncertainty* and *stochastic uncertainty*.

2.4 Understand risk menagement and impact of financial cost

For reasons that will be clearer in the following, we take risk to be defined as variance of the utility function over a given set of outcomes. When utility is defined only on business impact (including the financial perspective of the business) then risk becomes a measure of the uncertainty of business impact estimates for given courses of action. Correct decisions can reduce product costs, management costs, and eliminate potential problems that may adversely affect the company's competitiveness. Most researchers recognize that to maintain high competitiveness of companies in an actively changing global environment, it is necessary to strategically create effective risk management mechanisms in the short term [7].

Taking the dual approach to monetizing intangibles, one could consider the financial perspective is just one of the many dimensions that the business would want to optimize for business-driven IT management solutions. To formalize the attitude of the decision-maker to specific risks we suggest building a risk matrix. Risk matrix is built via crossing qualitative categories of risk-forming parame-ters and matching them to generated risk categories. In the next stage, the risk matrix is coded by numerical values. This makes it possible to approximate the risk function and to solve the above-formulated risk management problem. To approximate the risk matrix, it is proposed to use the continuous additive–multiplicative approach to aggregation and requirements for developing a strategy and mechanism for countering risks [7 –11].

III. APPLICATION AND RESULTS

3.1 Risk Matrix Interpretation

Our model defines a perspective as a first class object, not limiting its usage to the traditional balance scorecard model. Perspectives do not represent a partition over the set of objectives defined. An objective can belong to more than one perspective. When building viable AIM solution, like Comparison of Numerical Methods for SWW Equations [16], it is important to validate this hypothesis by carrying out experiments to determine the sensitivity of any measure of the goodness of the decisions suggested to the complexity of the methods used for determining workable figures for the alignment.

The most important features and trends in the results should be described but should not interpreted in detail. Results should be clear and concise. A customer perspective could contain objectives defining targets over some KPIs representing quantitative measures of the customer satisfaction (measures of TCE: total customer experience), and so on.

Therefore, we chose a set of 4 risk categories and a 4-point scale corresponding to them in the present study (Table 1).

Table 1. The categories of risks used and the corresponding strategies for responding to them.

Risk Significant Score	Generalized Risk Category	Risk Response Strategy
4	extreme risk (e)	avoidance
3	high risk (h)	transfer
2	medium risk (m)	mitigation
1	negligible risk (n)	acceptance

We also use 4 categories of risk-forming parameters (Table 2) and a 4-point scale corresponding to them.

Table 2. The categories of risk-forming parameters.

Risk Significant Score	Generalized Risk Category	Risk Response Strategy
4	very likely (v)	disastrous (d)
3	Likely (l)	critical (c)
2	possible (p)	significant (s)
1	unlikely (u)	acceptable (a)

The following categories could be used: frequent can occur several times in a year; occasional—can occur several times in 1 or 2 years; uncommon—can occur several times in 2 or 5 years; and remote—can occur once in 5 or 30 years [15].

3.2 Numerical Risk Matrix Interpolation

If you're working with a quantitative risk matrix (e.g., likelihood vs. severity scored 1–5), interpolation might refer to estimating risk levels between discrete matrix cells using mathematical or statistical methods. In the numerically coded risk matrix, the risk probability categories are shown along the vertical axis (variables $X_L \in L$ (likelihood) and the value of hazard classes (categories) that considers the severity of consequences from the onset of risks is along the horizontal axis ($X_C \in C$ (consequences). The sets of numerical values of the introduced variables are given as L = {1, 2, 3, 4} and C = {1, 2, 3, 4}. The sets L and C are the basis of the numerical risk matrix (Table 3).

Table 3. An example of a numerically coded risk matrix form, the risk significant scores; 4 (extremal risk); 3 (high risk); 2 (medium risk); and 1 (negligible risk)

					X_l-axis
	4(e)	4(e)	4(e)	4(e)	4(v)
	4(e)	3(h)	3(h)	2(m)	3(l)
	3(h)	3(h)	2(m)	2(m)	2(p)
	2(m)	2(m)	2(m)	1(n)	1(u)
X**c**-axis	4(d)	3(c)	2(s)	1(a)	

The elements of the matrix are scores corresponding to the generalized risk categories. Risk categories are defined as generalized, since they simultaneously consider both the likelihood of a risk occurring and the severity of its consequences. The risk level will be written as X_R, $X_R \in R$, R = {1, 2, 3, 4}.

The sets of 4 risk categories {n, m, h, and e} (see Table 1), 4 categories of risk-forming parameters {a, s, c, and d} and {u, p, l, and v} (see Table 2) and the 4-point scale corresponding to them is used only in this particular case for the study. In general, it is possible to use other scales and number of categories. The risk matrix (see Table 3) is a subset of the Cartesian product, $X_L \times X_C \times X_R$, which determines the mapping from the set XL × XC to the set X_R: M: $X_L \times X_C \rightarrow X_R$.

Accordingly, the risk matrix can be interpreted as a function of two variables, $X_R = M(X_L, X_C)$. (1)

Notation: Let us denote the matrix element given by row **r** and column **c** as $\mathbf{j_i}$ in general. These notations will be used to approximate the risk matrix.

3.2. Approximation using math aggregation method

To approximate the risk matrix, it is proposed to use the continuous additive–multiplicative approach to aggregation proposed in [13]:

$$X_R(X_L, X_C) = j_3 + \gamma 1(j_4 - j_3) + \gamma_2(j_5 - j_3) + \gamma_1 \gamma_2 (j_3 + j_6 - j_4 - j5), \quad (2)$$

where, γ_1 is the remainder of dividing the risk-forming parameter X_L by 1, and γ_2 is the remainder of dividing the risk-forming parameter X_C by eq.1:

$$\gamma_1 = \mathrm{mod}(X_L;1), \gamma_2 = \mathrm{mod}(X_C;1), \quad (3)$$

j_3, j_4, j5 and j_6 are the pivot elements of the risk matrix. The pivot elements are determined according to Equations (2)–(3), where the integer value of the risk-forming parameter XL is taken as row r, and the integer value XC is taken as column c:

$$r = X_L = \dim(X_L), \qquad c = X_C = \dim(X_C) \tag{4}$$

The advantage of this aggregation method is the continuity, monotonicity and piece-wise smoothness of the function (16).

3.3 Visual or Graphical Interpolation

In a heatmap-style risk matrix, interpolation might mean smoothing or estimating color gradients between severity/likelihood zones to visualize trends better. The risk matrix was interpolated using the additive–multiplicative procedure to aggregation [6–11] or [17–19]. The risk matrix (or risk map or diagram) is a graphic representation of risks divided into groups of priorities according to which we further work with single risks. The risk matrix enables the evaluation and assessment of risks according to two criteria (it's axes).

Let's assume you have a quantitative risk matrix, where risk is calculated as:

Risk = Likelihood × Impact

Both Likelihood and Impact are rated on a scale (e.g., 1 to 5), and you want to interpolate [22-28] values between these discrete levels to smooth out the risk surface—possibly for plotting a heatmap or for more refined assessment.

Fig. 1 possible matrix representation of risks by numbers

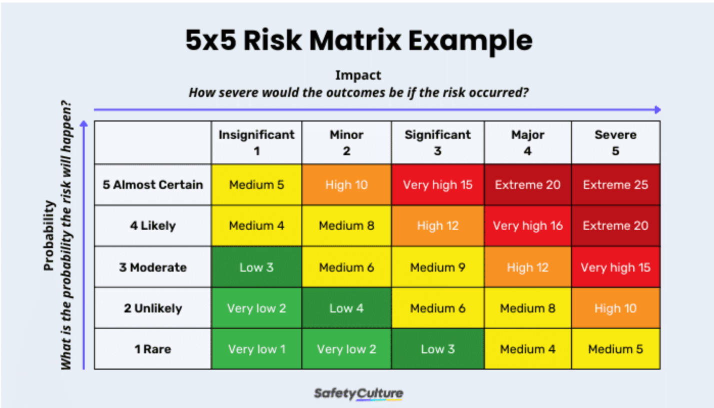


Fig. 2 possible matrix representation of risks by outcomes

Fig. 3 Possible matrix representation of risks by results

3.4 Simple code using bilinear interpolation across a Unimodular risk matrix

We use an Unimodular Risk Matrix [29]. In this context, “unimodular” usually refers to a matrix with a single peak (maximum) — meaning the risk increases up to a central point and then decreases in other directions. This creates a “mountain” or “hill”-like shape, which is helpful in modelling localized high-risk zones.

3.4.1 Generating unimodular matrices of any size

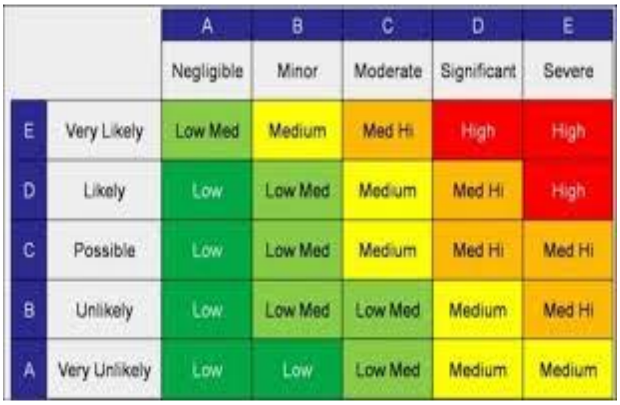
```
def generate_unimodular_matrix(n, peak=9):

    """Generates a symmetric unimodular matrix with a central peak."""
    center = n // 2
    matrix = np.zeros((n, n))
    for i in range(n):
        for j in range(n):
            dist = abs(i - center) + abs(j - center)
            matrix[i, j] = max(1, peak - dist * 2)  # tweak decay rate here
    return matrix
```

3.4.2 The code in Python for iinterpolated Unimodular Risk Matrix 5x5

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.interpolate import interp2d

# Define 5x5 grid
scale = np.array([1, 2, 3, 4, 5])

# Unimodular 5x5 Risk Matrix
risk_matrix = np.array([
    [1, 2, 3, 2, 1],
    [2, 4, 6, 4, 2],
    [3, 6, 9, 6, 3],
    [2, 4, 6, 4, 2],
    [1, 2, 3, 2, 1]
])

# Interpolator
interpolator = interp2d(scale, scale, risk_matrix, kind='linear')

# Fine grid for smooth interpolation
fine_grid = np.linspace(1, 5, 200)
fine_risk = interpolator(fine_grid, fine_grid)

# Plot heatmap
plt.figure(figsize=(6, 5))
plt.contourf(fine_grid, fine_grid, fine_risk, levels=100, cmap='plasma')
plt.colorbar(label='Risk Level')
plt.title('Interpolated 5x5 Unimodular Risk Matrix')
plt.xlabel('Impact')
plt.ylabel('Likelihood')
plt.grid(True)
plt.show()
```

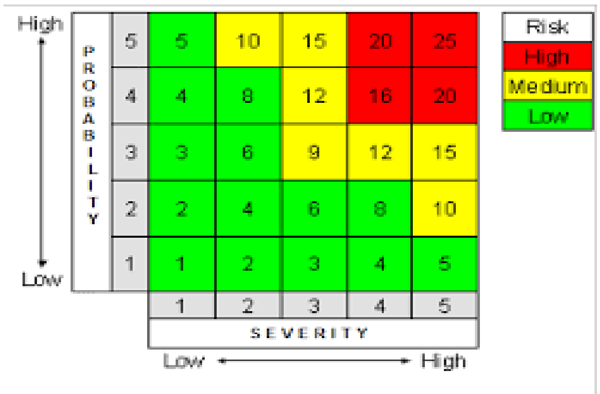
3.4.3 Output is visualized elsewhere, i.e. in Python

One of the most common matrix of a risk assessment matrix is the 5×5 unimodular risk matrix. In this case, we will use five different likelihood ratings



		A	B	C	D	E
		Negligible	Minor	Moderate	Significant	Severe
E	Very Likely	Low Med	Medium	Med Hi	High	High
D	Likely	Low	Low Med	Medium	Med Hi	High
C	Possible	Low	Low Med	Medium	Med Hi	Med Hi
B	Unlikely	Low	Low Med	Low Med	Medium	Med Hi
A	Very Unlikely	Low	Low	Low Med	Medium	Medium

Fig. 4 Result according matrix 5×5 risk assessment matrix



PROBABILITY	1	2	3	4	5
5	5	10	15	20	25
4	4	8	12	16	20
3	3	6	9	12	15
2	2	4	6	8	10
1	1	2	3	4	5

Fig. 5 Probability result according matrix 5×5 risk matrix

5x5 RISK MATRIX

LIKELIHOOD ↓ / SEVERITY →	1	2	3	4	5
1	LOW 1	LOW 2	LOW 3	MEDIUM 4	MEDIUM 5
2	LOW 2	MEDIUM 4	MEDIUM 6	HIGH 8	HIGH 10
3	LOW 3	MEDIUM 6	HIGH 9	HIGH 12	EXTREME 15
4	MEDIUM 4	HIGH 8	HIGH 12	HIGH 16	EXTREME 20
5	MEDIUM 5	HIGH 10	EXTREME 15	EXTREME 20	EXTREME 25

Fig. 6 Matrix 5×5 risk assessment matrix

IV. DISCUSSION

Our treatment of the quantitative aspects of urgency as a first order derivative of impact is anyway a first step in the right direction as it enables ways of making trade-offs between impact and urgency that had not been made explicit before. This could be done for example by defining utility functions that take into account both expected business impact and (expected or instant) urgency. Decision makers could be characterized by their "urgency sensitivity" and their profile could be used for tuning the relative importance of perceived impact vs. urgency in defining utility functions.

Here, the 5×5 risk assessment matrix works very well and the results as follow:

- The five rows represent the likelihood or probability of the risk occurring, while the columns represent the severity (effect) of the consequences.
- Each cell in the matrix represents a level of risk, with the highest risk in the top-right corner and the lowest risk in the bottom-left corner.
- The likelihood and severity of a risk occurring are usually rated as low, low/medium, medium/high, or high.
- These ratings are assigned based on the expert knowledge of the risk assessor, and they can be adjusted based on the specific project or situation.
- The **maximum risk is 9** at the center (row 3, column 3) for our result.
- Risk decreases symmetrically as you move away from the center.

V. CONCLUSION

The main way in which our methodology advances the state of the art is that it provides a coherent way of dealing with both tangible and intangible objectives in a way that is natural to people who are used to instruments such as the balanced scorecard.

The added advantage of our methodology is that the objectives that are used to compute alignment (and therefore business impact) are the same that IT executives are used to negotiating over.

It is more natural to argue over the relative importance of a small number of objectives than it would be to argue over any possible arbitrary monetization of outcomes.

The importance of probability distributions, approximations, errors, and interpolation techniques has been emphasized in this paper, as they are crucial for the success of simulation methods in optimization problems. Furthermore, we have discussed the advantages and disadvantages some methods of simulation to provide a comprehensive understanding of its applicability and importance in this field. Looking ahead, as technology continues to advance, there are many opportunities to further develop and implement these methods to improve the effectiveness and efficiency of computer science applications.

From our definition of AIM, our survey of the state of the art of AIM research and exploration of open research challenges, it is apparent that the whole space that we touch on in this chapter is way too vast for anyone to try and embrace it all. In this research we will focus our contributions in the space of AIM decision support over IT Service Management processes**.**

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