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Research Article

Micro Reuleaux Polygons Geometric Models

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Abstract – In geometry, a Reuleaux polygon is a curve of constant width made up of circular arcs of constan radius. These shapes are named after their prototypical example, the Reuleaux triangle, which in turn is named after 19th-century German engineer Franz Reuleaux.The Reuleaux triangle can be constructed from an equilateral triangle by connecting each pair of adjacent vertices with a circular arc centered on the opposing vertex, and Reuleaux polygons can be formed by a similar construction from any regular polygon with an odd number of sides as well as certain irregular polygons. Every curve of constant width can be accurately approximated by Reuleaux polygons. The Reuleaux polygons based on regular polygons are the only curves of constant width whose boundaries are formed by finitely many circular arcs of equal length. Sometimes in Reuleaux polygons, each curve of constant width may not have the same width but may have similar widths. This creates an irregular Reuleaux polygon. In other words; every curve of constant width can be approximated arbitrarily closely by a (possibly irregular) Reuleaux polygon of the same width. It is possible to come across examples polygons of Reuleaux in nature. According to Plateau's laws, the circular arcs in two-dimensional soap bubble clusters meet at 120° angles, the same angle found at the corners of a Reuleaux triangle. Based on this fact, it is possible to construct clusters in which some of the bubbles take the form of a Reuleaux triangle. The shape was first isolated in crystal form in 2014 as Reuleaux triangle disks. Basic bismuth nitrate disks with the Reuleaux triangle shape were formed from the hydrolysis and precipitation of bismuth nitrate in an ethanol–water system in the presence of 2,3-bis(2-pyridyl)pyrazine. In this study, we investigated the examples of Reuleaux polygons geometry models that we can easily see with the naked eye in nature in micro structures. We have detected these geometric models in the cells where we can observe with the help of microscope. We saw that Some of these cells we examine are regular and inregular reuleaux polygon. We took microscopic photos of these geometric models.

Keywords – Reuleaux, polygon geometric model, micro

I. Introduction

Reuleaux polygon is a curve of constant width made up of circular arcs of constan radius. These shapes are named after their prototypical example, the Reuleaux triangle, which in turn is named after 19th-century German engineer Franz Reuleaux [1], [2],[3].

The Reuleaux triangle can be constructed from an equilateral triangle by connecting each pair of adjacent vertices with a circular arc centered on the opposing vertex, and Reuleaux polygons can be formed by a similar construction from any regular polygon with an odd number of sides as well as certain irregular polygons. Every curve of constant width can be accurately approximated by Reuleaux polygons. The Reuleaux polygons based on regular polygons are the only curves of constant width whose boundaries are formed by finitely many circular arcs of equal length [4],[5], [6].

The constant width of these reuleaux shapes allows their use as coins that can be used in coin-operated machines [7]. Although coins of this type in general circulation usually have more than three sides, a Reuleaux triangle has been used for a commemorative coin from Bermuda[8],[9], [10],[11]. Similar methods can be used to enclose an arbitrary simple polygon within a curve of constant width, whose width equals the diameter of the given polygon. The resulting shape consists of circular arcs (at most as many as sides of the polygon), can be constructed algorithmically in linear time, and can be drawn with compass and straightedge. Although the Reuleaux polygons all have an odd number of circular-arc sides, it is possible to construct constant-width shapes with an even number of circular-arc sides of varying radii [12],[13],[14].

It is possible to come across examples polygons of Reuleaux in nature. According to Plateau's laws, the circular arcs in two-dimensional soap bubble clusters meet at 120° angles, the same angle found at the corners of a Reuleaux triangle. Based on this fact, it is possible to construct clusters in which some of the bubbles take the form of a Reuleaux triangle [12], [15]. The shape was first isolated in crystal form in 2014 as Reuleaux triangle disks. Basic bismuth nitrate disks with the Reuleaux triangle shape were formed from the hydrolysis and precipitation of bismuth nitrate in an ethanol–water system in the presence of 2,3-bis(2-pyridyl)pyrazine[13].

A Reuleaux triangle [ʁœlo] is a curved triangle with constant width, the simplest and best known curve of constant width other than the circle [7]. It is formed from the intersection of three circular disks, each having its center on the boundary of the other two. Constant width means that the separation of every two parallel supporting lines is the same, independent of their orientation. Because its width is constant, the Reuleaux triangle is one answer to the question "Other than a circle, what shape can a manhole cover be made so that it cannot fall down through the hole [8]. They are named after Franz Reuleaux, a 19th-century German engineer who pioneered the study of machines for translating one type of motion into another, and who used Reuleaux triangles in his designs [9],[10]. However, these shapes were known before his time, for instance by the designers of Gothic church windows, by Leonardo da Vinci, who used it for a map projection, and by Leonhard Euler in his study of constant-width shapes. Other applications of the Reuleaux triangle include giving the shape to guitar picks, fire hydrant nuts, pencils, and drill bits for drilling filletedsquare holes, as well as in graphic design in the shapes of some signs and corporate logos [16].

Among constant-width shapes with a given width, the Reuleaux triangle has the minimum area and the sharpest (smallest) possible angle (120°) at its corners. By several numerical measures it is the farthest from being centrally symmetric. It provides the largest constant-width shape avoiding the points of an integer lattice, and is closely related to the shape of the quadrilateral maximizing the ratio of perimeter to diameter. It can perform a complete rotation within a square while at all times touching all four sides of the square, and has the smallest possible area of shapes with this property. However, although it covers most of the square in this rotation process, it fails to cover a small fraction of the square's area, near its corners. Because of this property of rotating within a square, the Reuleaux triangle is also sometimes known as the Reuleaux rotor.

II. MATERIALS AND METHOD

Some micro samples were used as study material. These micro samples were indicated in the photographs of their microstructures. In order to obtain the microstructures to be used in defining the geometric models of these samples, cross-sections of 10-20 μm were taken from different parts of the plant samples with the help of a microtome. These sections obtained were colored with safranin and fast green double staining. The preparations prepared from the sections were examined using microscope objectives and their photographs were taken at different magnifications. Cells of geometric modeling were marked on these original photographs. In mathematical evaluations, mathematical concepts determined in the light of the literature were used to describe the models of microstructures. In the study, literature information about geometric structures and their mathematical formulas was evaluated. detail the materials and methods used when conducting the study. The citations you make from different sources must be given and referenced in references [2],[3],[5], [12],[13],[14].

III. RESULTS

In this study, microscopic observations were made to see the micro structures of some plant parts. It was determined that the cells that make up the micro structures obtained from these observations have different reuleaux polygons. These shapes (reuleaux polygons) are named after their prototypical example, the Reuleaux triangle (Fig. 1). We have found that some of the cells we observe have regular and irregular polygon models of this geometric model called Reuleaux Polygon in geometry. Some of the cells that has the Reuleaux polygons geometric models have been shown by marking on photographs (Fig. 2).

The Reuleaux polygons based on regular polygons are the only curves of constant width whose boundaries are formed by finitely many circular arcs of equal length (Fig.3). Sometimes in Reuleaux polygons, each curve of constant width may not have the same width but may have similar widths. This creates an irregular Reuleaux polygon (Fig.4). In other words; every curve of constant width can be approximated arbitrarily closely. Reuleaux polygons The Reuleaux triangle can be generalized to regular or irregular polygons with an odd number of sides, yielding a Reuleaux polygon, a curve of constant width formed from circular arcs of constant radius [11]. If P is a convex polygon with an odd number of sides, in which each vertex is equidistant to the two opposite vertices and closer to all other vertices, then replacing each side of P by an arc centered at its opposite vertex produces a Reuleaux polygon. As a special case, this construction is possible for every regular polygon with an odd number of sides. Every Reuleaux polygon must have an odd number of circular-arc sides, and can be constructed in this way from a polygon, the convex hull of its arc endpoints. However, it is possible for other curves of constant width to be made of an even number of arcs with varying radii.

A regular Reuleaux polygon has sides of equal length. More generally, when a Reuleaux polygon has sides that can be split into arcs of equal length, the convex hull of the arc endpoints is a Reinhardt polygon. These polygons are optimal in multiple ways: they have the largest possible perimeter for their diameter, the largest possible width for their diameter, and the largest possible width for their perimeter (Fig.3) [4].

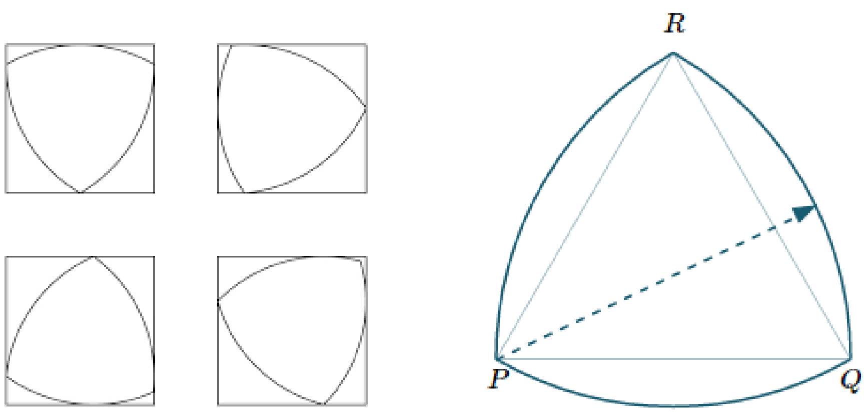


Fig.1 a reuleaux triangle replaces the sides of an equilateral triangle by circular arcs [1],[3].

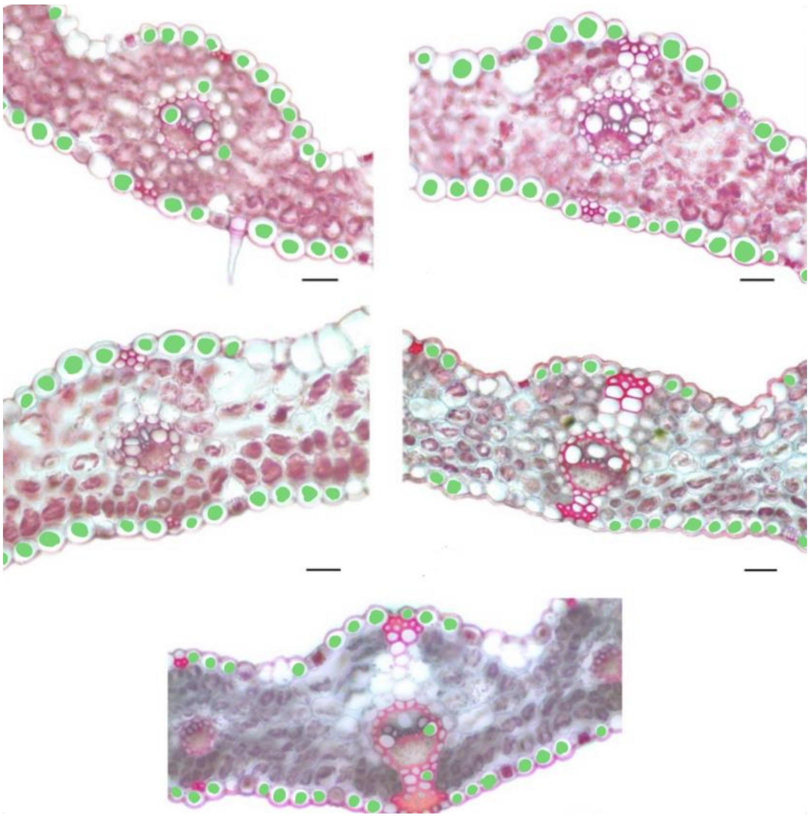
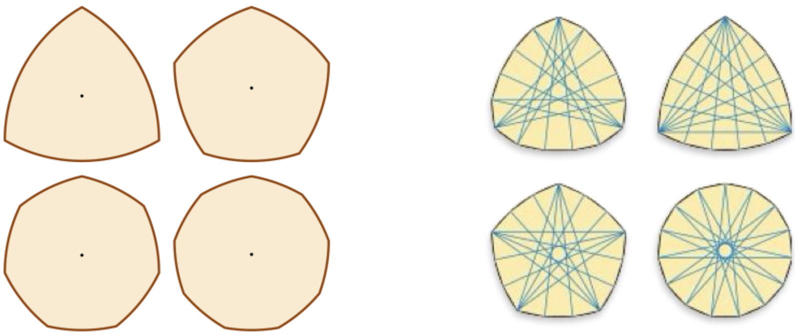


Fig. 2 In the original photographs of microstructures, cells with a regular and irregular reuleaux polygon geometric models (scala bar :20 mikron).



a b

Fig. 3 Reuleaux polygons (all of the same with) based upon regular polygons

a: with three, file, seven and nine sides. b: Four 15-sided Reinhardt polygons, formed from four different Reuleaux polygons with 9, 3, 5, and 15 sides [1],[2].

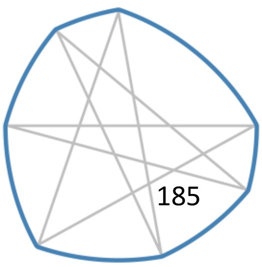


Fig. 4 Reuleaux polygons (all of the different with) based upon irregular polygon [3],[4].

IV. DISCUSSION

In our research, we determined that the microstructures (the cell) of some plants whose microscopic structures we examined have different geometrical models. We observed that some of these geometric models of microstructures belong to different types of the geometric concept of Reuleaux polygons (regular and irregular). In this study, the shape of the cells in the microstructures of plants was tried to be revealed by using the definitions corresponding to the geometrical rules. In this study, a different perspective was provided by evaluating the micro morphological structures of some plants mathematically. Thus, it creates a new comparison opportunity for future research on the relevant subject.

V. CONCLUSION

Similarly, geometric models of plant micro morphological structures can help us understand how microstructure determines important properties. It can also help us to develop predictive models of known the geometric model of microstructures.

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