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Computational Analysis of MHD Mixed Convection Jeffrey Nanofluid Flow Past a Stretching Sheet With Thermal Radiation

Saira Kanwal [1] and Siddra Rana [*1]

[1]Department of Mathematics, Faculty of Basic Sciences, University of Wah Wah Cantt, Pakistan

[*](sidra.rana@uow.edu.pk)

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Abstract **–** This work considers the effects of thermal radiation, Brownian and thermophoretic diffusion, and viscous dissipation in the mixed convection magnetohydrodynamic (MHD) flow of a Jeffrey nanofluid over a nonlinear stretching surface. A set of ordinary differential equations was created from the governing partial differential equations for momentum, energy, and concentration using the proper similarity transformations. These nonlinear equations are splved numerically under predetermined boundary conditions. The results show that increasing the magnetic parameter suppresses the velocity field because of the increased Lorentz force. Thermal radiation and viscous dissipation enhance the temperature distribution. The results shed light on a number of industrial procedures, such as viscoelastic nanofluid heat transfer systems, wire coating, and polymer extrusion.

Keywords – Jeffrey nano fluid, Mixed convection magnetohydrodynamic, thermal radiation, Brownian and thermophoretic diffusion.

I. INTRODUCTION

The study of fluid dynamics is both complex and captivating, playing a vital role in numerous aspects of daily life. Significant advancements in fluid dynamics were achieved in the 20th century, and recently the behavior of non-Newtonian Nano fluids has lately drawn interest of researchers. Blood flow in our arteries and veins are an interesting example of non-Newtonian fluid. Non-Newtonian fluids have significantly impacted on our industry. The study of non-Newtonian fluids, especially which are modeled by Jeffery fluid model, is more important because of its wide applications in industries and daily products. The Jeffery fluid model was introduced by George Barker Jeffery in 1922, is essential in representing the unique characteristic of Non-Newtonian fluid, which is not followed by Newton's law of viscosity. The Jeffery fluid model is an impressive and significant framework used to represent the behavior of such fluid, particularly when examining flow over a nonlinear stretching surface. This model considered the complex fluid viscosity and normal stress difference, which is essential for accurately representing the behavior of flow of many real world Non-Newtonian fluids. Abdul Rehman [1]s considered Jeffery fluid flow with transversal magnetic field is present. They found solution for highly nonlinear problems. Sandeep et al. [2] investigated MHD Jeffery Nano fluid over a stretching surface with magnetic field. Sadiq [3] have discussed heat transmission of Nano fluid over a surface extended sheet.

The term "nanofluids" was first proposed by Choi [4]at Argonne National Laboratory in the United States in 1995 [5] as a way of common heat transfer liquids. The MHD influence is considered to enhance the rate of cooling and properties of final products in the metallurgical operation such as copper wire drawing, annealing and thinning. The effects of this study have significant importance for the design and development of industrial process involving Non-Newtonian fluids. Engineers and Scientist can expand more efficient and effective process, leading to improved product quality and performance by understanding the Jeffery fluid's heat transfer features and flow dynamics a stretched surfac [6].

Numerous studies have focused on the Jeffery fluid model [7] over stretching surfaces, highlighting various physical effects and employing different numerical techniques. Riaz [8] discussed properties of thermal and flow of a Jeffrey fluid. Naz [9] examined the impacts of thermal radiation and magnetic fields on the movement of a Jeffery fluid across stretched sheet. Ahmed [10]worked on impact of annular sector duct fluid on MHD Jeffrey. They used finite difference approach to show how these variable features affect the rates of heat transfer and flow. Qasim [11] investigated the Jeffery fluid's mass and heat transmission properties over a stretched surface when a porous material was present. By applying the shooting method for numerical solutions, they provided detailed insights into the role of nanoparticles in modifying thermal and flow fluids. The influence of magnetohydrodynamics (MHD) and thermal radiations on non-Newtonian fluid has been extensively researched because to their practical significance in high temperature and electromagnetic applications. H.Basha [12] explored mechanisms of sheet heat and mass transport on Jeffrey fluid past on stretchablesheet. Bhadauria [13] worked on thermal rariadtion on Nano fluids. Shaheen [14] discovered Electro-osmotic propulsion of Jeffrey fluid including effect of nonlinear radiation and heat source. Sadiq [15] enhanced his for estimating thin effects on flow of Jeffrey fluid model. Ramesh [16] worked on numerical solution of unsteady flows of Jeffrey fluid between plates which are parallel. Riaz [17] investigated on exact solutions for thermo magnetized unsteady Jeffrey fluid.

II. MATERIALS AND METHOD

A. Mathematical Formulation

Consider two-dimensional incompressible steady Jeffery fluid flow across a sheet which is stretched nonlinearly in the existence of uniform magnetic field. Suppose that the sheet is being stretched along x-axis maintaining equilibrium at origin. The sheet stretched nonlinearly due to applied forces along the x-axis hence generating the flow. The direction of motion along the stretching surface is placed with the x-axis, whereas the y-axis is perpendicular to it. The velocity at which the sheet is stretched once the origin has been fixed is U_w. The magnetic field B_0 is applied down the y-axis and is the fixed magnetic strength. Although the surface temperature T_w deviate from ambient temperatures it is considered as constants. The concentration of the fluid $C_w$is proposed to be constant. For $y \to \infty$ the concentration and temperature of the fluid is represented as C_∞ and T_∞ respectively.

The relevant continuity, momentum, energy equations and concentration equations for two-dimensional Jeffery fluid are as follow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{v}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_2\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)\right]+g\beta_T(\mathrm{T}-T_\infty)-\frac{\sigma {B_0}^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{v}{C_p(1+\lambda_1)}\left[(\frac{\partial u}{\partial y})^2+\lambda_2(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2})\right]-\frac{\partial q_r}{\partial y}, \tag{3}$$

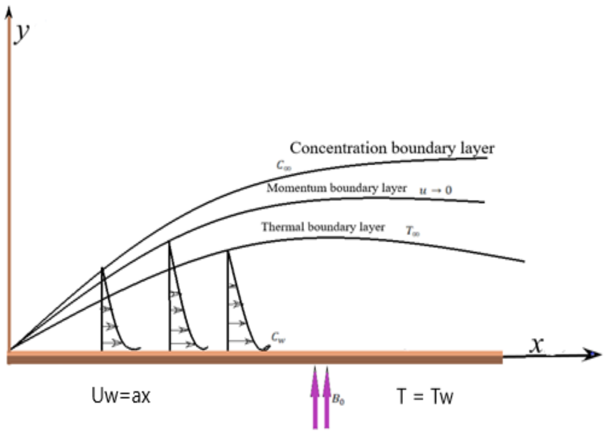

Fig.1. Flow model geometry

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} - k_0(\ C - C_\infty)\ . \quad (4)$$

The appropriate conditions of boundary are as follow,

$$\left.\begin{array}{l} \mathrm{u} = U_w\ , \quad \mathrm{v} = 0, \quad \mathrm{T} = T_w,\ \mathrm{C} = 0 \quad \text{at}\ \ \mathrm{y} = 0, \\ \mathrm{u} \to 0, \quad \frac{\partial \mathrm{u}}{\partial \mathrm{y}} \to 0\ ,\ \ \mathrm{T} \to T_\infty\ \ ,\mathrm{C} \to C_\infty \quad \text{at}\ \ \mathrm{y} \to \infty\ . \end{array}\right\} \quad (5)$$

The definition of radiative heat flow q_r is defined as [18].

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (6)$$

$$u = axf'(\eta), \quad v = -\sqrt{av}f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\ ,\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ \eta = \sqrt{\frac{a}{v}}\ y\ ,$$

$$f''' + \beta(f''^2 - ff^{iv}) + (1 + \lambda_1)\left[ff'' - f'^2 - Mf' + \lambda\theta\right] = 0, \quad (7)$$

$$(1 + \lambda_1)[(1 + N_r)\theta'' + Prf\theta'] + EcPr\left[f''^2 + \beta(ff''^2 - ff''f''')\right] = 0, \quad (8)$$

$$\phi'' + L_e f\phi' + \frac{N_t}{N_b}\theta'' + L_e\gamma\phi'. \quad (9)$$

$$\left\{\begin{array}{l} f(0) = 0,\ f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1, \\ \quad f'(\infty) \to 0, \quad \theta(\infty) \to 0\ , \quad \phi(\infty) \to 0. \end{array}\right\} \quad (10)$$

Here $\Pr = \frac{v}{\alpha}$ is Prandtl number, $N_r = \frac{16\sigma^* T_\infty{}^3}{3kk^*}$ is symbolized as radiation parameter, $M = \frac{\sigma B_0{}^2}{a\rho}$ is magnetic parameter, $Ec = \frac{U_w{}^2}{C_{p(T_W - T_\infty)}}$ Eckert number.

B. *Computational Scheme and Solution Methodology:*

The following transformation is used to convert higher order differential equations into first order so that to apply the RK mFehlberg method for solution.

$$\begin{pmatrix} f \\ f' \\ f'' \\ f''' \\ f^{iv} \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ yy_1 \end{pmatrix}, \begin{pmatrix} \theta \\ \theta' \\ \theta'' \end{pmatrix} = \begin{pmatrix} y_5 \\ y_6 \\ yy_5 \end{pmatrix}, \quad \begin{pmatrix} \phi \\ \phi' \\ \phi'' \end{pmatrix} = \begin{pmatrix} y_7 \\ y_8 \\ yy_7 \end{pmatrix}$$

By using above equations, we have

$$yy_1 = \frac{1}{\beta y_1} [y_4 + \beta {y_3}^2 + (1+\lambda_1)\{y_1 y_3 - {y_2}^2 - y_2 M + \lambda y_5\}] \tag{11}$$

$$yy_5 = \frac{-E_c P_r}{(1+N_r)(1+\lambda_1)} \{{y_3}^2 + \beta(y_2 {y_3}^2 - y_1 y_3 y_4)\} - \frac{P_r}{(1+N_r)} y_1 y_6, \tag{12}$$

$$yy_7 = -L_e y_5 y_8 - \frac{N_t}{N_b} yy_5 + L_e \gamma y_7. \tag{13}$$

$$At\ \eta = 0, y_1 = 0, y_2 = 1, y_5 = 1, y_7 = 1,$$

$$At\ \eta = \infty, \quad y_2 = 0 \quad , y_5 = 0, \quad y_7 = 0$$

The solution for the equation from is obtained graphically.

C. *Figures and Tables*

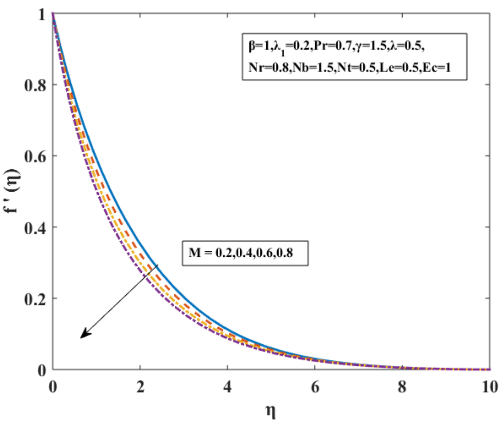

Fig. 2. Impression of magnatic parameter M on

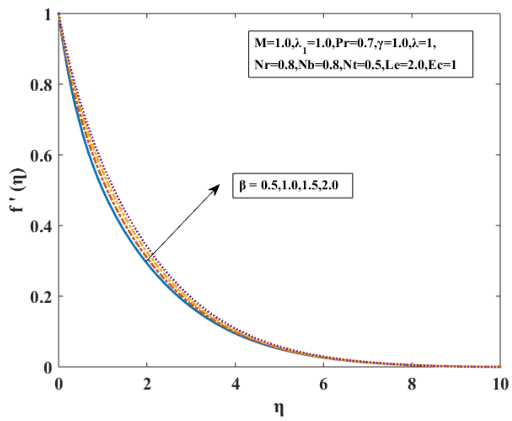

Fig.3. Impression of Deborah number β

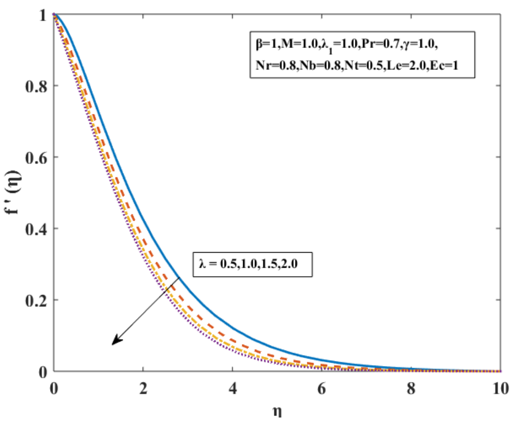

Fig. 4.Impression of mixed convection parameter λ

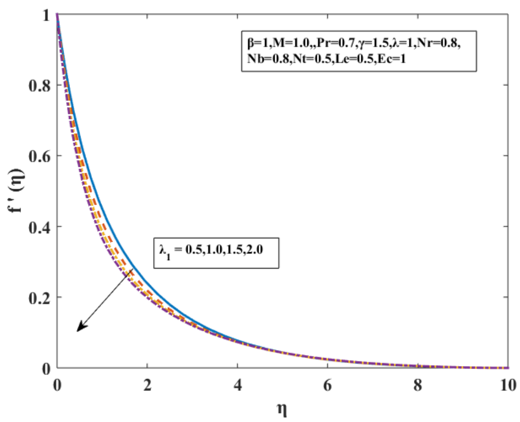

Fig.5Impression of mixed convection parameter λ_1

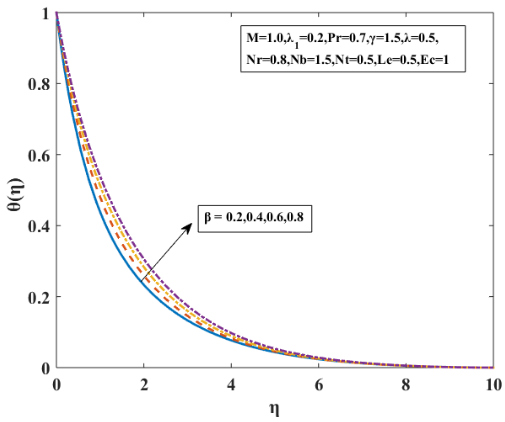

Fig..6 Impression of Deborah number β

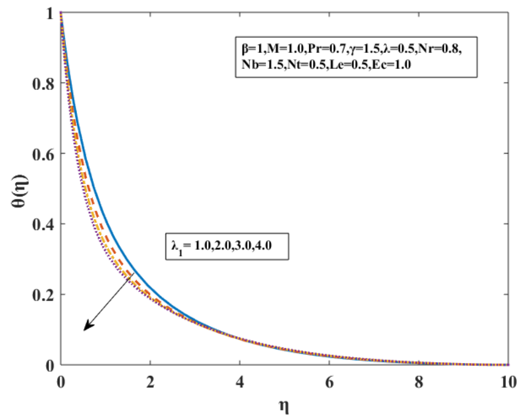

Fig.7 Impression of mixed convection parameter λ_1

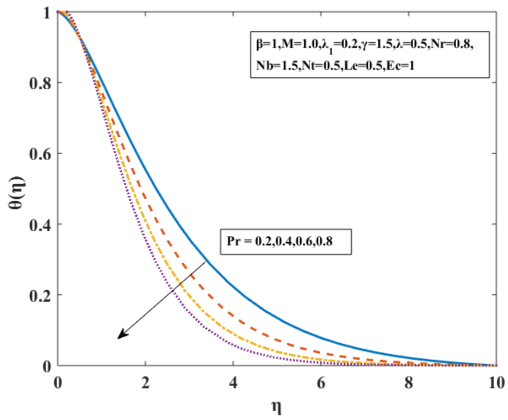

Fig.8 Impression of Prandtle number Pr

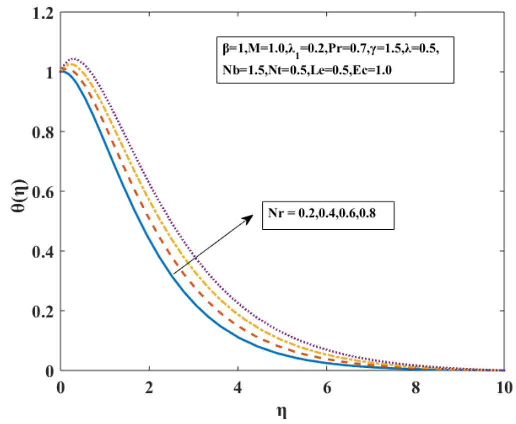

Fig.9 Impression of thermal radiation parameter Nr

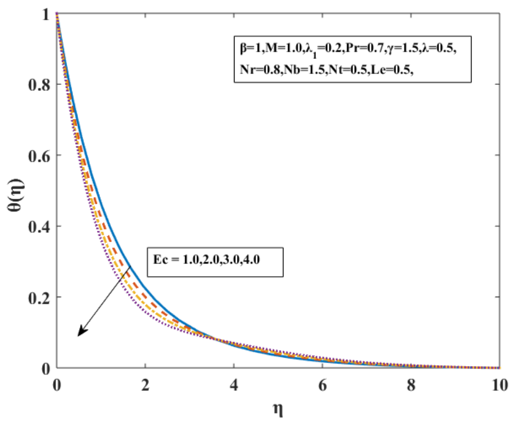

Fig.10 Impression of Eckert number Ec

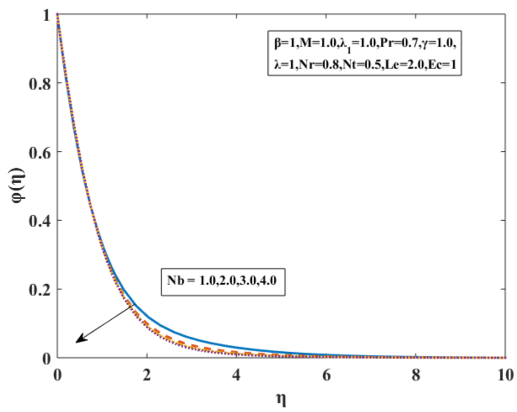

Fig.11 Impression of Brownian motion parameter Nb

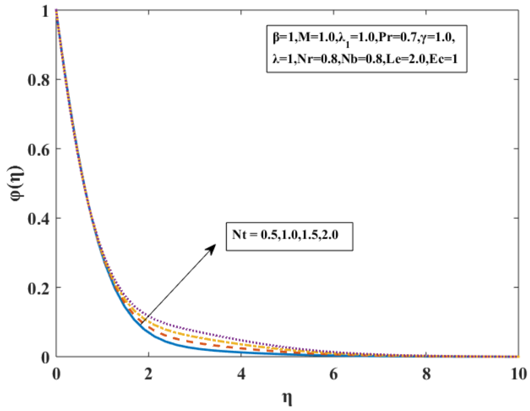

Fig.12. Impression of thermophoresis parameter N_t

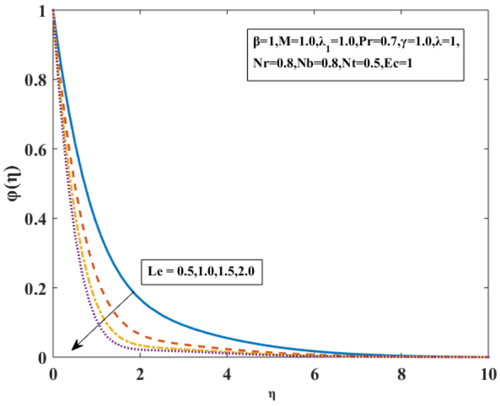

Fig.13 Impression of Lewis number Le

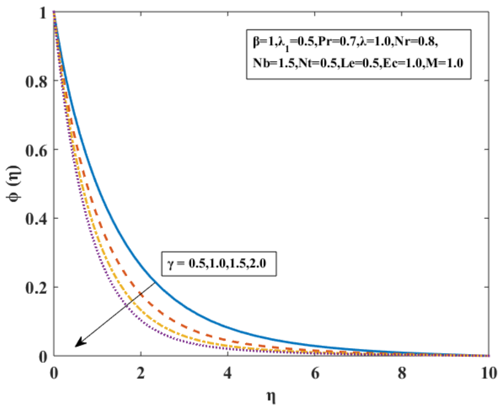

Fig.14 Impression of porosity medium γ

III. RESULTS & *DISCUSSION*

This section is designed to show the results obtained through numerical solution.

Fig.2 displays the impact of magnetic field parameter on the velocity profile. It is seen that the variation in magnetic parameter M have a considerable impact on $f'(\eta)$. As the values of M increases the resultant Lorentz force become greater and act as a resistive force against the flow of fluid. This resistance reduced the velocity of fluid with in the boundary layer. Hence velocity profile decreases continuously for greater values of M.

Fig.3 plotted the behavior of velocity profile for increasing values of Deborah number β. As Deborah number β grows the velocity profile increases. It is observed that the velocity and boundary layer thickness are increasing functions of the Deborah number β.

The variation in velocity profile for mixed convection parameter is plotted in fig.4. The mixed convection parameter effects the velocity profile indicating the intensity of buoyancy force verses viscous forces. The existence of thermal buoyancy leads to increase the flow over the surface. As λ grows the velocity profile decrease as higher values of λ decrease the influences of buoyant forces.

Fig.5 depicts the variation in velocity profile towards various values of parameter λ_1, which is the ratio of relaxation and retardation time. An increase in λ_1 indicates a more elastic reaction of the fluid. This increase in λ_1 resist the flow deformation resulting in a reduced velocity profile. Higher values of λ_1 reduced the motion of fluid at boundary layer due to elastic and viscous resistance.

Figure.6 depicts the behavior of Deborah number on temperature distribution. It is seen that the temperature profile goes up for increasing values of Deborah number β. This is because viscoelastic effect increases internal friction and more heat is generated. This heat dissipation thickens the thermal boundary layer increasing the temperature dispersion throughout the fluid.

The temperature distribution of the flow also declines as λ_1 increases as shown in fig.7. Higher values of λ_1 diminishes the fluid capacity to transmit heat effectively and the boundary layer become thinner. The elastic nature of fluid reduces energy dissipation and temperature distribution decreases.

The impact of Prandtl number on temperature distribution ids shown in fig.8. The Prandtl number is the ratio of kinematic viscosity to thermal diffusivity. As the values of Prandtl number increases the capacity of fluid to transfer heat decreases which represents the reduction in thermal diffusivity. This result in a narrow thermal boundary layer and lower the temperature profile. Physically it means that the fluid with greater values of Pr prefer momentum diffusion over heat diffusion in this way less heat is transfer into the fluid.

Thermal radiation parameter Nr significantly affect the temperature profile as shown in fig.9. It is seen that temperature profile increases as Nr increases. This is because Nr added more heat to the fluid. A thicker thermal boundary layer and more significant temperature differential is seen due to thermal energy.

The Eckert number Ec measure the impact of viscous dissipation, has significant influence on temperature profile. Fig.10 shows the variation in temperature distribution for Eckert number. The temperature profile decrease for greater values of Ec as viscous dissipation causes a noticeable conversion of mechanical energy into thermal energy. This thermal energy decrease the temperature distribution.

Fig.11 shows how the Brownian motion parameter effect the concentration profile. It is seen that rising Nb reduces the concentration profile. This phenomenon is due to enhancement in Brownian motion which allows the particles spread more equally through the fluid. The increased in random mobility declines the concentration profile by spreading the particle uniformly and reducing buildup in certain regions.

Fig.12 shows the impact of thermophoresis parameter on concentration profile. The study found that higher values of Nt leads to a lower concentration profile. The reason is impact of thermophoretic force which moves particles from cooler to hotter region. This movement of particles accelerates the redistribution of particles throughout the fluid resulting in a decrease concentration profile.

Fig.13 examines how the variation in Lewis number Le affect the concentration profile. The concentration distribution decreases for increasing values of Le. Physically the Lewis number is ratio of thermal diffusivity to molecular diffusivity. Higher values of Le indicates the lower molcular diffusivity. The decrease in molecular diffusivity decreases the concentration profile.

Fig.14 shows the concentration profile for different values of γ. The concentration profile decreases for increasing values of γ, it denotes the external source or sink or a chemical reaction parameter. By removing solute particles from the fluid, the concentration boundary layer becomes thinner and concentration profile reduces. This decrease happen as a result of the reaction or sink reducing the accumulation of particles near the surface.

The influence of various parameters of interest on the skin friction coefficient, Local Nusselt number and Sherwood number is shown in tables.

IV. *CONCLUSION*

This section presents a thorough analysis of the numerical solution of the Jeffrey fluid model on a nonlinear stretching sheet. The main findings from this work can be as follow:

As the magnetic parameter increases, the velocity field reduces, whereas the temperature profile shows the opposite tendency. The higher values of Prandtl number indicates better heat transfer rates as it decreases the thickness of thermal boundary layer and temperature profile decreases. Increasing elastic effect, the velocity and temperature profile increases as Deborah number rises. By increasing the thermal radiation parameter, the heat transfer increases and concentration profile decreases. The concentration profile reduces for increasing values of Lewis number.

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