

MHD Natural Convection over a Vertical Wedge in a Darcy–Forchheimer Porous Medium

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(Received: 27 February 2026, Accepted: 10 March 2026)

(8th International Conference on Scientific and Academic Research ICSAR 2026, February 26-27, 2026)

ATIF/REFERENCE: Matar, M. I. & Mohammad, S. A. (2026). MHD Natural Convection over a Vertical Wedge in a Darcy–Forchheimer Porous Medium, *International Journal of Advanced Natural Sciences and Engineering Researches*, 10(3), 55-62.

Abstract – This study addresses heat and mass transfer under magnetohydrodynamic convection over a vertical wedge submerged in a porous medium. The governing equations were non-dimensionalized and solved numerically using the finite difference method. The effects of several physical parameters were investigated, including the power-law index, inertia coefficient, buoyancy ratio, Lewis number, thermal gradient coefficient, and Hartmann number, on velocity, temperature, concentration, and Nusselt and Sherwood numbers. The results showed that increasing the buoyancy ratio enhances heat and mass transfer, while increasing the inertia coefficient and Hartmann number reduces them. The results were validated by comparing them with the Darcy case, where the error percentage was 0.045%, confirming the accuracy of the numerical model.

Keywords – Magnetohydrodynamics (MHD), Natural Convection, Vertical Wedge, Heat And Mass Transfer.

I. INTRODUCTION

Multiple media contain spaces (pores) that allow the movement of fluids (whether liquids or gases). These materials are found in nature, such as black and soil, and are then used in applications such as filtration as well as the energy and biomedical fields. Understanding multimedia fundamentals is essential in many scientific disciplines, including hydrology, petroleum engineering, and materials science [1,2]. Solids have a network of mesh or insulated meshes so that liquid can flow through the mesh and are called mesh media. This distance is characterized by polarities such as porosity (porosity) and permeability (permeability), including the size of the pores, and external forces such as degrees of pressure and solubility on the movement of fluids through these pores. There are many different natural and artificial forms of multimedia that have a large number of applications in science and industry as shown in Figure (1). Nath, G. et al [3] An experimental and analytical study of natural convection heat transfer from an isothermal vertical wall immersed in a highly permeable porous medium (air-saturated polyurethane foam). By combining experimental measurements and analytical modeling, the researchers evaluated heat transfer rates by taking into account the inertia effect. They concluded that heat transfer and fluid speed are clearly affected by the Graschoff number (Gr). However, the effect of the Graschoff number (Gr) on heat transfer is very small in the case of fluid suction. It is found that the maximum velocity occurs at the wall and increases with

increasing the Grashof number (Gr) or fluid injection. (Murthy, p. and Singh, p.) [4] did, where they conducted an experimental study that dealt with the effect of viscous dissipation on the non-Darcy natural convection system along an isothermal vertical wall immersed in a saturated porous medium. The researchers noticed that viscous dissipation reduces the rate of heat transfer. (Kladias, N. and Prasad, V.) [5] numerically investigated the phenomenon of natural convection of Darcy flow in porous horizontal layers (rectangular) heated from below, focusing on the effect of inertia and the change in porosity. Using the Darcy-Brinkman-Forchheimer (DBF) model, he concluded that the speed of rotational flow increases, meaning that the fluid inside the layer does not move in a straight line from the bottom to the top only, but rather rotates in closed paths resembling vortices or cells as a result of the difference in temperature and density within the medium. , thermal stratification in the upper and lower regions is enhanced with increasing Darcy number (Da). In addition, temperature gradients become steeper near the upper left and lower right. It is clear that in any given system the flow intensity and thermal activity increase as the Darcy number increases, as a result of the increased permeability of the porous medium, which leads to reduced flow resistance.

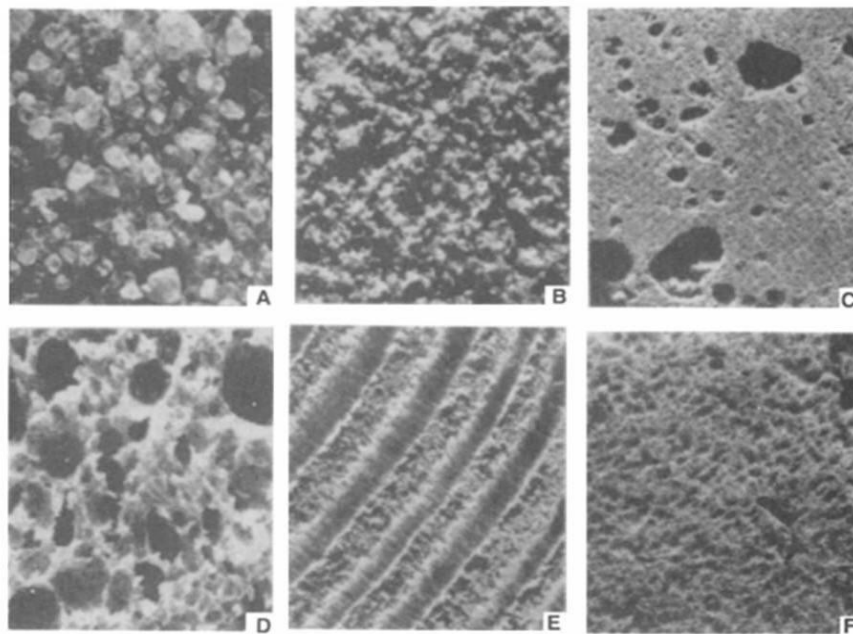


Fig.1. Examples of natural porous materials: A) Coastal sand, B) Sandstone, C) Limestone, D) Rye bread, E) Wood, F) Human lung.[6]

II. MATHEMATICAL FORMULATION

To begin studying any physical issue, one must first identify the hypotheses that lead to a clear and logical modeling of the given problem. In this study, all physical quantities, such as velocity, temperature, concentration, and others, are treated based on the average volume. To derive the governing equations for these quantities, it is assumed that there is a vertical edge immersed in a porous medium saturated with fluid, as shown in Figure (2).

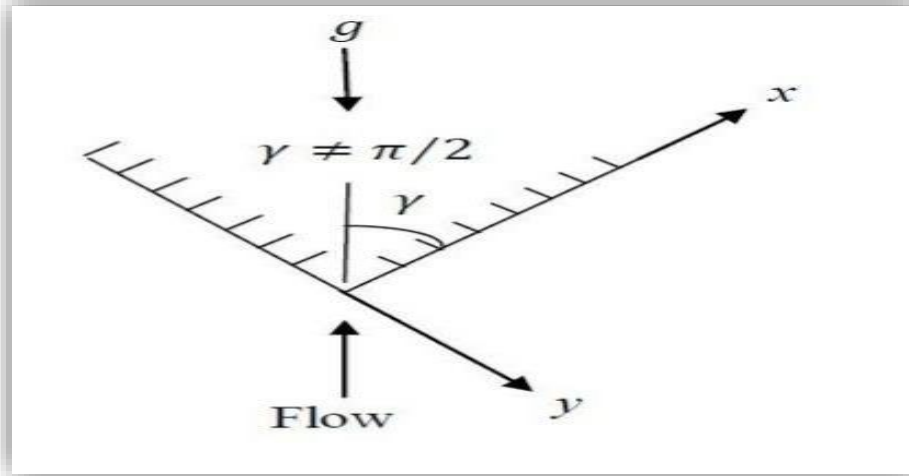


Fig.2. illustrates the physical description of the problem

A. Continuity Equation Since the flow is two-dimensional, steady, and the fluid is incompressible, the continuity equation takes the following form [7],[8]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Since (u) is the velocity component in the (x) direction and (v) is the velocity component in the (y) direction

B. Momentum Equation Based on the previously mentioned assumptions and taking into account the effect of fluid inertia, the momentum conservation equation in the x-direction is expressed as follows.

$$u = \frac{-K}{\mu} \left(\frac{\partial p}{\partial x} + \rho g_x \right) - \frac{c\sqrt{K}}{\nu} u^2 + \frac{\sigma\beta^{02}u}{\phi} \quad (2)$$

C. Energy Equation The energy equation will be converted to the nonlocal form.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

D. Concentration Equation The concentration equation will be transformed into the Laplace form.

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

E. Boundary condition

$$\text{at } y = 0: v = 0, T = T_w(x) = T_{\infty}(0) + Ax^m, C = C_w(x)C_{\infty}(0) + Ax^m$$

$$\text{at } y = \infty: u = 0, T = T_{\infty}(x) = T_{\infty}(0) + Bx^m, C = C_{\infty}(0) \tag{5}$$

F. Dimensionless Variables

$$\theta(\eta) = \frac{T - T_{\infty}(x)}{T_w(x) - T_{\infty,0}}, S = \frac{T_{\infty}(x) - T_{\infty,0}}{T_w(x) - T_{\infty,0}}, \Phi(\eta) = \frac{C - C_{\infty,0}}{C_w - C_{\infty,0}} \tag{6}$$

G. Dimensionless Variables

$$[1 + 2\Gamma f' + M] f'' + (-\theta' - N\Phi') = 0 \tag{7}$$

where

$$\Gamma = \frac{\left(c\sqrt{K} * \left(Ra_x^{\frac{1}{2}} \right)^2 * \alpha \right)}{\nu}$$

Γ : moment of inertia

$$M = \frac{(\sigma\beta_0^2 K)}{\mu\phi}$$

M: square of the Hartmann Number

$$N = \frac{\beta_C(C_w - C_{\infty})}{\beta_T(T_w - T_{\infty})}$$

N: Buoyancy Ratio

H. Dimensionless Energy Equation

$$\theta'' + \frac{1}{2}f\theta'(m + 1) - mf'\theta = -Smf' \tag{8}$$

I. Dimensionless Concentration Equation

$$-\frac{1}{Le}\Phi'' - \frac{1}{2}f\Phi'(n + 1) + n f'\Phi = 0 \tag{9}$$

$$\frac{1}{Le} = \frac{D}{\alpha}$$

Le: Louis number

III.RESULT AND DISCUSSION

A. The extent of the parameters studied: To analyze the effect of variables on the flow phenomena of heat and mass, specific parameter values were entered into the program, and their effect on the velocity diagram, temperature distribution diagram, concentration factor diagram, spot and rate Nusselt number, spot and rate Sherwood number, and on the local shear stress were studied. The values of m were taken, which represent the exponent of the law of temperature and concentration (0, 0.5, 1, 2). The value of Γ (the inertia effect factor) was taken (10, 1, 0.1, 0). The value of N (buoyancy factor) was taken (8, 5, 1, 0). The effect of the parameter Le (Lewis number) is studied for values (100, 10, 5, 0.5). The values of the parameter S (coefficient of thermal stratification) were changed from (0 to 0.3, with a difference of 0.1). And the values of the parameter M (the square of Hartmann's number) were changed from (0 to 3, with a difference of 1).

B. Effect of the power law (m) on Darcy flow:

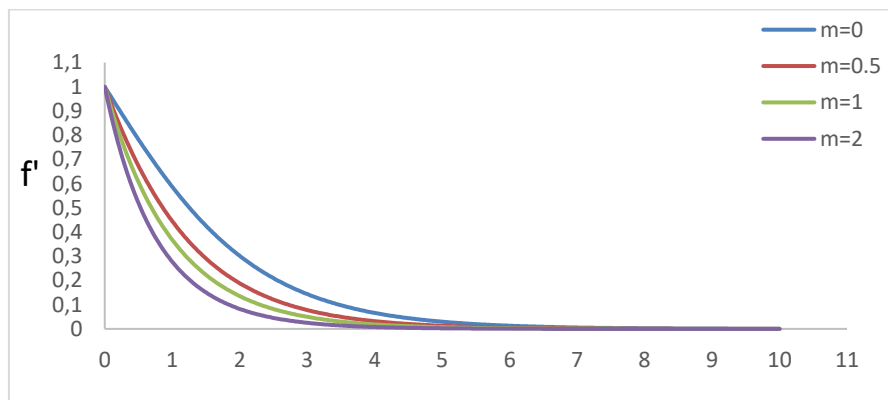


Fig .3. Effect of the parameter m on the non-dimensional flow velocity of Darcy flow when: $\Gamma = 0$, $N = 0$, $Le = 1$, $S = 0$, $M = 0$

Figure (3) illustrates the relationship between the far-field flow velocity and the similarity variable. It is observed that the far-field flow velocity of a Darcian flow decreases with an increase in the similarity variable η and the parameter m . The maximum far-field flow velocity starts at the surface and gradually decreases with distance from the surface. The reason for these decreases is that an increase in the parameter m enhances the effect of the surface temperature, which generates greater hydrodynamic resistance, leading to a reduction in the far-field flow velocity.

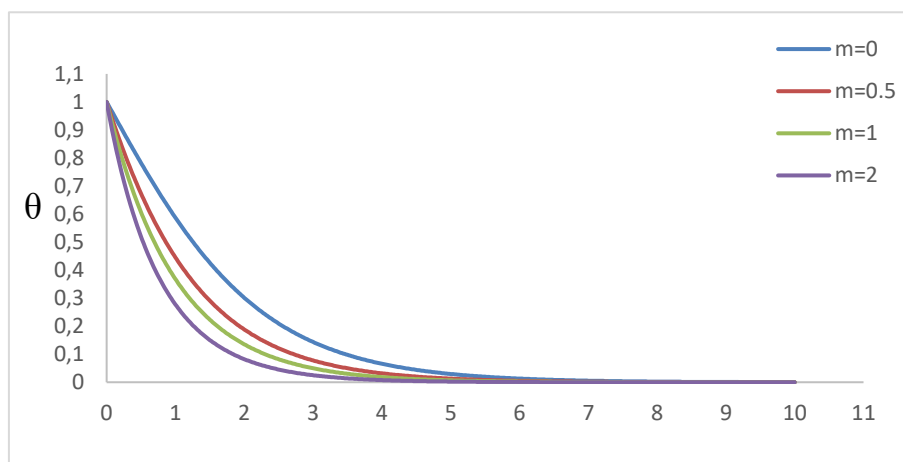


Fig .4. Effect of the parameter m on the remote temperature of Darcy flow when: $\Gamma=0$, $N=0$, $Le=1$, $S=0$, $M=0$

Figure (4) shows the relationship between the dimensional temperature and the analogue variable and it is noted that the dimensional temperature of the Darcy flow decreases with the increase of the η analogue variable and the parameter m . The reason for the decrease is the increase in the rate of heat transfer from

the surface to the fluid with the increase in the parameter m , which leads to a contraction in the thickness of the thermal boundary layer, due to the more efficient heat transfer as the fluid absorbs heat and reaches the temperature of the free fluid T_∞ with an increase in the analogue variable η .

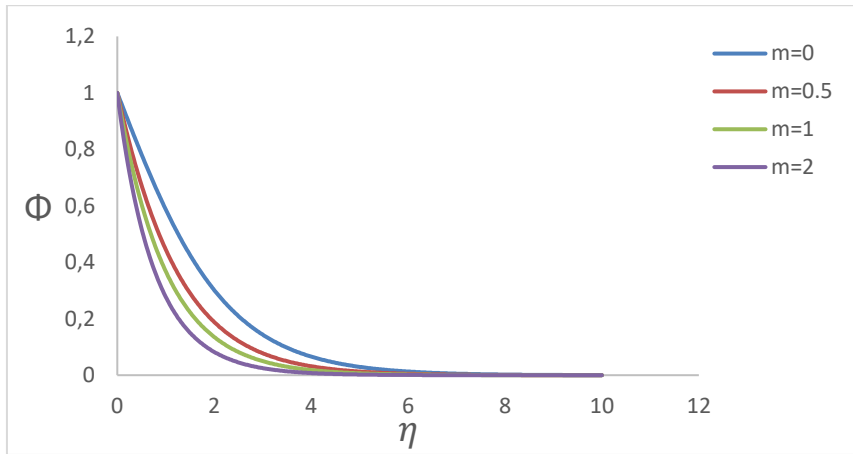


Fig .5. Effect of the parameter m on the Darcy flow local concentration coefficient when: $\Gamma=0, N=0, Le=1, S=0, M=0,$

Figure (5) illustrates the relationship between the local concentration coefficient and the similarity variable. It is observed that the local concentration parameter for a Darcian flow decreases with an increase in the similarity variable η and the parameter m . The reason for these decreases is the increase in the mass transfer rate with the increase of parameter m , which leads to a reduction in the boundary layer thickness of the concentration due to more efficient mass transfer.

C. Effect of Inertia (Γ):

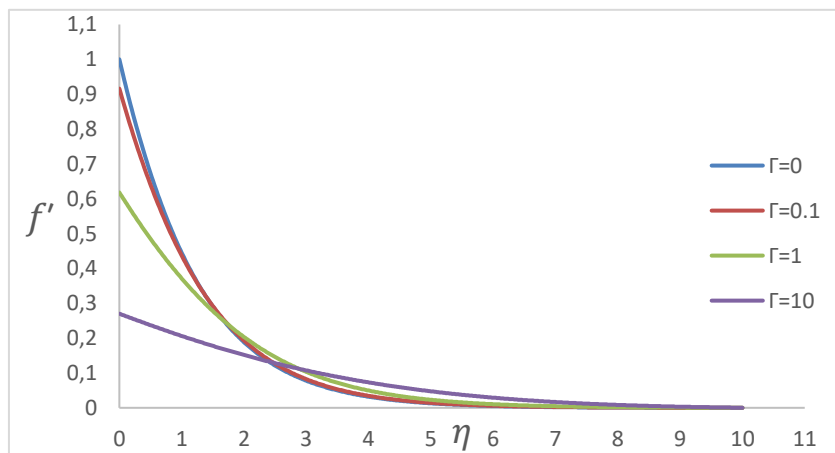


Fig .6. Effect of the non-local flow parameter Γ on velocity $m=0.5, N=0, Le=1, S=0, M=0$

Figure (6) illustrates the relationship between the non-dimensional flow velocity and the similarity variable. It is observed that the non-dimensional flow velocity decreases with the increase of the similarity variable η and the increase of the parameter Γ . We note that as the parameter Γ increases, the velocity decreases significantly near the wall and then begins to increase away from the wall.

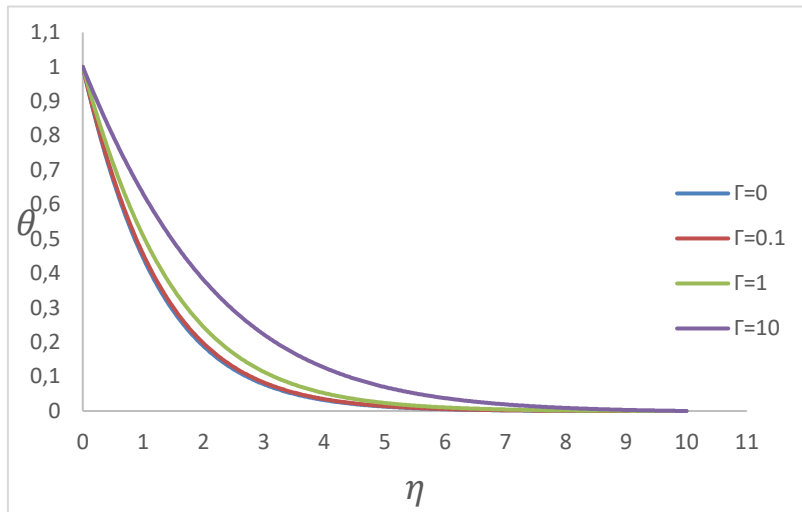
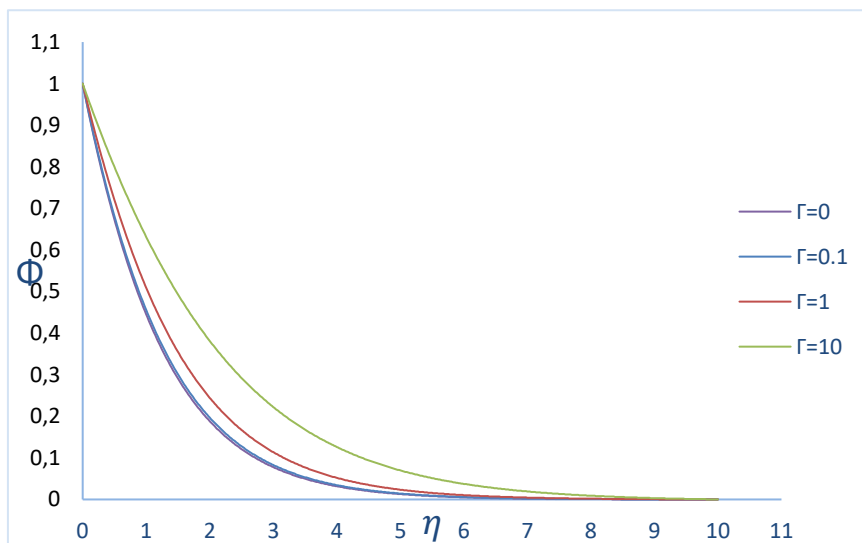


Fig .7. Effect of the heat transfer parameter when: Γ on the degree $m=0.5, N=0, Le=1, S=0, M=0$

Figure (7) illustrates the relationship between the local temperature and the similarity variable. It is observed that the local temperature increases with the increase of the similarity variable η and the parameter Γ . The reason for this increase is that the rise in the parameter Γ increases the flow resistance and affects the velocity distribution. We also notice an increase in the thermal boundary layer thickness, which reduces the heat transfer efficiency away from the surface, and this in turn leads to an increase in local temperatures within the thermal boundary layer.



Fi.8. Effect of the parameter Γ on the coefficient when the backward concentration $m=0.5, N=0, Le=1, S=0, M=0$

In light of Figure (8), we see the relationship between the posterior concentration coefficient and the analog variable. It is noted that the posterior concentration parameter increases with the increase of the analog variable η and the parameter Γ . We also observe an increase in the thickness of the boundary layer associated with the concentration, which is attributed to the same reasons explained during the discussion of Figure (8).

IV. CONCLUSION

The results showed that increasing the power law index m leads to an improvement in local heat and mass transfer, with optimal performance at $m = 0.5$, while average values decrease with further increase. It was also found that increasing the inertia coefficient Γ reduces heat and mass transfer rates due to increased flow resistance. Conversely, increasing the buoyancy ratio N results in a significant enhancement in the

Nusselt and Sherwood numbers and shear stress. The results indicated that the Lewis number Le significantly affects mass transfer without a noticeable impact on heat transfer. Additionally, increasing the thermal slip coefficient S and the square of the Hartmann number M leads to a decrease in heat and mass transfer rates due to the effects of thermal gradient and magnetic field. Overall, it is evident that the interaction between the physical parameters plays a crucial role in controlling flow behavior and heat and mass transfer in a Darcy–Forchheimer porous medium.

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