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Research Article

A triangular finite element based on assumed strains for membrane structures

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Abstract **–** A simple triangle strain-based element has been developed for plane stress and plane strain issues. This element has three nodes. Each of the three nodes has three degrees of freedom. The developed element can be applied to a variety of practical issues. Some membrane analysis problems are used to evaluate its performance. The obtained findings show that the present element performs well and accurately.

Keywords – Finite element method, Plane strain, Plane stress, Strain approach, Drilling rotation, Triangular element, Linearanalysis

I. Introduction

It was shown that numerical techniques such as finite element, spectral, finite volume, finite difference, and discrete element method are powerful and effective computational tools for solving real and complex engineering issues. However, due to its strong mathematical basis and intrinsic capabilities, the finite element approach is gaining popularity, leading to the rising usage of this technique in a variety of applications.

Turner et al. [1] use the displacement technique in standard elements to create the linear (constant-strain) triangle and bilinear rectangle, while Taig [2] creates the conventional bilinear quadrilateral. Both have been widely used in two-dimensional structures as plane-stress, plane-strain. However, the computational experience quickly revealed that these elements are too stiff for issues where linear strain gradients dominate the response. Furthermore, when the rate of aspect worsens, over-rigidity increases fast. Mesh distortion and the bending problem are two instances of such phenomena.

Much time and effort have gone into enhancing them or developing new simple elements. Other solutions, such as hybrid stress elements [3-5] assumed strain or improved assumed strain elements [6-8], quasi-conforming elements [9-10] generalized conforming elements [6], [11], [12], have been shown to have particular benefits over traditional finite elements.

The fundamental motivation for scientific study in solid mechanics is the creation of efficient and simple finite elements for structural analysis. The strain-based technique was used to create a class of components. This method generates a displacement field that is enhanced with higher-order terms. This approach yields elements with no shear locking or parasitic shear. In this method, strain states are classified as rigid body movements, continuous strain, and higher-order strain states.

II. FORMULATION OF THE DEVELOPED ELEMENT

The SBTH (Strain Based Three Node Element) is a suggested element that is a triangle with three degrees of freedom at each node, corresponding to two translations (u, v) and an extra in-plane rotation degree of freedom (Figure 1).

In the Cartesian coordinate system, the strain–displacement relations of an element for plan elasticity may be represented as:

$$\begin{cases} \varepsilon_x = \dfrac{du}{dx} \\ \varepsilon_y = \dfrac{dv}{dx} \\ \gamma_{xy} = \dfrac{du}{dy} + \dfrac{dv}{dx} \end{cases} \quad (1)$$

where u and v are displacements in the x and y axes, respectively;($\varepsilon_x, \varepsilon_y$) are the normal strains and γ_{xy} is the shear strain.

Equation (1)'s strain components must fulfil the current compatibility equation:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \cdot \partial y} = 0 \quad (2)$$

Setting the three deformations in equation (1) to zero, followed by integration, produces the rigid body modes displacement field, which is as follows:

$$\begin{cases} u = a_1 - a_3 y \\ v = a_2 + a_3 x \\ \theta = a_3 \end{cases} \quad (3)$$

for the element's drilling degree of freedom, the following equation is used:

$$\theta = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right) \quad (4)$$

Figure 1 shows the geometry of the proposed element ‘‘SBTH’’ (Strain Based three-node element) and the corresponding nodal displacements

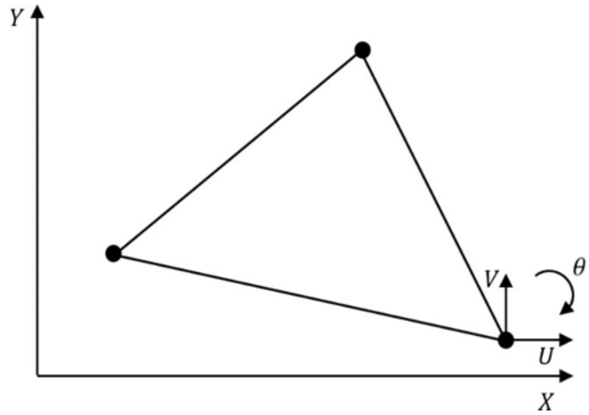

Figure 1: Strain Based Three node element

The (SBTH) element has nine degrees of freedom, and his strain field is expressed as follows:

$$\varepsilon = [Q]\, a \quad (5)$$

were $\lceil \lfloor Q \rceil \rfloor$ presents the matrix relating the strain fields to the unknown constants

We get the displacement field by integrating the strain field:

$$u = [T]\ a \quad (6)$$

The stiffness matrix $[K_e]$ is given by:

$$[K_e] = (\int\int [B]^T [D][B] dxdy) \quad (7)$$

where:

$$[B] = [Q][C]^{-1} \quad (8)$$

$$[K_e] = [C]^{-T} (\int\int [Q]^T \cdot [D] \cdot [Q] dxdy) [C]^{-1} = [C]^{-T} [K_0][C]^{-1}$$
(15)

By using numerical integration:

$$[K_0] = \int_{-1}^{1}\int_{-1}^{1} [Q]^T [D][Q] \det|J| \, d\xi d\eta \quad (9)$$

Where [Q], [J], and [D] is the strain, the Jacobean, and the elasticity matrices, respectively, and [C] is the matrix that relates nodal displacements to the degrees of freedom.

III. NUMERICAL VALIDATION

Several tests are selected to assess the quality of the element using various analyses. A comparison study is carried out between the current element and the following elements:

Q4	**Standard four-node quadrilateral element.**
CST	Standard eight-node quadrilateral element.
LST	Quadrilateral element with six nodes.

MCNEAL's beam

The susceptibility of the proposed element to mesh distortion is tested using the McNeal beam depicted in Figure 2

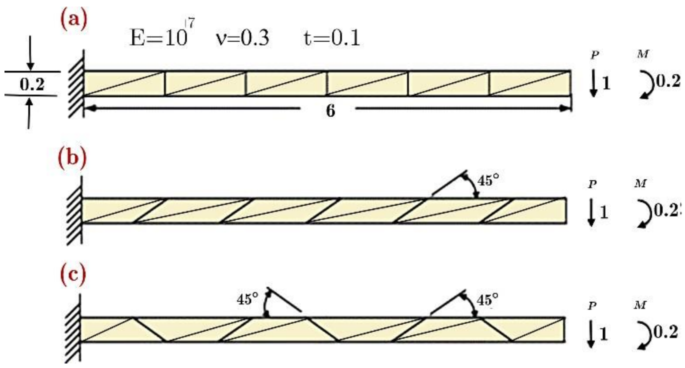

Figure 2. McNeal's cantilever beam (a) rectangular (b) trapezoidal (c) parallelogram

Three different meshes are used: rectangular, parallelogram, and trapezoidal. The suggested MacNeal and Harder [13] test is widely accepted as a standard for determining mesh distortion sensitivity. There are two types of loads considered: pure bending and transverse linear bending. Figure 2 depicts the necessary mechanical and geometrical data. Table 1 compares the results of the proposed element to those of the other elements.

Table 1. Normalized deflection at the tip of theMcNeal's beam

ELEMENT	Load P			Load M		
	Mesh Type			Mesh Type		
	Rectangular(a)	Parallelogram (b)	Trapezoidal (c)	Rectangular (a)	Parallelogram(b)	Trapezoidal (c)
Q4	0.093	0.035	0.003	0.093	0.031	0.022
CST	0.783	0.714	0.709	0.812	0.771	0.724
LST	0.983	0.970	0.961	0.993	0.994	0.992
SBTH	**0.937**	**0.798**	**0.816**	**0.944**	**0.963**	**0.944**
Reference solutions[13]	**- 0.1081**			**- 0.0054**		

SBTH element has a neglected sensitivity in all mesh types, and better accuracy is seen in situations (b) and (c) in comparison to the other element. However, the transverse shears locking caused by the over-rigidity of the Q4 and CST elements has an effect on their results.

Beam in-plane bending

To evaluate our element in the console beam issue [14], [15] exposed to a uniform vertical stress, as illustrated in Figure 3. Five meshes are used to calculate the vertical displacement at the beam's free end. Figure 3:

As a reference solution, Timoshenko's beam theory was used:

$$V_c^{ref} = \frac{L^3}{3EI} + \frac{6PL_z}{5GA} \tag{10}$$

Table 2 displays the results of the SBTH element for a variety of meshes (M1, M2, M3, M4, and M5). The obtained findings are compared to some membrane element outcomes in the literature

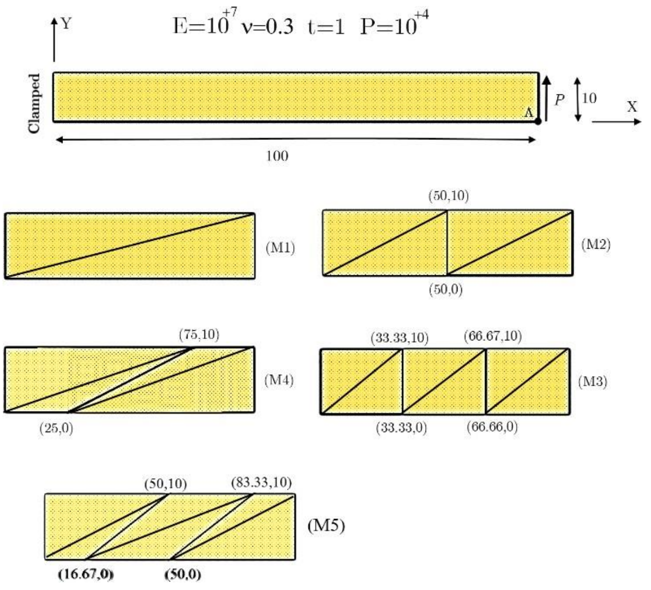

Figure 3. Beam in-plane bending (Data andmeshes)

Table 2. Vertical displacement of a beam in planebending

Mesh Type	Q4[14]	CST	LST	SBTH
M1	0.10	0.05	3.00	**2.87**
M2	0.38	0.13	3.7	**3.61**
M3	0.75	0.25	3.84	**3.74**
M4	0.12	0.06	3.02	**2.92**
M5	0.22	0.10	3.09	**2.98**
$V_{ref} = 4.03$				

SBTH and LST produce more accurate results than the Q4 and CST elements and is less susceptible to deformed meshes than other membrane elements for M4 and M5 meshes, according to the results.

Cook's skew beam

The non-prismatic beam is a common benchmark issue for assessing planar elements. The reference solution is produced using the CPS8 element of ABAQUS with a 64 × 64 mesh due to the absence of an analytical solution. Figure 4 depicts the mechanical parameters, geometrical data, and loading data used in the treated structure; The used meshes are indicated in Figure 5, Tables 3 shows the findings of the vertical deflection at point C.

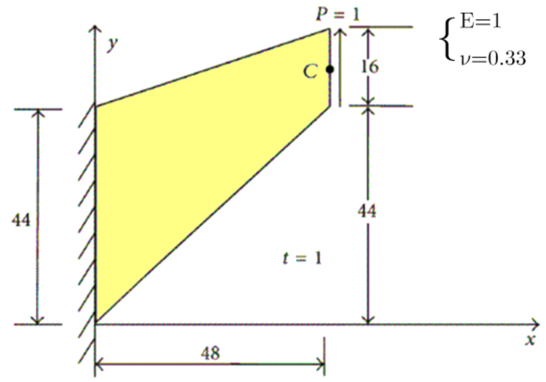

Figure 4. Cook's skew beam

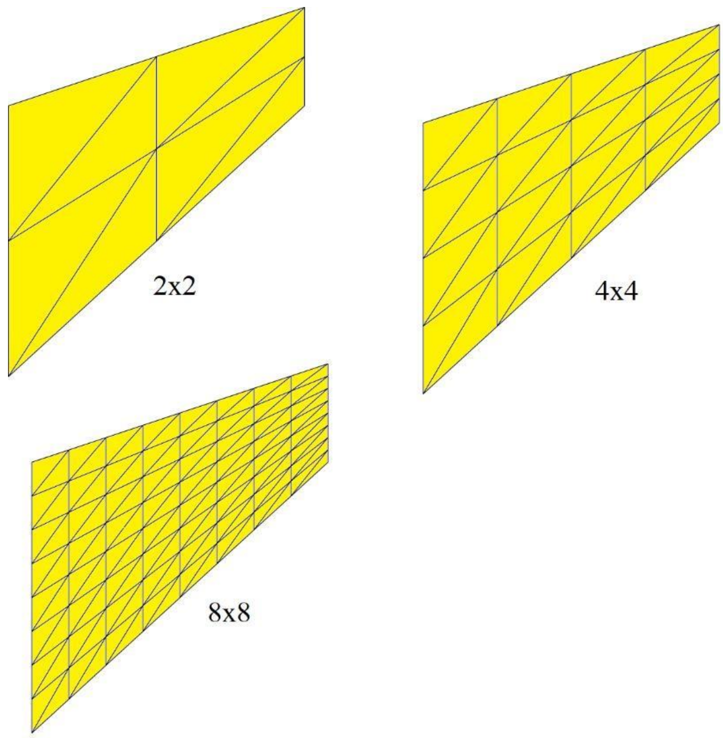

Figure 5 Cook's skew beam Meshes

Table 3. Tip vertical deflection of the Cook's skewbeam

	Vertical displacement at point C		
	Mesh		
Element	2×2	4×4	8×8
Q4	11.80	18.29	22.08
CST	25.65	24.27	24.01
LST	21.05	23.02	23.69
SBTH	**20.83**	**23.54**	**23.65**
Reference solution [16]	**23.9652**		

The SBTH element provides an excellent accord with the reference solution, unlike Q4 and CST elements, even when the mesh is coarse.

IV. CONCLUSION

The assumed strain approach suggests a novel triangular plane element in this work. The rigid body movements, constant strain, and application of compatibility conditions to the assumed strain field ensure and maximize monotonic convergence to the solution.

The formulated element comprises three nodes with nine degrees of freedom. Each node has two translations and in-plane rotation (u,v,θ). The SBTH triangular element performs well in all numerical examples, is immune to mesh distortion, and has a good convergence characteristic. The accuracy of the suggested membrane element was often close to that of the linear strain triangular element LST. Furthermore, when the bending is dominant, the suggested element's numerical results are consistent and offer better results.

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