

Mathematical Model and A Solution Method of the Establishment of Logistics Centres

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Abstract – In this article, I will give an integer non-linear programming model of the first phase of the three-phase method. The objective function of this model has got an indefinite quadratic form. This problem has not got any exact algorithm. But the variables of the model are special, as these are integer variables, and their values are 0 or 1. If we substitute these variables with new special variables, and change some conditions, the new model will be linear integer programming model. The components of the original objective function are rational numbers (these components are cost components), so it can give a new objective function with integer coefficients. The optimum of this new function will correspond with the original objective function. The new model with the new objective function has got an exact solving method this time.

Keywords – Non-Linear Programming, Integer Programming, Linear Programming, Gomory Cut Method, Section Plane, Gradient Method, Cutting Plane

I. INTRODUCTION

In earlier articles I have shown that the problem of establishment has got a mathematical programming model [2,3]. The establishment problem is not a special centre problem. The centre problem has not got general solving method. But, in special case like our problem good approximate method can be given. In earlier articles I gave an effective heuristic algorithm with three phases for this problem. But the question is that the problem has got an exact solving method. To answer this question, I have to examine the third phase only, because of another two phases has got exact solving method [2,3,9].

In this article I show that the centre problem with three phases has got a non-linear condition system with a non-linear objective function [1]. This problem cannot solve generally. But the variables of the problem are specially, especially are integer with value 0 or 1. If we substitute these variables with a new variables, and modify the condition system and the objective function then we will get an linear integer programming model. The components of the original objective function have got rational values, so we make a new objective function with same optimum. Regarding this objective function, the problem has got suitable exact solving method [10].

As follows I give the model and the solving method.

II. MATERIALS AND METHOD

A. The condition system of the problem

The condition system equals the condition system of the model [1,2], so we use this system. In this article I do not detail the meaning of the components, I gave the meaning of the components earlier, and I will not use the meaning of the components.

In sort of the model of the problem is the following:

$$x_{ki}^v \geq 0; x_{ki}^v = int \leftrightarrow (k = 1, \dots, n; i = 1, \dots, p_0; v = 1, \dots, m) \quad (1)$$

$$y_{kj}^\mu \geq 0; y_{kj}^\mu = int (k = 1, \dots, n; j = 1, \dots, r_0; \mu = 1, \dots, w) \quad (2)$$

$$\sum_{k=1}^n x_{ki}^v = 1; (i = 1, \dots, p_0; v = 1, \dots, m) \quad (3)$$

$$\sum_{j=1}^{r_0} y_{kj}^\mu = 1; (k = 1, \dots, n; \mu = 1, \dots, w) \quad (4)$$

$$\sum_{i=1}^{p_0} x_{ki}^v q_{iv} \leq c_{ki}^v; (k = 1, \dots, n; v = 1, \dots, m) \quad (5)$$

$$\sum_{i=1}^{p_0} x_{ki}^v q_{iv} \geq c^v; (k = 1, \dots, n; v = 1, \dots, m) \quad (6)$$

$$\sum_{k=1}^n y_{kj}^\mu \sum_{v=1}^m \sum_{i=1}^{p_0} x_{ki}^v d_{i\mu} \leq b_{j\mu}; (j = 1, \dots, r_0; \mu = 1, \dots, w) \quad (7)$$

$$K_{red}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{p_0} \sum_{k=1}^n \sum_{v=1}^m x_{ki}^v \sum_{j=1}^{r_0} \sum_{\mu=1}^w k_{kj\mu\epsilon}^{AS} (\sum_{t=1}^m q_{it} a_{t\mu}) l'_{kj} y_{kj}^\mu + \sum_{i=1}^{p_0} \sum_{v=1}^m \sum_{k=1}^n (k_{kvi}^{BS} c_{ki}^v l'_{ki} + k_{kv}^M c_{ki}^v) x_{ki}^v \rightarrow min \quad (8)$$

Denote

$$\mathbf{x} = [x_{ki}^v], \mathbf{y} = [y_{kj}^\mu]. \quad (9)$$

This minimization problem is a non-linear programming problem.

B. Converting the problem to a linear integer problem

Solving this problem, we have to make a linear condition system with a linear objective function. Substitute the components y with components y'. Components y are used in three conditions [(2),(4),(7)] and objective function (8). We have to convert these elements. Denote

$$y_{lijk}^{v\mu} = x_{li}^v y_{kj}^\mu; (k = 1, \dots, n; i = 1, \dots, p_0; j = 1, \dots, r_0; l = 1, \dots, n; \mu = 1, \dots, w; v = 1, \dots, m), \mathbf{y}' = [y_{lijk}^{v\mu}] \quad (10)$$

1. CONVERTING THE CONDITION (2)

Regard the condition (2). This will change in the new condition system. The new condition is

$$y_{lijk}^{v\mu} \geq 0; y_{lijk}^{v\mu} = int \quad (11)$$

because the components x and y are non-negative. These variables are integer (with value 0 or 1), because product of two integer variable is integer (with value 0 or 1).

2. CONVERTING THE CONDITION (4)

Using (10) and (3)

$$\sum_{l=1}^n y_{lijk}^{v\mu} = \sum_{l=1}^n x_{li}^v y_{kj}^\mu = y_{kj}^\mu \cdot \sum_{l=1}^n x_{li}^v = y_{kj}^\mu \quad (12)$$

we get the expression

$$\sum_{l=1}^n y_{lijk}^{v\mu} = y_{kj}^\mu; i \in \{1, \dots, p_0\}. \quad (13)$$

This is the converting expression. Using the condition (2) (4), we get the equation

$$\sum_{j=1}^{r_0} \sum_{l=1}^n y_{lijk}^{v\mu} = 1 \quad (14)$$

Regarding the condition (2) we get the expression

$$\sum_{j=1}^{r_0} y_{lijk}^{v\mu} = \sum_{j=1}^{r_0} x_{li}^v y_{kj}^\mu = x_{li}^v \sum_{j=1}^{r_0} y_{kj}^\mu = x_{li}^v \quad (15)$$

Converting (16), the new condition (4) is

$$-x_{li}^v + \sum_{j=1}^{r_0} y_{lijk}^{v\mu} = 0 \quad (16)$$

3.3. CONVERTING THE CONDITION (7)

Left side of the condition (7) is non-linear:

$$\sum_{k=1}^n y_{kj}^\mu \sum_{v=1}^m \sum_{i=1}^{p_0} x_{ki}^v d_{i\mu} \leq b_{j\mu} \quad (17)$$

Converting this expression for this way:

$$\sum_{k=1}^n \sum_{v=1}^m \sum_{i=1}^{p_0} x_{ki}^v y_{kj}^\mu d_{i\mu} \leq b_{j\mu} \quad (18)$$

Using the expression (10), get

$$\sum_{k=1}^n \sum_{v=1}^m \sum_{i=1}^{p_0} y_{lijk}^{v\mu} d_{i\mu} \leq b_{j\mu} \quad (19)$$

The left side of the new condition depends on $y_{lijk}^{v\mu}$ linearly. All conditions are linearly now.

4. THE OBJECTIVE FUNCTION

Regard the objective function (8) (this is the reduced cost function). Sort this function to the following form:

$$K_{red}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{p_0} \sum_{k=1}^n \sum_{v=1}^m \sum_{j=1}^{r_0} \sum_{\mu=1}^w k_{kj\mu\epsilon}^{AS} (\sum_{t=1}^m q_{it} a_{t\mu}) l'_{kj} x_{ki}^v y_{kj}^\mu + \sum_{i=1}^{p_0} \sum_{v=1}^m \sum_{k=1}^n (k_{kvi}^{BS} c_{ki}^v l'_{ki} + k_{kv}^M c_{ki}^v) x_{ki}^v. \quad (20)$$

Apply the expression (10):

$$\tilde{K}_{red}(\mathbf{x}, \mathbf{y}') = \sum_{i=1}^{p_0} \sum_{k=1}^n \sum_{v=1}^m \sum_{j=1}^{r_0} \sum_{\mu=1}^w k_{kj\mu\epsilon}^{AS} (\sum_{t=1}^m q_{it} a_{t\mu}) l'_{kj} y_{lijk}^{v\mu} + \sum_{i=1}^{p_0} \sum_{v=1}^m \sum_{k=1}^n (k_{kvi}^{BS} c_{ki}^v l'_{ki} + k_{kv}^M c_{ki}^v) x_{ki}^v. \quad (21)$$

This function is an objective function of the linear integer programming problem. The completely new model is the following:

$$\begin{aligned}
 &x_{ki}^v \geq 0; x_{ki}^v = int \Leftrightarrow (k = 1, \dots, n; i = 1, \dots, p_0; v = 1, \dots, m) \\
 &\quad y'_{lijk}{}^{v\mu} \geq 0; y'_{lijk}{}^{v\mu} = int \\
 &(l = 1, \dots, n; k = 1, \dots, n; i = 1, \dots, p_0; j = 1, \dots, r_0; \mu = 1, \dots, w; \\
 &\quad v = 1, \dots, m) \\
 &\quad \sum_{k=1}^n x_{ki}^v = 1; (i = 1, \dots, p_0; v = 1, \dots, m) \\
 &\quad \sum_{j=1}^{r_0} \sum_{l=1}^n y'_{lijk}{}^{v\mu} = 1; (i = 1, \dots, p_0; k = 1, \dots, n; v = 1, \dots, m; \mu \\
 &\quad = 1, \dots, w) \\
 &\quad -x_{li}^v + \sum_{j=1}^{r_0} y'_{lijk}{}^{v\mu} = 0; (l = 1, \dots, n; k = 1, \dots, n; i \\
 &\quad = 1, \dots, p_0; \mu = 1, \dots, w; v = 1, \dots, m) \\
 &\quad \sum_{i=1}^{p_0} x_{ki}^v q_{iv} \leq c_k^v; (k = 1, \dots, n; v = 1, \dots, m) \\
 &\quad \sum_{i=1}^{p_0} x_{ki}^v q_{iv} \geq c^v; (k = 1, \dots, n; v = 1, \dots, m) \\
 &\quad \sum_{k=1}^n \sum_{v=1}^m \sum_{i=1}^{p_0} y'_{lijk}{}^{v\mu} d_{i\mu} \leq b_{j\mu}; \\
 &\quad (j = 1, \dots, r_0; \mu = 1, \dots, w). \\
 &\quad \hat{K}_{red}(\mathbf{x}, \mathbf{y}') = \\
 &\quad \sum_{i=1}^{p_0} \sum_{k=1}^n \sum_{v=1}^m \sum_{j=1}^{r_0} \sum_{\mu=1}^w k_{kj\mu\epsilon}^{AS} (\sum_{t=1}^m q_{it} a_{t\mu}) l'_{kj} y'_{lijk}{}^{v\mu} + \\
 &\quad \sum_{i=1}^{p_0} \sum_{v=1}^m \sum_{k=1}^n (k_{kvi}^{BS} c_{ki}^v l'_{ki} + k_{kv}^M c_{ki}^v) x_{ki}^v \rightarrow min
 \end{aligned} \tag{22}$$

C. Theorem

The problem (22) has got an optimum if, an only if the original problem has got optimum. Optimum of the original problem can give from the optimum of the problem (22) The optimal value of the objective function of the problem (22) is equal to the optimal value of the objective function of the original problem [16-18].

D. Proof

Denote L_N set of possible results of the original problem and L_E set of possible results of the problem (22). Choose a $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \in L_N$. Convert this possible result to $\begin{bmatrix} \mathbf{x} \\ \mathbf{y}' \end{bmatrix}$ using (10). According to (11)-(19), this vector satisfies the condition system of problem (22), so $\begin{bmatrix} \mathbf{x} \\ \mathbf{y}' \end{bmatrix} \in L_E$.

It will be true if the sets are exchanged. Now choose $\begin{bmatrix} \mathbf{x} \\ \mathbf{y}' \end{bmatrix} \in L_E$. Use the converting expression.

The given vector $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$ satisfies the conditions (1).

Non-negative and integer \mathbf{x} and \mathbf{y}' satisfy the condition (2). The truth of the condition (3) follows from (13,16). The condition (7) is transformed from (19,18,17) with equivalent steps.

Suppose that the vector $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{y}_0 \end{bmatrix} \in L_N$ is an optimal solution of original problem. Then

$$K_{red}(\mathbf{x}_0, \mathbf{y}_0) \leq K_{red}(\mathbf{x}, \mathbf{y}), \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \in L_N \tag{23}$$

Make a vector $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{y}'_0 \end{bmatrix} \in L_E$ according to (10).

$$K_{red}(\mathbf{x}_0, \mathbf{y}_0) = \hat{K}_{red}(\mathbf{x}_0, \mathbf{y}'_0). \tag{24}$$

Every vector $\begin{bmatrix} \mathbf{x} \\ \mathbf{y}' \end{bmatrix} \in L_E$ has got a vector $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \in L_N, i \in \{1, \dots, p_0\}$.

This satisfies this expression

$$K_{red}(\mathbf{x}, \mathbf{y}) = \hat{K}_{red}(\mathbf{x}, \mathbf{y}') \tag{25}$$

Then

$$\hat{K}_{red}(\mathbf{x}_0, \mathbf{y}'_0) = K_{red}(\mathbf{x}_0, \mathbf{y}_0) \leq K_{red}(\mathbf{x}, \mathbf{y}) = \hat{K}_{red}(\mathbf{x}, \mathbf{y}') \tag{26}$$

It follows from the foregoing that the vector $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{y}'_0 \end{bmatrix} \in L_E$ is optimal solution of (22). This statement is true reverse. Suppose that the vector $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{y}'_0 \end{bmatrix} \in L_E$ is optimal solution of (22), and then according to previous sequence of ideas, the vector $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{y}_0 \end{bmatrix} \in L_N$ is an optimal solution of original problem. //

Follows from this theorem, that the value of the variables x_{ki}^v is equal the value of the origin variables \mathbf{x} , the value of the origin variables y_{kj}^μ can be determined by converting expression (13). It follows from this that we must solve only problem (22).

The components of the objective function are costs components. The cost components are rational numbers. Multiple the components of the objective function by a suitable constant. Let this constant be the lowest common multiple of the denominators of the cost components:

$$K'_{red} = k \cdot \hat{K}_{red} \tag{27}$$

where k the lowest common multiple of the denominators of the cost components.

Then all new components of the objective function are integer numbers. Denote

$$\tilde{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y}' \end{bmatrix} \tag{28}$$

$$\tilde{c}^* = [\tilde{c}_1^*, \tilde{c}_2^*] \tag{29}$$

where

$$\tilde{c}_1^* = k \cdot [(\sum_{v=1}^m \sum_{k=1}^n (k_{kvi}^{BS} c_{ki}^v l'_{ki} + k_{kv}^M c_{ki}^v) x_{ki}^v)], \tag{30}$$

$$\tilde{c}_2^* = k \cdot [k_{kj\mu\epsilon}^{AS} (\sum_{t=1}^m q_{it} a_{t\mu}) l'_{kj}]. \tag{31}$$

The model (22) can be transforming the next form [10]:

$$\begin{aligned} \tilde{x} &\in \{0; 1\} \\ \tilde{A}\tilde{x} &\leq \tilde{b} \quad \tilde{c}_i = \text{int} \end{aligned} \tag{32}$$

$$\tilde{f}(\tilde{x}) = \tilde{c}^* \tilde{x} \rightarrow \max$$

The objective function is limited on the set of the possible results. Proof of this fact is very simplified. The values of the components \tilde{x} are 0 or 1, the values of the components \tilde{c} are integer numbers. It follows

$$\tilde{f}(\tilde{x}) = \tilde{c}^* \tilde{x} \leq \tilde{c}^* \mathbf{1} = K \tag{33}$$

E. The solving algorithm

This problem already has got an exact solving method. One of these methods is g-cut method for integer programming problems. The other method is the Gradient Method. This method is based on method of the cutting planes [11]. I describe this method in more detailed.

STEP 1.

Regard the continuous problem

$$\begin{aligned} \mathbf{0} &\leq \tilde{x} \leq \mathbf{1} \\ \tilde{A}\tilde{x} &\leq \tilde{b} \end{aligned} \tag{34}$$

$$\tilde{g}(\tilde{x}) = \tilde{x}^* (\tilde{x} - \mathbf{1}) \rightarrow \max$$

and solve this problem. This problem has got exact solving method, because

$$\tilde{g}(\tilde{x}) = \tilde{x}^* (\tilde{x} - \mathbf{1}) = \tilde{x}^* \tilde{x} - \tilde{x}^* \mathbf{1} = \tilde{x}^* E \tilde{x} - \tilde{x}^* \mathbf{1} \tag{35}$$

Matrix of the quadratic form is positive definite (identity matrix E), then in this case the problem can be solved [10].

Try to find the local minimum of the problem with the Gradient Method, after then restrict the set with the cutting plane method. Do this step until the set L will be empty. Choose the maximum of the local minimums. It is the optimum. Function \tilde{g} is limited on the set of possible results (\tilde{L}), because

$$\tilde{g}(\tilde{x}) \leq 0, \tilde{x} \in \tilde{L} \tag{36}$$

In this case if the problem has got possible result, then objective function \tilde{g} (is limited on \tilde{L}) has got optimal result.

STEP 2.

It can be two cases:

- a) $\tilde{g}(\tilde{x}_0) = 0,$
- b) $\tilde{g}(\tilde{x}_0) < 0.$

In case b) the integer problem has not got any possible results. In case a) it has got any possible result. In this case add the condition

$$\tilde{f}(\tilde{x}) \geq \tilde{f}(\tilde{x}_0) + 1 \tag{37}$$

to the problem, and solve the problem below again with objective function \tilde{g} :

$$\begin{aligned} \mathbf{0} &\leq \tilde{x} \leq \mathbf{1} \\ \tilde{A}\tilde{x} &\leq \tilde{b} \\ \tilde{f}(\tilde{x}) &\geq \tilde{f}(\tilde{x}_0) + 1 \end{aligned} \tag{38}$$

$$\tilde{g}(\tilde{x}) = \tilde{x}^* (\tilde{x} - \mathbf{1}) \rightarrow \max$$

STEP 3.

Do this procedure from step 4.2. until will be case b). It is bound to happen, because function \tilde{f} is limited on the set of possible results of condition system (32). If it happens in iterating step i , then the optimal result will be \tilde{x}_{i-1} of the previous step.

III. CONCLUSION AND DISCUSSION

In this article I show the model and solving method of the inside phase of establishment of the postponed assembly plants of the multinational enterprise using reduced cost. I converted the origin non-linear problem to a linear integer programming problem. I used solving method [11] for this problem. This method gives the optimal result if it exists. This method gives the optimum, but this method is not effective furthermore the size of the problem changes form

$$(p_0 \cdot m + r_0 \cdot w + 2n \cdot m + r_0 \cdot w) \times [n \cdot (p_0 \cdot m + r_0 \cdot w)] \tag{39}$$

to

$$(p_0 \cdot m + p_0 \cdot n + m \cdot w + p_0 \cdot n^2 \cdot w \cdot m + 2n \cdot m + r_0 \cdot w) \times [n \cdot p_0 \cdot m \cdot (1 + r_0 \cdot w)] \tag{40}$$

Now I don't know count of the steps of the problem, but the algorithm is finite [2,11]. From the early analyzing it seems the method is very complicated ineffective. Earlier [3] I gave a new effective heuristic algorithm for this problem. Based on the above and [2] I can tell it is suitable the heuristic algorithm for the solving of the establishment problem, where the maximum order of the efficient matrix is only

$$\max(n \cdot p_0; n \cdot r_0; w \cdot m) \tag{41}$$

But very important fact, the establishment problem has got exact solving method.

REFERENCES

- [1] M. S. Granovetter. (1965) Getting a job: A study of contracts and careers. Chicago: University of Chicago Press.
- [2] R. L. Kahn and E. Boulding, (Eds.). (1964) Power and conflict in organizations. Glencoe, IL: Free Press.
- [3] I. Nonaka, (1991) The knowledge-creating company. Harvard Business Review, 69(6): 96–104.
- [4] B. Sevitt and J. G. March. (1988) Organizational learning. In W. R. Scott & J. F. Short (Eds.), Annual review of sociology, vol. 14: 319–340. Palo Alto, CA: Annual Reviews.
- [5] M. H. Smith. (1980) A multidimensional approach to individual differences in empathy. Unpublished doctoral dissertation, University of Texas, Austin.
- [6] M. Gubán and J. Cselényi. (2004) Quadratic linear-programming model to establish delayed assembling plants oriented by logistics, Logistics Networks. Models, Methods and Applications (Ed. T. Bányai, J. Cselényi) University of Miskolc, ISBN 963 661 641 8 pp 279-288
- [7] M. Gubán. (2004) Késleltetett (kihelyezett) összeszerelő üzemek logisztika orientált telepítésére szolgáló matematikai modellek és módszerek fejlesztése globalizált termelés esetén PhD. értekezés. Miskolci Egyetem,
- [8] M.Gubán, J. Cselényi and D.cVadász. (2003) Comparing method of mathematical programming and heuristic method to establish delayed assembly plants oriented by logistics and examination of these methods. 4th Workshop on European Scientific and Industrial Collaboration May, T. Tóth, P. Bikfalvi, J. Göndri Nagy (Ed.) Published by Institute of Information Science University of Miskolc, ISBN 963 661 570 5 Miskolc Vol. II pp. 587-594,
- [9] M. Gubán and J.cCselényi. (2004) The method and analysis of establishment of logistic-oriented postponement assembly plants, Chapter 25, DAAAM International Scientific Book, 2004, Wien, B. Katalinic (Ed.), Published by DAAAM International, ISBN 3 901509 38 0, ISSN 1726 9687, Vienna, Austria pp 255-264
- [10] M. Gubán, J. Cselényi and L. Kovács (2001) Methodes for establish of delayed assembling plants oriented by logistics MISKOLCER GESPRÄCHE 2001 Seminarband. 13-14. September, Published by University of Miskolc ISBN 963 661493 8, Miskolc pp.77-83
- [11] M. Gubán. (2000) Késleltetett összeszerelő üzemek logisztikaorientált optimális telepítésére szolgáló matematikai modellek. Magyar Tudomány napja. Doktoranduszok fóruma. Miskolci Egyetem, Gépészmérnöki kar szekciókiadványa október 30. pp 19-24
- [12] M. Gubán. (2001) Heuristic algorithm to establish delayed assembling plants oriented by logistics. 3rd International Conference of PhD Students. University of Miskolc. 13-19 August ISBN 963 661 480 6 pp 71-76
- [13] M. Gubán and J. Cselényi. (2001) Mathematical model and heuristic algorithm to establish delayed assembling plants oriented by logistics. Miskolczer Gesprache
- [14] L. Csernyák. (Ed.) (1990) Operációkutatás I.-II., Nemzeti Tankönyvkiadó.
- [15] B. Krekó. (1972) Optimumszámítás (Nemlineáris programozás), Közgazdasági és Jogi Könyvkiadó, Budapest, KG-1802-k-7275 pp 411-415
- [16] M. Gubán and J. Udvaros. (2022) A Path Planning Model with a Genetic Algorithm for Stock Inventory Using a Swarm of Drones. Drones. 6(11):364. <https://doi.org/10.3390/drones6110364>
- [17] L. Radácsi, M. Gubán, L. Szabó and J. Udvaros. (2022) A Path Planning Model for Stock Inventory Using a Drone. Mathematics. 10(16):2899. <https://doi.org/10.3390/math10162899>
- [18] O. Takáč, D. Hrubý and V. Cviklovič. (2011) Possibilities of navigation of mobile agricultural robots on the principle of the geometrical objects detections. 1, Bucharest: UNIV Agricultural Sciences & Veterinary Medicine Bucharest, p. 206-208. ISSN: 2284-7995.