

Random number-based Brownian motion and practical examples of its implementing in high school computer science lessons

Dávid Paksi*, Norbert Annuš², Iveta Štempeľová³ and Daniel Dancaš⁴

¹Department of Informatics, J. Selye University, Slovakia

²Department of Informatics, J. Selye University, Slovakia

³University of Veterinary Medicine and Pharmacy in Košice, Slovakia; Masaryk University, Czech Republic

⁴Department of Biology, J. Selye University, Slovakia

*(paksid@ujv.sk) Email of the corresponding author

(Received: 13 May 2023, Accepted: 30 May 2023)

(DOI: 10.59287/ijanser.792)

(1st International Conference on Contemporary Academic Research ICCAR 2023, May 17-19, 2023)

ATIF/REFERENCE: Paksi, D., Annuš, N., Štempeľová, I. & Dancaš, D. (2023). Random number-based Brownian motion and practical examples of its implementing in high school computer science lessons. *International Journal of Advanced Natural Sciences and Engineering Researches*, 7(4), 463-467.

Abstract – Brownian motion is defined in many literature sources as the random motion of microscopic particles in a liquid or gaseous medium. The cause of this motion has long been unknown. The explanation of Brownian motion is that molecules in solution are constantly colliding due to thermal motion and these are random. Over time, this motion has also become the basis for many problems and practical exercises in the natural sciences and has become popular in computer science. Brownian motion simulation and its variations have many positive features. They not only allow the understanding of natural phenomena but especially allow the understanding of random processes. In the classroom, we can practice generating random numbers (e.g. using our own random number generator and testing it), creating various commonly occurring phenomena with a random character (the shape of lightning, a drop running down glass, the formation of copper crystals or a snowflake), or even linking them to statistics (statistical evaluation of the results of work). The tasks presented in our paper are designed for two-dimensional simulations. However, the same procedures are also well applicable to three-dimensional simulations. This can be an assignment for students, for example, for a midterm project. In this way, we can also use Brownian motion for project-based learning and contribute to collaborative teamwork.

Keywords –Computer Science, Education, Brownian Motion, Simulation, Random Numbers

I. INTRODUCTION

Nowadays, there is a great emphasis on practical computer science in education as well. The current state of development of computer science as a discipline is finding its place in more and more areas. [9] This boom also brings with it many complications that must also be taken into account in the school environment - for example, children's

rights and child protection. [20] Many of the modern practices used in practice are also finding their way into the teaching process. This is made possible, for example, by the affordability of both hardware and software products. Such include, for example, the various UAV implementations with their associated cultural heritage protection which is very modern. [2][4][5][6][7][8][9][10] The use of UAVs can

result in, for example, a 3D model. These implementations are nicely described in several publications, e.g.: [12][14] With emphasis also on the economic aspect, which has also gained in popularity in recent years, especially after the pandemic period. However, [22] in these implementations, basic simulations are often forgotten, while they can also help to foster cross-curricular relationships, e.g. with mathematics or biology. Brownian motion is one such example, which can also be implemented in a playful way in many educational domains and innovative techniques. [6][11][13][14][15][16][17][21] Examples include random UAV flight planning, random processes in microworlds, etc. [3][7] But for this we need to understand the basic principles of using random variables. And this is the area we want to address in our publication. We want to point out the support of both algorithmic thinking but also a deeper understanding of physical processes using simulations. But for this, a sufficient knowledge of programming is necessary, and therefore our work builds already on the basic knowledge of programming. [20][23] In the simulation of random processes, the wandering task is often used which we will also use. Random wandering was first investigated by the Scottish botanist Robert Brown in 1827 using experimental tools. It was one of the first random processes to be simulated on a computer. Brown observed that the smaller the particles and the warmer the liquid, the more intense the motion. [1] And we will try to simulate this movement.

II. MATERIAL AND METHODS

Assume that the random wandering of the particle occurs along a line. The particle takes equal steps of length l with probability p to the right and with probability $1-p$ to the left, i.e:

$$\Delta x = \begin{cases} +l: & p \text{ with probabilities} \\ -l: & 1 - p \text{ with probabilities} \end{cases}$$

Let x_t denote the position of the particle after step t . In this case:

$$x_{t+1} = x_t + \Delta x$$

It is therefore valid:

$$x_t = \sum_{i=1}^t \Delta x$$

Using the x_t function, we can now graphically visualize the wander, as shown in the following figure, where t is on the x -axis and x_t on the y -axis.

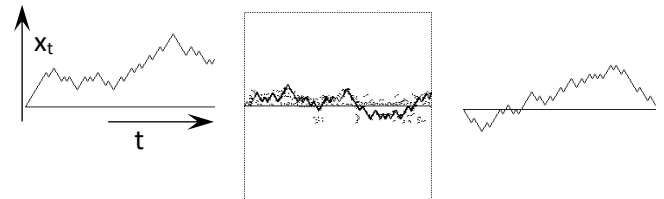


Fig. 1 Some simulations of particle motion

III. RESULTS

The basic simulation mentioned above is well known. However, it is not suitable for practical demonstrations and simulations in the classroom. Therefore, we will adapt this basic principle to a two-dimensional simulation - to make it clearer. And this is the first task for students to create a program that represents the Brownian motion principle. Students should understand that in a two-dimensional presentation, they will have to generate four random numbers. A common question from students is that what good is this? We need to explain to them that it simulates the uniform and disordered motion of microscopic particles in a gas or liquid. Wandering was one of the first processes to be simulated by computer. Let's assume that this wandering takes place on a square grid. We can do this by making each node of the square grid a pixel. Furthermore, we can simplify the task by moving in only four directions in the pixel space, i.e. left, right, up and down - with probabilities p_1, p_2, p_3 and p_4 . Of course, $\sum p_i = 1$. The displacement travelled must also be equal to the size of the pixel, so that wandering from one pixel to another can only occur towards one of the pixels around it. Generate a random number $\xi \in (0, 1)$ with uniform distribution. Furthermore:

- if $\xi \leq p_1$ - step left,
- if $p_1 < \xi \leq p_2$ - step right,
- if $p_2 < \xi \leq p_3$ - step up,
- if $p_3 < \xi \leq p_4$ - step down.

Fig. 2 Simulation of Brownian motion on the grid with different step distances

You can also try to simulate Brownian motion in three dimensions with your students. This could be a joint project or homework. In this case, it will be useful for us to present the time steps of the simulation with colour changes.

Another task could be to simulate the movement of water droplets on window glass. The water droplets move down the glass due to gravity, but the surface of the glass also causes them to move left and right. The movement is not isotropic. We have 4 directions: up, down, right and left, with probabilities $p_1 = 0.6$, $p_2 = 0.1$, $p_3 = 0.15$ and $p_4 = 0.15$. Of course, $\sum p_i = 1$. We can apply the simplifications above to this simulation. The result of such an experiment is illustrated in the figure. In further tests, we can try to vary the probabilities and observe how the trajectories of the water droplets change. In the same way, we could observe a small probability at each step, e.g. ≈ 0.03 . If this were to happen, two water droplets would be created from one water droplet and both would continue on their paths. In this way, we could even simulate lightning.

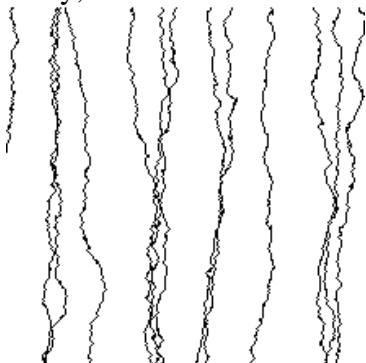


Fig. 3 Simulation of water droplets [1]

The third task is Diffusion-limited aggregation. This is a more complex problem, which not all students may be able to solve easily. It is therefore very important for the teacher to explain the exercise to the students. Fluffing and particle diffusion are the basis of many growth processes. One of the most beautiful examples is the formation of snowflakes or crystals. The individual steps could be described as follows:

- Place a nucleus in the centre of the square lattice.
- The particles wander around the grid, and when they approach the nucleus to the point where they are in the vicinity of the nucleus,



they stop wandering and at that position they land on the nucleus - they stay there.

- It is important that the particles arrive isotropically - from a great distance and from all directions.
- There should only be one particle wandering on the grid at a time - the particles should not be able to influence each other.
- It is easy for particles to move away from the nucleus rather than towards it. Such cases would reduce the optimality of the simulation, so if the particle moves away from the nucleus, at a certain distance it is ignored, no longer counted and a new particle is launched. If an *Image* type component is used for the simulation, the boundary is the edges of the Image.

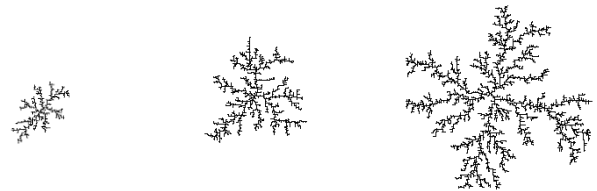


Fig. 4 The shape obtained using the DLA algorithm after 40 000 000, 55 000 000 000 and 95 000 000 particles are launched [1]

Remember that as the particles adhere, the shape will also increase. We should expect this and launch the particles further and further away. Shapes created using diffusion-limited aggregation are some of the most beautiful. They have many variants and are used for statistical calculations.

The last exercise that you can do with your students may involve different modifications of the previous exercise. Two examples of these are shown in the following figure.

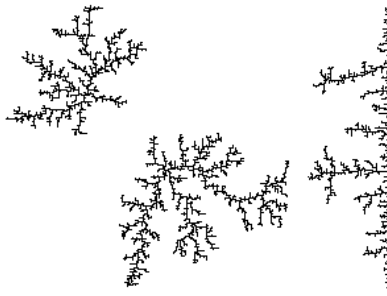


Fig. 5 The two snowflakes created using the DLA algorithm (two seeds were placed at the beginning of the growth process) and the simulation of the growth process on a straight line [1]

IV. CONCLUSION

We have deliberately avoided showing program codes because we think that these are easy to compile from the mathematical equations we have presented. Nevertheless, we believe that by implementing the techniques mentioned above, we will contribute to the popularization of Brownian motion and to the understanding of its contribution in the natural sciences. As mentioned in the introduction of this paper, we are already building on some knowledge of programming. Thus, we do not recommend implementing these tasks in courses on programming fundamentals, but in classes that have already covered these fundamentals.

REFERENCES

- [1] Takáč, O. MODELLEZÉS ÉS SZIMULÁCIÓ, Komárno: J. Selye University, 2017.
- [2] Takáč O, Végh L. USAGE OF UAVS IN THE PROTECTION OF CULTURAL HERITAGE IN THE TEACHING OF COMPUTER SCIENCE. In: INTED2021 Proceedings. 15th International Technology, Education and Development Conference. IATED; 8-9 March, 2021 2021:9987-9992. doi:10.21125/inted.2021.2084
- [3] Czakóová, K. MICROWORLD ENVIRONMENT OF SMALL LANGUAGE AS „LIVING LABORATORY” FOR DEVELOPING EDUCATIONAL GAMES AND APPLICATIONS. In. Proceedings of the 13th International Scientific Conference „eLearning and Software for Education“ : Could technology support learning efficiency? Volume 1, DOI: 10.12753/2066-026X-17-042, 2017/1, p. 286-291. Bucharest : “CAROL I” National Defence University Publishing House, 2017. ISSN 2066-026X ISSN-L, 2066-026X, ISSN CD 2343 – 7669.
- [4] Udvaros J, Takáč O. DEVELOPING COMPUTATIONAL THINKING BY MICROCONTROLLERS. In: ICERI2020 Proceedings. 13th annual International Conference of Education, Research and Innovation. IATED; 9-10 November, 2020 2020:6877-6882. doi:10.21125/iceri.2020.1474
- [5] Czakóová, K. GAME-BASED PROGRAMMING IN PRIMARY SCHOOL INFORMATICS. In. INTED 2021 : Proceedings of the 15th International Technology, Education and Development Conference. DOI: 10.21125/inted.2021.1134 , p. 5627-5632, Valencia : IATED Academy, 2021. ISBN 978-84-09-27666-0. ISSN 2340-1079.
- [6] Végh L, Takáč O. ONLINE GAMES TO INTRODUCING COMPUTER PROGRAMMING TO CHILDREN. In: INTED2021 Proceedings. 15th International Technology, Education and Development Conference. IATED; 8-9 March, 2021 2021:10007-10015. doi:10.21125/inted.2021.2091
- [7] Czakóová, K. DEVELOPING ALGORITHMIC THINKING BY EDUCATIONAL COMPUTER GAMES. In. Proceedings of the 16th International Scientific Conference: “eLearning and Software for Education : eLearning sustainment for never-ending learning. Volume 1, DOI: 10.12753/2066-026X-20-003, 2020/1, p. 26-33. Bucharest : “CAROL I” National Defence University Editura, Universitara, 2020. ISSN 2066-026X, ISSN-L 2066-026X, ISSN CD 2343 – 7669. Scopus
- [8] Gubo S, Kmet T, Molnar A, Takac O. A MULTI-RANGE APPROACH FOR CULTURAL HERITAGE SURVEY: A CASE STUDY OF A MEDIEVAL CHURCH IN SLOVAKIA. In: 2020 IEEE 18th World Symposium on Applied Machine Intelligence and Informatics (SAMI). ; 2020:000117-000122. doi:10.1109/SAMI48414.2020.9108724
- [9] Štempeľová I, Takáč O. IMPLEMENTATION OF ELEMENTS OF INTELLIGENT AGRICULTURE IN INFORMATICS LESSONS WITH REGARDS TO THE SUPPORT OF INTERSUBJECT RELATIONSHIPS. In: INTED2023 Proceedings. 17th International Technology, Education and Development Conference. IATED; 6-8 March, 2023 2023:5968-5973. doi:10.21125/inted.2023.1571
- [10] Tumor B, Takáč O, Molnár A, Gubo Š, Várkonyi-Kóczy AR. SHAPE RECOGNITION IN DRONE IMAGES USING SIMPLIFIED FUZZY INDEXING TABLES. In: 2020 IEEE 18th World Symposium on Applied Machine Intelligence and Informatics (SAMI).; 2020:129-134. doi:10.1109/SAMI48414.2020.9108735
- [11] Cóká, M., Czakóová, K. INNOVATIONS IN EDUCATION THROUGH THE APPLICATION OF RASPBERRY PI DEVICES AND MODERN TEACHING STRATEGIES. In. INTED 2021 : Proceedings of the 15th International Technology, Education and Development Conference. DOI: 10.21125/inted.2021.1327, p. 6653-6658, Valencia : IATED Academy, 2021. ISBN 978-84-09-27666-0. ISSN 2340-1079.
- [12] Takáč O, Végh L. CREATION OF 3D MODELS OF REAL OBJECTS IN THE TEACHING OF COMPUTER SCIENCE. In: ICERI2021 Proceedings. 14th annual International Conference of Education, Research and Innovation. IATED; 8-9 November, 2021 2021:5723-5727. doi:10.21125/iceri.2021.1291

- [13] Czakóová, K., Udvaros, J. APPLICATIONS AND GAMES FOR THE DEVELOPMENT OF ALGORITHMIC THINKING IN FAVOR OF EXPERIENTIAL LEARNING. In: EDULEARN21 : Proceedings of the 13th International Conference on Education and New Learning Technologies. DOI: 10.21125/edulearn.2021.1389, p. 6873-6879, Valencia : IATED Academy, 2021. ISBN 978-84-09-31267-2. ISSN 2340-1117. Formal Methods in Informatics. <https://doi.org/10.33039/ami.2023.04.001>
- [14] Takáč O, Végh L. POSSIBILITIES OF USING PHOTOGRAMMETRY IN THE TEACHING PROCESS. In: EDULEARN21 Proceedings. 13th International Conference on Education and New Learning Technologies. IATED; 5-6 July, 2021 2021:9237-9242. doi:10.21125/edulearn.2021.1860
- [15] Bódi S. (2021). A GYERMEKEK JOGAI, In: Alapjogok, Az emberi jogok alkotmányos védelme Magyarországon, szerk. Bódi S. Schweitzer G., Budapest, Ludovika kiadó, 451-466. ISBN 978-963-531-399-0.
- [16] Végh L, Takáč O. MOBILE CODING GAMES TO LEARN THE BASICS OF COMPUTER PROGRAMMING. In: EDULEARN21 Proceedings. 13th International Conference on Education and New Learning Technologies. IATED; 5-6 July, 2021 2021:7791-7799. doi:10.21125/edulearn.2021.1590
- [17] O. Takáč, D. Hrubý, V. Cviklovič. POSSIBILITIES OF NAVIGATION OF MOBILE AGRICULTURAL ROBOTS ON THE PRINCIPLE OF THE GEOMETRICAL OBJECTS DETECTIONS. 1, Bucharest : UNIV Agricultural Sciences & Veterinary Medicine Bucharest, 2011. p. 206-208. ISSN: 2284-7995.
- [18] J. Udvaros, N. Forman, L. Szabó, K. Szabó (2022) THE IMPORTANCE OF TEACHING DRONES IN LOGISTICS, ICERI2022 Proceedings, pp. 3286-3290. doi: 10.21125/iceri.2022.0811
- [19] J. Udvaros and K. Czakóová. (2021) USING TEACHING METHODS BASED ON VISUALIZING BY TINKERCAD IN TEACHING PROGRAMMING, ICERI2021 Proceedings, pp. 5913-5917. <https://doi.org/10.21125/iceri.2021.1333>
- [20] J. Udvaros and M. Gubán. (2016). DEMONSTRATION THE CLASS, OBJECTS AND INHERITANCE CONCEPTS BY SOFTWARE. Acta Didactica Napocensia, 9 (1). pp. 23-34. ISSN 2065-1430
- [21] M. T. Fülöp, J. Udvaros, Á. Gubán, and Á. SÁNDOR. (2022) DEVELOPMENT OF COMPUTATIONAL THINKING USING MICROCONTROLLERS INTEGRATED INTO OOP (OBJECT-ORIENTED PROGRAMMING). Sustainability, 14 12, 7218, <https://doi.org/10.3390/su14127218>.
- [22] J. Udvaros, Á. Gubán and M. Gubán. (2019) METHODS OF ARTIFICIAL INTELLIGENCE IN ECONOMICAL AND LOGISTICAL EDUCATION. eLearning and Software for Education Conference, pp. 414-421. <http://dx.doi.org/10.12753/2066-026x-19-055>
- [23] J. Udvaros, N. Forman, D.É. Dobák. (2023) APPLICATION AND IMPACT OF ELECTRONIC SOLUTIONS IN TEACHING PROGRAMMING. Annales Mathematicae et Informaticae, Special issue on